

Numerical Analysis of Sufficient Condition on Larger Rate Volume of CIS/non-SIC over IIS/SIC in 3-User NOMA

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삼중 사용자 비직교 다중 접속에서 IIS/SIC에 대한 CIS/non-SIC의 확대 전송률 용적의 충분조건의 수치 해석

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Abstract Since a sufficient condition on the larger rate volume of 3-user correlated information sources (CIS)/non-successive interference cancellation (SIC) non-orthogonal multiple access (NOMA) over independent information sources (IIS)/SIC NOMA has not been investigated, this paper analyzes such a sufficient condition. First, we demonstrate that the rate volume of 3-user CIS/SIC NOMA is the same as a portion of the rate volume of 3-user IIS/SIC NOMA. Then, by identifying a dominant rate region, we calculate the sufficient condition on the larger rate volume of 3-user CIS/non-SIC NOMA over 3-user IIS/SIC NOMA. We also show that with such condition, the rate volume of 3-user CIS/non-SIC NOMA can be larger than that of 3-user IIS/SIC NOMA.

Key Words : 5G, NOMA, Successive interference cancellation, Correlation coefficient, Power allocation

요약 삼중 사용자 CIS/non-SIC 비직교 다중 접속의 IIS/SIC 비직교 다중 접속에 대한 확대된 전송률 용적의 충분조건에 대한 연구가 미비하여, 본 논문은 그러한 충분조건을 분석한다. 먼저, 삼중 상관 정보원/SIC 비직교 다중 접속의 전송률 용적이 삼중 독립 정보원/SIC 비직교 다중 접속의 전송률 용적의 일부와 일치하는 것을 입증한다. 다음, 우세 전송률 영역을 확인함으로써, 삼중 상관 정보원/non-SIC 비직교 다중 접속이 삼중 독립 정보원/SIC 비직교 다중 접속에 대한 확대된 전송률 용적의 충분조건을 계산한다. 또한, 분석된 조건을 기반으로, 삼중 상관 정보원/non-SIC 비직교 다중 접속의 전송률 용적이 삼중 독립 정보원/SIC 비직교 다중 접속의 전송률 용적보다 확대될 수 있음을 보여준다.

주제어 : 5G, 비직교 다중 접속, 연속 간섭 제거, 상관관계 계수, 전력 할당

1. Introduction

Non-orthogonal multiple access (NOMA) in the fifth-generation (5G) mobile network is considered as a promising technology, owing to its large spectral efficiency and lower latency[1,2]. NOMA allows superposition coding

(SC) and successive interference cancellation (SIC)[3,4]. Also NOMA can increase spectral efficiency by sharing channel resources[5]. In addition, the bit-error rate (BER) of NOMA networks was derived in [6]. Local oscillator imperfection was investigated [7]. The BER with randomly generated signals was studied [8]. The

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exact BER expression was investigated for the two or three-user cases[9]. The average symbol error rate (SER) expression was derived in [10]. The importance of SIC of NOMA was considered [11]. The secure NOMA-enabled mobile network was investigated [12]. In [13], the physical layer security was also considered. The intelligent reflecting surface (IRS) NOMA was presented in [14]. The mutual-aid NOMA scheme was studied [15]. The higher order modulation schemes in visible light communication (VLC) systems were investigated [16]. Various receivers' structures for NOMA were investigated [17]. Impacts of channel estimation errors were studied [18]. Asymmetric binary pulse amplitude modulation NOMA was proposed in [19]. Total power for achieving 1+1 capacity region was calculated in NOMA[20]. Correlated information sources (CIS) have been studied for NOMA[21]. Correlated SC was proposed for lossless NOMA implementation without SIC[22].

In this paper, we calculate a sufficient condition on the larger rate volume of 3-user CIS/non-SIC NOMA with respect to 3-user independent information sources(IIS)/SIC NOMA, especially the values of correlation coefficients of CIS, based on the achievable data rates in [21]. First, we demonstrate numerically that the rate volume of 3-user CIS/SIC NOMA is the same as a portion of the rate volume of 3-user IIS/SIC NOMA. Then, by identifying a dominant rate region, we calculate the sufficient condition on the larger rate volume of 3-user CIS/non-SIC NOMA over 3-user IIS/SIC NOMA. We also show that with such condition, the rate volume of 3-user CIS/non-SIC NOMA can be larger than that of 3-user IIS/SIC NOMA.

