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Estimation of Hovering Flight Time of Battery-Powered Multicopters

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Abstract

The estimation of hovering flight time of multicopters using the battery power propulsion system is important for the development and design of the aircraft and its operation. For a given operational weight, the maximum possible battery weight can be decided using both a conventional energy density method and a new Peukert law. In the present study, the hovering flight time is predicted using both methods. The specific data of multicopters in the published literatures were employed for the computation of the hovering flight time. The results were validated with the measured data. The effect of figure of merit of propeller, battery discharging process on the hovering flight time was evaluated, Finally, the effect of the battery cell and package connection types on the hovering time was investigated. It was found that the combination of serial battery cell connections and parallel package connection is the bast in the endurance maximization aspect. As the cell number increases in a package, the hovering flight time is increased. There exists the max. battery ratio for the given takeoff gross weight.

Key Words : Endurance, Multicopter, Peukert Law, Aircraft Performance

1. Introduction

Multicopters can fly autonomously with its hovering capability and high maneuverability. Because of the attractive easy operational characteristics, multicopters are quite popular in civilian applications such as aerial photography, plant protection, and package delivery[1].

One of drawbacks for commercial applications of the multicopters is the limited endurance because most of the multicopters use batteries as the power source for propulsion. The energy density of battery is still low compared to its fossil-fuel counterpart. There have been continuous efforts to increase the endurance by developing batteries with higher energy density with safe operation.

When developing the multicopters, endurance are given as one of those important design requirements. During the initial conceptual design process, a selection of the optimal battery weight requires a simple estimation for the given payload and empty weight. The conventional estimation of endurance for fuel-powered aircraft is well established with the Breguet equations[2]. It assumes that the weight of the aircraft decreases by the weight of the burned fuel which means the reduction in the required power for sustaining the flight. In the electric propulsion using the batteries, the battery weight does not change during the mission. Thus, a new approach for the estimation of the endurance is required.

From the literature survey, it was found that the new approach for the estimation of endurance was mainly initiated in the process of developing micro air vehicles(MAVs) and unmanned aerial

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vehicles(UAVs)[3,4]. The MAVs and UAVs with fixed and rotary wings have different dynamics with each other. However, these aircraft has the common feature that they use the electric propulsion composed of a motor, a motor driver, and a propeller(or rotors). MAVs have small-sized electric components because their wing sizes are small and the weight is limited by their small wing sizes. Endurance is typically estimated by equating an available energy with the required power consumption. The power efficiencies can be calculated considering the losses in the subsystems with the motor model, the motor driver model (or ESC model), the performance data of the propeller(or rotor).

Traub[5] applied the Peukert's effect on the battery discharge process to the theoretical derivation of endurance of battery-powered aircraft. Peukert[6] performed tests on lead-acid batteries and discharging them with a constant current. For a one-ampere discharge rate, Peukert's law can be expressed as $C = I^{pk} t$ where C is the capacity at a one-ampere discharge rate(ampere hours), I is the actual discharge current(amperes), t is the actual time to discharge the battery(hours), and pk is the Peukert constant(dimensionless). The equation is developed to account for the intrinsic losses associated with discharging batteries at elevated currents. When discharging a battery at higher currents, the internal cell resistance increases. Peukert[6] introduced a discharge parameter(pk) for representing the losses which is also dependent on battery type, temperature, ages and cycles. In case of the lead-acid batteries, the value of pk for flooded batteries lies within the range 1.2–1.5. For gel batteries the range is from 1.1 to 1.25 and for AGM batteries from 1.05 to 1.15[7].

Basically, the Peukert effect represents the fact that higher the current draw, the less the effective battery capacity. Thus, higher values of discharge parameter indicate a smaller efficiency at higher currents. In turn, Peukert's effect states benefits at low currents with the battery capacity delivered higher than the nominal one.

Avanzini and Giulietti[8] theoretically extended the Traub's approach[5] to the prediction of best range condition for battery-powered fixed-wing aircraft under the hypothesis that Peukert's law is valid for the battery discharge process of constant current. Typically, the discharge parameter n is larger than 1 and, thus, the best range velocity decreases, lying between minimum drag and minimum required power airspeed.