The remainder of this paper is organized as follows. In Section 2, the system and channel

model are described. The achievable rate volumes for 3-user NOMA are presented in Section 3. A sufficient condition on larger rate volume of 3-user CIS/non-SIC NOMA over 3-user IIS/SIC NOMA is calculated in Section 4. The numerical results are presented and discussed in Section 5. Finally, the conclusions are presented in Section 6.

2. System and Channel Model

In a cellular downlink NOMA network, three users are assumed to experience block fading. A base station and three users are within a cell. The complex channel coefficient between the m th user and base station is denoted by h_m , $m=1,2,3$. The channels are sorted as $|h_1| \geq |h_2| \geq |h_3|$. The base station sends the superimposed signal $x = \sum_{m=1}^3 \sqrt{\beta_m P_A} c_m$, where c_m is the message for the m th user with average unit power, $E[|c_m|^2] = 1$, where $E[u]$ represents the expectation of the random variable(RV) u , β_m is the power allocation coefficient for CIS (we use α_m for IIS), with $\sum_{m=1}^3 \beta_m = 1$, and P_A is the average total allocated power. The correlation coefficient between the i th and j th users is denoted by $\rho_{i,j} = E[c_i c_j^*]$. For a average total transmitted power P at the base station, P_A is scaled as the base station is denoted by

$$P_A = \frac{P}{\sum_{i=1}^3 \sum_{j=1}^3 \rho_{i,j} \sqrt{\beta_i} \sqrt{\beta_j}}. \quad (1)$$

The observation at the m th user is given by

$$y_m = h_m x + n_m, \quad (2)$$

where $n_m \sim N(0, N_0/2)$ is additive white Gaussian

noise(AWGN) at the m th user. Note that purely-real complex correlation coefficients (i.e., $\rho_{i,j} = \text{Re}[\rho_{i,j}]$) are assumed in the rest of this paper.

3. Achievable Rate Volume for 3-User NOMA In this section, we summarize the rate volumes for three different 3-user NOMA schemes.

3.1 3-User IIS/SIC NOMA

In this subsection, the achievable rate volume of IIS NOMA is given by [1]

$$R_1^{(IIS/SIC)} = \log_2 \left(1 + \frac{|h_1|^2 P \alpha_1}{\sigma^2} \right), \quad (3)$$

$$R_2^{(IIS/SIC)} = \log_2 \left(1 + \frac{|h_2|^2 P \alpha_2}{|h_2|^2 P \alpha_1 + \sigma^2} \right), \quad (4)$$

and

$$R_3^{(IIS/SIC)} = \log_2 \left(1 + \frac{|h_3|^2 P \alpha_3}{|h_3|^2 P \alpha_1 + |h_3|^2 P \alpha_2 + \sigma^2} \right). \quad (5)$$

3.2 3-User CIS/SIC NOMA

In this subsection, the achievable rate volume of CIS/SIC NOMA is expressed as [21]

$$R_1^{(CIS/SIC)} = \log_2 \left(1 + \frac{|h_1|^2 P_A \beta_1 (1 - \rho_{1,2,3}^2)}{\sigma^2} \right), \quad (6)$$

$$R_2^{(CIS/SIC)} = \log_2 \left(\frac{|h_2|^2 P_A \left(\beta_1 (1 - \rho_{1,3}^2) + \beta_2 (1 - \rho_{2,3}^2) \right) + \sigma^2}{|h_2|^2 P_A \beta_1 (1 - \rho_{1,2,3}^2) + \sigma^2} \right), \quad (7)$$

and

$$R_3^{(CIS/SIC)} = \log_2 \left(\frac{|h_3|^2 P + \sigma^2}{|h_3|^2 P_A \left(\beta_1 (1 - \rho_{1,3}^2) + \beta_2 (1 - \rho_{2,3}^2) \right) + 2\sqrt{\beta_1} \sqrt{\beta_2} (\rho_{12} - \rho_{13})} + \sigma^2 \right),$$

(8)

where the conditional correlation coefficient $\rho_{1,2,3}$ is defined by

$$\rho_{1,2,3}^2 \equiv \frac{(\rho_{12} - \rho_{13})^2 + 2\rho_{12}\rho_{13}(1 - \rho_{23})}{1 - \rho_{23}^2}. \quad (9)$$