Traub[9] performed an experimental investigation in order to validate the endurance estimates for a battery powered flight vehicle using a radio controlled model aircraft flown inside a wind tunnel. The endurance derived in his previous publication[5] is not correct because the voltage drops during the discharge process. The voltage drop requires the increase in the battery supply current, which results in the reduction of the effective battery capacity. It was proved that the modified theoretical estimates predicted the endurance within 3% of that measured experimentally.

When it comes to the case of multicopters in hovering flight mode, the constant voltage assumption is valid. Gatti et al.[10] established an analytical framework for predicting the hovering flight time of multicopters by imposing the balance between required and available power. It was assumed that battery discharge voltage is constant and rotor figure of merit do not change much. The closed-form analytical solutions for the best endurance battery capacity was found for two cases(1.figure of merit is constant, 2.the required power by the payload is small compared to the required power for the hovering condition). The approximate expression of the best endurance capacity was compared with an experimental flight data using a six-rotor platform. A total of twenty hovering flights are completed with identical weight/energy ratio along four different nominal capacity. It was found that the theoretical estimation of endurance matches with the measured data.

A more elaborated model for the battery discharging process was studied by changing the voltage and effective battery capacity with iterative process with time stepping. Cheng et al.[11] used the Kriging method to model the discharge characteristics of Li-Po batteries under standard conditions. A battery discharge model is implemented through a numerical integration. They showed that their model was more effective and accurate, on the prediction of the endurance, than the previously published methods. Hwang et al.[12] also used the elaborated discharge model with iterative process. The effective capacity and voltage drop are calculated to determine the battery discharge for a constant power discharge. They calculated the effect of the number of batteries on the endurance. They found that their model was accurate(average error of 2.3%) compared with the measured data.

A new concept of extending the endurance by dumping or diving the batteries has been suggested. Chang and Yu[13] firstly suggested the battery dumping strategy amd they found that 1) increasing the battery weight ratio doesn't guarantee improved endurance because of increased weight. Thus, there is an optimal battery weight ratio. Battery dumping of exhausted battery packs can extend the endurance with the battery weight ratio greater than a threshold value. Feng et al.[14] extended Chang and Yu's estimation by including Traub's method. They found that Peukert effect has an significant impact on the success of the battery dumping strategy(28% when n=1.0 and 3% when n=1.3). The benefits of battery dumping are greater for a large battery weight ratio with low Peukert constant.

Abdilla et al.[15] studied the extension of endurance of battery-powered rotorcraft by sub-dividing the monolithic on-board battery into multiple smaller capacity batteries which are sequentially discharged and released. They found that the discarding of consumed battery mass can reduce the propulsive power required. However, the introduction of the additional parasitic mass for battery switching and release mechanism can limit the endurance with a threshold payload.

As shown in the previous literature survey, the endurance of the multicopter(like the other aircraft with electric propulsion) is limited by the battery characteristics as the energy storage. The optimal battery weight fraction is one of crucial parameters for the design of the multicopters. During the operation, users can also want to change payload heavier than the values suggested by the developer, while maintaining the takeoff cross weight. It is possible by reducing the battery capacity but it costs endurance. In the present paper, the maximum endurance of multicopters are estimated using Peukert law and the usable energy method. The effect of battery pack configuration on the endurance is investigated.

2. Problem statement and Solution

For multicopters equipped with a payload for specific missions, the maximum take-off(total) weight will not change, and it can be splitted into three groups as follows[10].

$$Wto = W_0 + W_b = W_{co} + W_p + W_b \tag{1}$$

where W_{co} is the empty weight that includes a) the frame weight (structure and rigging), b) the driving system weight (motors, regulators, and propellers), and c) the avionics weight (autopilot and communication system), W_p is the payload weight, and W_b is the battery weight. W_0 is the zero-capacity weight that represents the take-off weight without the battery system.

The total battery power required for completing the mission flight should be calclulated by considering the required power for each segment of the mission profiles. In the present study, as an initial and conceptual phase, we only consider the hovering mission. By considering only the hovering flight, we can also assume that the battery power and the rotor figure of merit do not change much. The battery power required for hovering can be splitted into a required power for hover without payload and electronic equipments, and a required power with payload and electronic equipments.

$$P_b = P_h + P_{ap} \tag{2}$$

The required hovering power can be derived using a momentum theory as follows[16].

$$P_h = \frac{T^{3/2}}{f\sqrt{2\rho A_t}} = \frac{Wto^{3/2}}{f\sqrt{2\rho A_t}} \tag{3}$$

where f is the figure of merit of the rotor, ρ is air density, A_t is the total disc area of the rotors. After naming $\sqrt{2\rho A_t} = \lambda$ for convenience[10], and by imposing the balance between the required power for hovering and the available power from the battery, the current draw from the battery can be derived as follows[10].

$$I = \frac{P_b}{V} = \frac{1}{V}\left(\frac{Wto^{3/2}}{\lambda f} + P_{ap}\right) \tag{4}$$

where V is the battery voltage. As shown in the literature surve, the battery voltage can change as the current or power change during the discharging process. We follow the assumption that the battery discharge voltage is constant or at most linear, and rotor figure of merit do not change much(the previous study of Gatti et al.[10], they proved the validity of the assumption by comparing the theoretical result with the measured data from experimental flight). From the definition of the discharge ratio, $I = dC/dt$, the endurance can be obtained by integrating the change of the capacitance[10].

$$\frac{dt}{dC} = \frac{1}{I} = \frac{V\lambda f}{Wto^{3/2} + P_{ap}\lambda f} \tag{5}$$

$$t = \int_0^C \frac{V\lambda f}{Wto^{3/2} + P_{ap}\lambda f}\, dC \tag{6}$$

The Peukert effect can be integrated into above equation for a nominal capacity C_0 as follows[5]

$$C = \eta C_0 \left(\frac{\eta C_0}{It_0}\right)^{pk-1} \tag{7}$$

where pk is the Peukert coefficient,t_0 is the rated discharge time, and η expresses the fraction of the nominal capacity in the linear part during the discharging process. When calculating the Eq. (6), it is assumed that the integrand do not change during hovering, thereby adopting the equivalent constant voltage of $V_e = (V_f + V_0)/2$. It considers the linear characteristics of the voltage change from initial to final voltages. Finally, the hovering flight time can be explicitly represented as follows[10].

$$t1 = \frac{V_e \lambda f}{Wto^{3/2} + P_{ap}\lambda f}\eta C_0 \left(\frac{\eta C_0}{It_0}\right)^{pk-1} \tag{8}$$

When calculating the hovering flight time in Eq.(8), the figure of merit, f, should be given. It was assumed that the rotor figure of merit is a slowly-varying power function of the rotor thrust[9].

$$f = f_0 \left(\frac{W_{t0}}{T_0 n} \right)^m \tag{9}$$

where T_0 is the single rotor thrust at the rated percent of the throttle, and n is the number of the rotors. f_0 and m are the model parameters that can be obtained from the experimental data of manufacturers and/or by means of an experimental characterisation[10].

Hovering flight time of an electric vehicle can be directly linked to the usable energy battery(E_b) and average battery power output(P_b).

$$t2 = E_b / P_b \tag{10}$$

The usable energy of a battery can be defined as

$$E_b = C_o v \eta_b f_{DOD} \tag{11}$$

where $\eta_b$is the battery efficiency that is proportional to temperature, output current, internal battery resistance. Its value is within the range from 60% to 75%. v is the nominal voltage between the leads, and f_{DOD} is the battery depth of discharge that represents the ratio between the real battery usable energy and the total storage energy. The battery depth of discharge is one of most important parameters for hovering flight time and a discharge of at least 80% DOD is taken as a deep discharge.

The usable energy of a battery can also defined as a function of the battery specific energy (E_{spec})[17]. It is a nominal battery energy per unit mass(gravimetric energy density).

$$E_b = E_{spec} m_b \eta_b f_{DOD}, \ E_{spec} = C_o v / m_b \tag{12}$$

The equation is useful in the early conceptual design stage along with one more important parameter, the energy density(the nominal battery energy per unit volume). It determines the battery size required to achieve a given electric range with the energy consumption of the vehicle.