3.3 3-User CIS/non-SIC NOMA

In this subsection, the achievable rate volume of CIS/non-SIC NOMA is expressed as [21]

$$R_1^{(CIS/non-SIC)} = \log_2 \left(\frac{|h_1|^2 P + \sigma^2}{|h_1|^2 P_A \left(\beta_2 (1 - \rho_{2,1}^2) + \beta_3 (1 - \rho_{3,1}^2) \right) + 2\sqrt{\beta_2} \sqrt{\beta_3} (\rho_{2,3} - \rho_{2,1}\rho_{1,3})} + \sigma^2 \right), \quad (10)$$

$$R_2^{(CIS/non-SIC)} = \log_2 \left(\frac{|h_2|^2 P + \sigma^2}{|h_2|^2 P_A \left(\beta_1 (1 - \rho_{1,2}^2) + \beta_3 (1 - \rho_{3,2}^2) \right) + 2\sqrt{\beta_1} \sqrt{\beta_3} (\rho_{1,3} - \rho_{1,2}\rho_{2,3})} + \sigma^2 \right), \quad (11)$$

and

$$R_3^{(CIS/non-SIC)} = \log_2 \left(\frac{|h_3|^2 P + \sigma^2}{|h_3|^2 P_A \left(\beta_1 (1 - \rho_{1,3}^2) + \beta_2 (1 - \rho_{2,3}^2) \right) + 2\sqrt{\beta_1} \sqrt{\beta_2} (\rho_{1,2} - \rho_{1,3}\rho_{2,3})} + \sigma^2 \right). \quad (12)$$

4. Calculation for Sufficient Condition on Larger Rate Volume of 3-User CIS/non-SIC NOMA over 3-User IIS/SIC NOMA

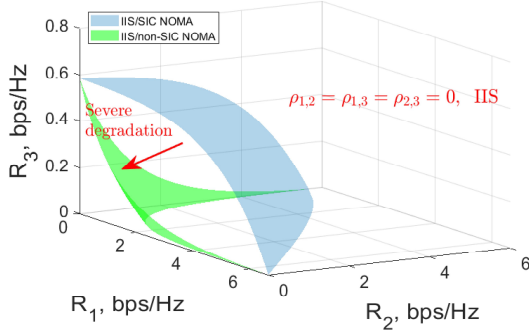


Fig. 1. Comparison of rate volumes of 3-user IIS/non-SIC NOMA and conventional 3-user IIS/SIC NOMA

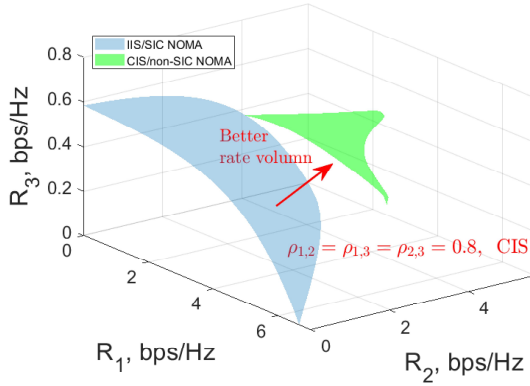


Fig. 2. Comparison of rate volumes of 3-user CIS/non-SIC NOMA and conventional 3-user IIS/SIC NOMA

First, we note that the rate volume of 3-user IIS/non-SIC NOMA is generally smaller than that of 3-user IIS/SIC NOMA, as shown in Fig. 1. It should be noted that $\rho_{1,2} = \rho_{1,3} = \rho_{2,3} = 0.0$ corresponds to IIS, i.e., all information sources are independent. However, for large correlations, it is observed in Fig. 2 that the rate volume of 3-user CIS/non-SIC NOMA can be larger than that of 3-user IIS/SIC NOMA. (Note that for Fig. 2, we choose sufficiently large $\rho_{1,2} = \rho_{1,3} = \rho_{2,3} = 0.8$.) Thus, we want to calculate a sufficient condition on larger rate volume of 3-user CIS/non-SIC NOMA over 3-user IIS/SIC NOMA. The problem can be expressed as

$$R_1^{(CIS/non-SIC)} \geq R_1^{(IIS/SIC)}, \quad (13)$$

$$R_2^{(CIS/non-SIC)} \geq R_2^{(IIS/SIC)}, \quad (14)$$

and

$$R_3^{(CIS/non-SIC)} \geq R_3^{(IIS/SIC)}. \quad (15)$$

However, the closed-form expression for such condition looks intractable. Thus, we need an intermediate step for this condition, as follows:

$$R_1^{(CIS/SIC)} = R_1^{(IIS/SIC)}, \quad (16)$$

$$R_2^{(CIS/SIC)} = R_2^{(IIS/SIC)}, \quad (17)$$

and

$$R_3^{(CIS/SIC)} = R_3^{(IIS/SIC)}. \quad (18)$$

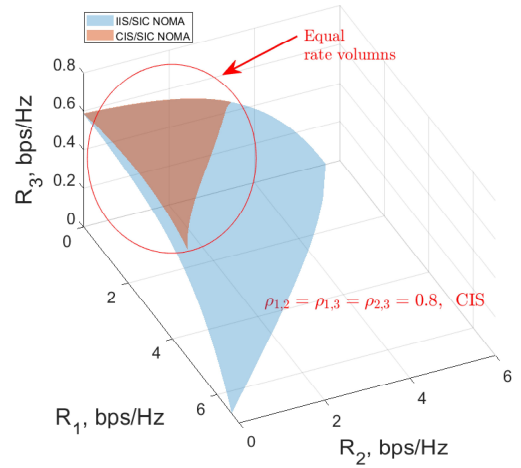


Fig. 3. Comparison of rate volumes of 3-user CIS/SIC NOMA and conventional 3-user IIS/SIC NOMA

Note that for a portion of power allocation ranges, the rate volume of 3-user CIS/SIC NOMA is equal to that of 3-user IIS/non-SIC NOMA, as shown in Fig. 3. (Note that for Fig. 3, we use the same values of $\rho_{1,2} = \rho_{1,3} = \rho_{2,3} = 0.8$ as

those in Fig. 2.) Although with the afore-mentioned intermediate step, for 2-user NOMA case, the closed-form expression has been derived, the condition for 3-user case looks still intractable. Thus, we try to analyze the condition numerically, instead of the analytical expression. With the intermediate step, we try to have

$$R_1^{(CIS/non-SIC)} \geq R_1^{(CIS/SIC)}, \quad (19)$$

$$R_2^{(CIS/non-SIC)} \geq R_2^{(CIS/SIC)}, \quad (20)$$

and

$$R_3^{(CIS/non-SIC)} \geq R_3^{(CIS/SIC)}. \quad (21)$$

Then we obtain the target in equation (13), (14), and (15).

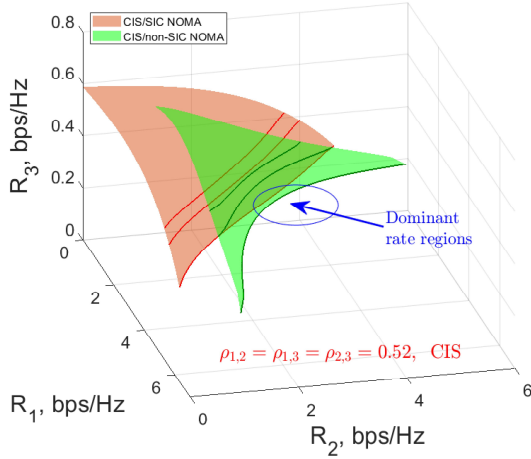


Fig. 4. Comparison of rate volumes of 3-user CIS/SIC NOMA and 3-user CIS/non-SIC NOMA

5. Numerical Results and Discussions

In this section, we present a numerical calculation for the approach in the previous section. For this calculation, we assume that the channel gains are $|h_1| = \sqrt{2}$, $|h_2| = 1$, and $|h_3| = 0.1$. In addition, the average total transmitted signal-to-noise power ratio (SNR) is $P/\sigma^2 = 50$.

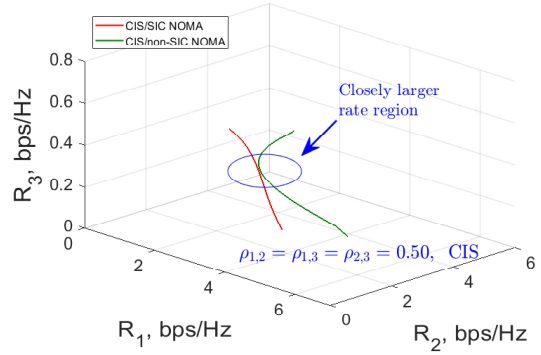


Fig. 5. Comparison of dominant rate regions of 3-user CIS/SIC NOMA and 3-user CIS/non-SIC NOMA