3. Effect of Battery Cell Numbers

In the present study, the parameters for calculating the hovering flight time of multicopters are used with the same values in Ref. 6(See Table 1). The battery weight is W_b =1.8833× N_c which is corresponding to $W_b = \alpha V_s C_0$, $V_s = 3.7 \times N_c$. Single battery pack has four cells. The nominal voltage of one pack is thus 14.8 V. Each pack has 10[Ah] nominal capacity.

Gatti et al.[10] computed the figure of merit of the rotor from the measured data of thrust and power. They approximated the figure of merit using the eq. 9. The equation fits with the measured data within the range of 50 % to 75 % of the throttle. We used the same value of the figure of merit presented in Ref.[10]. Using the data in Table 1 and Fig.1, and equations 8~10, the hovering flight time of a multicopter with six rotors are calculated by changing the battery weight ratio. As shown in the Fig.2, it was found that present results agree well with the results of Gatti et al[10] which were validated using an experimental flight data with a six-rotor platform(Fig. 5 in Ref.[10]) and with a total of twenty hovering flights.

In the table 1(which is shown in the reference), the discharge fraction(η) is set to 0.71. However, we found that the value of the discharge coefficient is 1.0 and it was confirmed through private communication with Gatti[18].

It is also shown that the energy density method predicts slightly longer hovering flight time than the Peukert method. Theoretically the increase in battery weight has the benefit of extending hovering flight time up to W_b/W_o=2. After then, the rise of the hovering flight time is very slow with the battery weight fraction stepping up. It can be illustrated as follows. From the Eq. 8, it can be inferred that, though the increase in the battery weight adds more battery capacity, it also affect the required power as a power of 3/2. The required current draw is also increased(See Eq.4). Thus it can be said that when the battery weight is small, the hovering flight time is increased proportional to the battery weight. Howver, as the total weight is increased, the rise of hovering flight time becomes slow. When the battery weight is larger than the cut-off battery weight fraction(in the present study, W_b/W_o=2), then the hovering flight time does not change much. The results have the limitation due to the assumption of the constant power during the hovering flight. If there is no power-consuming payload on board and aircraft systems only consist of autopilot and communication system with negligible power consumption, required power for payload can be neglected. In the conceptual design process where the selection of the driving system without detailed information on rotor features, a

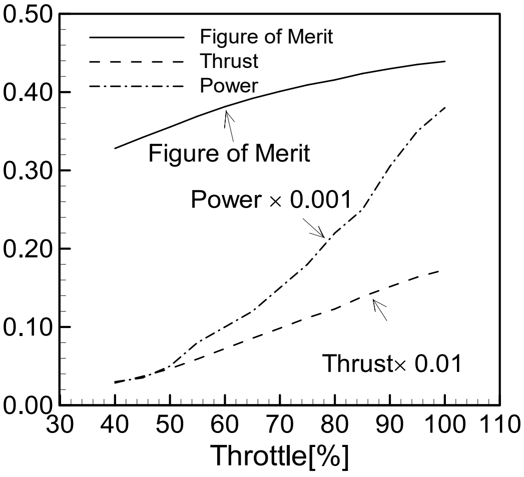


Fig. 1 Thrust, power and figure of merit reproduced from Ref.[10].

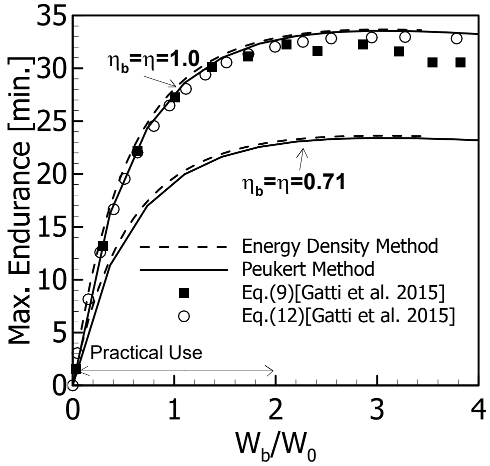


Fig. 2 Validation of the present method.

Table 1 Multi-rotor Platform Parameter[Ref.10]

Parameter	symbol	value
Number of rotors	n	6
Empty weight	W_{eo}	19.62 N
Payload weight	W_p	2.32 N
Avionics & payload power	P_{ap}	18 W
Propeller diameter	d_p	10 in
Nominal voltage	V_0	14.8 V
Nominal capacity	C_0	10-40 Ah
Rated discharge time	t_0	1 h
Weight/energy ratio	α	0.0509
Discharge fraction	η	0.71%
Peukert coefficient	pk	1.051

simple estimation is useful as a valuable starting guess during the aircraft sizing. If the rotor figure of merit does not change much within the operational range of motor throttle, the assumption that the value of m is zero is valid.

Figure 3 shows the effect of the figure of merit on the hovering flight time. In all computations, the rotor thrust at the reference percentage of throttle position is assumed to have the value of 7.3516 N. The proposed model of the figure of merit in Fig.1 was used. The solid lines shows the change of the hovering flight time obtained by using the figure of merit in Eq.9. The figure of merit f_O is the figure of merit at the reference throttle(For example 60%). The dashed lines with hollow symbols shows the change of hovering flight time when the figure of merit has constant value. As shown in the Ref.[10,17], the figure of merit is valid within 50%-70% range of the throttle setting. The mean value of figure of merit in the range of 65 to 100%, f_{65}, can be used for estimating the hovering time. We changed the values of f_{65} from 0.4068 to 0.6068. while maintaining the figure of merit as constant(f=f_{65}). As shown in the figure and in the Ref.[10], when m=0, the battery weight ratio for maximum endurance is $W_b/W_0=2$ [6]. With the fixed value of the figure of merit, the predicted maximum endurance increased faster than the case of changing figure of merit upto $W_b/W_0=2$. After then, the hovering flight time of the fixed figure of merit decreases where as the hovering flight time of the changing figure of merit increased continuously. It is shown in the figure that the hovering flight time using the fixed figure of merit over-estimates the hovering flight time when it is less than $W_b/W_0=2$, and under-estimates when it is larger than $W_b/W_0=2$. It can be also conjectured that, when the figure of merit does change, then the hovering flight time is increasing always as the battery weight fraction stepping up.

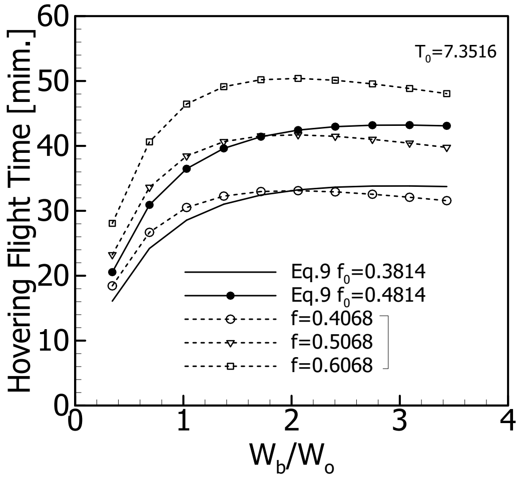


Fig. 3 Effect of the figure or merit for the hovering flight time

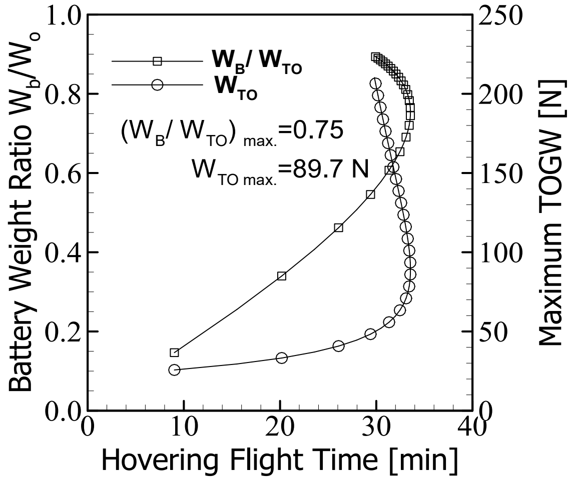
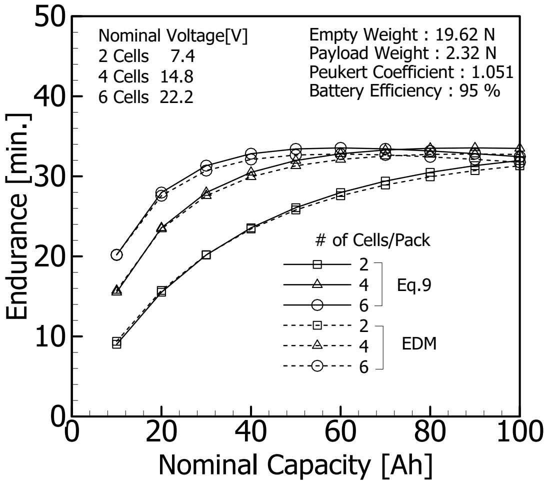


Fig. 4 Hovering flight time ($\eta=\eta_b=1.0$).