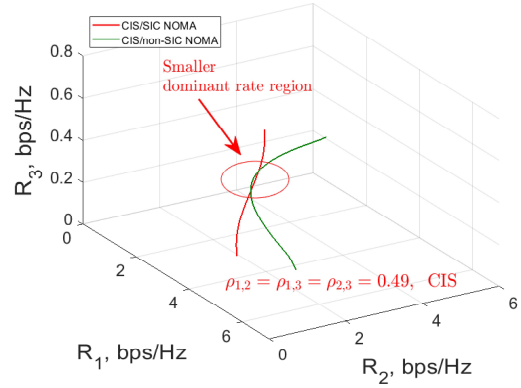


Fig. 6. Comparison of dominant rate regions of 3-user CIS/SIC NOMA and 3-user CIS/non-SIC NOMA.

First, in order to achieve the the target in equation (13), (14), and (15), we need to have the larger rate volume of 3-user CIS/non-SIC NOMA than that of 3-user CIS/SIC NOMA, especially for the entire power allocation ranges. For this, we have to identify the dominant rate region in the corresponding rate volume, as shown in Fig. 4. (Remark that we choose the values of $\rho_{1,2} = \rho_{1,3} = \rho_{2,3} = 0.52$ to highlight dominant rate regions.) In addition, for large correlation information, the non-SIC NOMA scheme is recommended, while for small correlation information, the SIC NOMA scheme is preferred.

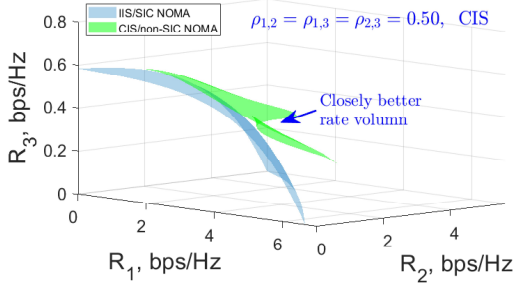


Fig. 7. Comparison of rate volumes of 3-user CIS/non-SIC NOMA and conventional 3-user IIS/SIC NOMA.

Now, we try to obtain a sufficient condition as close as possible, i.e., $\rho_{1,2} = \rho_{1,3} = \rho_{2,3} = 0.50$, as shown in Fig. 5. However, when $\rho_{1,2} = \rho_{1,3} = \rho_{2,3} = 0.49$, we observe the smaller dominant rate region of 3-user CIS/non-SIC NOMA than that of 3-user CIS/SIC NOMA, as shown in Fig. 6. Therefore, we have the tight sufficient condition $\rho_{1,2} = \rho_{1,3} = \rho_{2,3} = 0.50$. With such condition, we depict the rate volumes of 3-user IIS/SIC NOMA and 3-user CIS/non-SIC NOMA, as shown in Fig. 7. Note that when $\rho_{1,2} = \rho_{1,3} = \rho_{2,3} \geq 0.50$, we have the larger rate volume of 3-user CIS/non-SIC NOMA than that of 3-user IIS/SIC NOMA. One comment on our experiments is that when the sufficient condition is satisfied, CIS/non-SIC NOMA is more efficient than IIS/SIC NOMA for transmitting information, especially without SIC, which results in reduced receiver's complexity and low latency.

It is worth mentioning that although a tractable closed-form equation is currently difficult to obtain, the extension of the results to cases with $M \geq 4$ users could be achieved by heavy algebraic manipulation.

6. Conclusion

This paper calculated a sufficient condition

on the larger achievable rate volume of 3-user CIS/non-SIC NOMA over IIS/SIC NOMA. First, we demonstrated that the rate volume of 3-user CIS/SIC NOMA is the same as a part of the rate volume of 3-user IIS/SIC NOMA. Then, based on a dominant rate region, we calculated the sufficient condition on the larger rate volume of 3-user CIS/non-SIC NOMA over 3-user IIS/SIC NOMA. We also showed that with this condition, the rate volume of 3-user CIS/non-SIC NOMA can be larger than that of 3-user IIS/SIC NOMA.

As a direction of future researches, it would be significant to investigate the achievable rates for CIS NOMA with the number of users more than three.

As a result, CIS could be more efficient than IIS in transmitting information of NOMA systems in 5G mobile communications, especially without SIC complexity.

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