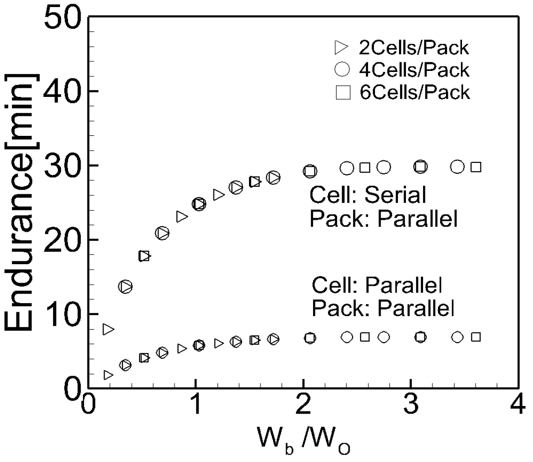
Figure 4 shows the max. endurance with the change of the battery weight and maximum take off gross weight. The optimal conditions for battery weight fraction to take-off gross weight for max. endurance are W_b/W_{T0}=0.75 and W_{T0} =89.7N.

When a pack is composed of N cells, the total battery weight can be written as W_b=1.8833$\times N_c \times N_P$, where N_P is the number of packs. The voltage of each pack(the cells are connected in serial inside a pack) is corresponding to $V_s = 3.7 \times N_c$. The nominal capacity of a pack in serial cell connection is the same as a capacity of a cell. If the pack is in parallel cell connection, the total capacity of a pack is the sum of all cells. This principle applies to the pack for serial/parallel connections. Ref. 6 studied the effect of the nominal capacity on the hovering flight time with the battery packs connected in parallel. Figure 5 shows the effect of connection types between cells and packs on the hovering flight time. It is shown in Fig. 5(a) that the estimated hovering flight time using the Peukert equation has almost similar value to the estimated value using the energy density method(Each pack has cells in serial. The packs are connected in parallel). Neglecting the structural weight for package, the packs with larger numbers of cells(connected in series) have higher value of hovering flight time. It means that the voltage of a pack with cells connected in serial increases with the number of cells stepping up, wile maintaining the nominal capacity. Thus, with the same capacity, the lower current draw due to the higher voltage has an effect of increasing the hovering time. As the nominal capacity increases, the difference in the hovering time for the number of cells are converged. This phenomenon can be illustrated as follows. The total battery weight is proportional to the number of total cells. Thus, though the nominal capacity is the same, the battery weight ratio of 6 cells in a pack is larger than the batter weight ratio of 2 or 4 cells in a pack. The increased total weight requires more power.

Figure 5(b) shows the effect of connection types in the pack on the hovering flight time. The packs are connected in parallel. The hovering flight time of the battery for different number of cells in a pack was on the same curve depending on the connection types of cells in a pack. The x-axis is the battery weight ratio. For the same battery weight, the connection type decides the total nominal capacity. It is shown that, at the same battery weight, a pack with its cells connected in series has longer hovering time than the pack with its cells in parallel. It can be explained as follows. The pack with its cells connected in serial has higher voltage and less nominal capacity than the pack with its cells connected in parallel. The so-called Peukert effect can increase the range and hovering flight time of a vehicle if the battery capacity is large with respect to the current required. Conversely,



(a) nominal capacity vs. hovering flight time($\eta = 1.0$)



(b) battery weight ratio vs. hovering flight time($\eta = 1.0$)

Fig. 5 Comparison of hovering flight time with parallel connections between the packs

the effective capacity is reduced if the current draw is close to the batteries' nominal capacity. For a constrained geometry and a fixed battery weight, as a fraction of the total aircraft weight, increasing battery capacity reduces performance due to greater required power and, consequently, current draw (which outweighs the capacity increase).

4. Conclusions

In the present study, the hovering flight time of multicopters is predicted using Peukert law and an energy density method. The specific data of multicopters in the published literatures were employed for the computation of the hovering flight time. The results were validated with the measured data. The effect of figure of merit of propeller, battery discharging process on the hovering flight time was evaluated, Finally, the effect of the battery cell and package connection types on the hovering time was investigated. It was found that the combination of serial battery cell connections and parallel package connection is the best in the endurance maximization aspect. As the cell number increases in a package, the hovering flight time is increased. There exists the max. battery ratio for the given takeoff gross weight.

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References

[1] J. Gundlach, "Designing unmanned aircraft systems: a comprehensive approach," American Institute of Aeronautics and Astronautics, Reston, VA, 2012.

[2] J. D., Anderson, Aircraft Performance, McGraw-Hill, New York, 1999, Chap. 5.

[3] D. A. Lawrence and K. Mohseni, "Efficiency analysis for long-duration electric MAVs," AIAA 2005-7090.

[4] G. Ohad and R. Aviv, "Optimizing electric propulsion systems for unmanned aerial vehicles," *AIAA Journal of Aircraft*, Vol. 46, No. 4, pp.1340-1353, July-August 2009.

[5] L. W. Traub, "Range and endurance estimates for battery-powered aircraft," *Journal of Aircraft*, Vol. 48, No.2, 2011.

[6] W. Peukert, "Über die Abhängigkeit der Kapazität von der Entladestromstärke bei Bleiakkumulatoren," *Elektrotechnische Zeitschrift*, Vol. 20, pp. 20–21, 1897.

[7] J. Garche, P.T. Moseley, E. Karten, "Lead-acid batteries for hybrid electric vehicles and battery electric vehicles," Advances in Battery Technologies for Electric Vehicles Edited by B. Scrosati, J. Garche, and W. Tillmetz, Woodhead Publishing, 2015.

[8] G. Avanzini and F. Giulietti, "Maximum range for battery-powered aircraft," *Journal of Aircraft*, Vol. 50, No.1, 2013.

[9] L. W. Traub, " Validation of endurance estimates for battery powered UAVs," *Aeronautical Journal*, Vol. 117, No. 1197, November 2013.

[10] M., Gatti, F., Giulietti, and M. Truci, "Maximum endurance for battery powered rotary-wing aircraft." *Aerospace Science Technology*, Vol.45, pp. 174-179, 2015.

[11] F. Cheng, H. Wang, P.Cui, "Rotorcraft flight endurance estimation based on a new battery

discharge model," *Chinese Journal of Aeronautics*, Vol. 30, No.4, pp.1561-1569, 2017.

[12] MH, Hwang, HR Cha, and SY Jung, " Practical endurance estimation for minimizing energy consumption of multirotor unmanned aerial vehicles," *Energies*, Vol. 11, No.9, 2221, 2018.

[13] T. Chang and H. Yu, "Improving electric powered UAVs' endurance by incorporating battery dumping concept," 2014 Asia-Pacific International Symposium, *Procedia Engineering*, Vol.99, pp.168–179,2015.

[14] X. Feng, Y. Sun, M. Chang, and J. Bai, "Endurance improvement by battery dumping strategy considering Peukert effect for electric-powered disposable UAVs," *Aeronautical Journal*, Vol. 125, No. 1283, pp.42-59, January 2021.

[15] A. Abdilla, A. Richards, and S. Burrow, "Endurance optimization of battery-powered rotorcraft," 16th Annual Conference, TAROS 2015, Liverpool, UK, September 8-10, pp.1-13, 2015.

[16] J. Gordon Leishman, Principles of Helicopter Aerodynamics, 2nd ed., Cambridge Aerospace Series, Cambridge University Press, Cambridge, New York, 2000.

[17] M. Gatti, Design and Prototyping High Endurance Multi-Rotor, Ph.D. Thesis, Alma Mater Studiorum Università di Bologna. Dottorato di ricerca in Meccanica e scienze avanzate dell'ingegneria, 27 Ciclo, 2015.

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