Original Article

MCNP-polimi simulation for the compressed-sensing based reconstruction in a coded-aperture imaging CAI extended to partially-coded field-of-view

Manhee Jeong a, Geehyun Kim b, *

a Department of Nuclear and Energy Engineering, Jeju National University, Jeju, 63243, Republic of Korea
b Department of Nuclear Engineering, Seoul National University, Seoul, 08826, Republic of Korea

ABSTRACT

This paper deals with accurate image reconstruction of gamma camera using a coded-aperture mask based on pixel-type CsI(Tl) scintillator coupled with silicon photomultipliers (SiPMs) array. Coded-aperture imaging (CAI) system typically has a smaller effective viewing angle than Compton camera. Thus, if the position of the gamma source to be searched is out of the fully-coded field-of-view (FCFOV) region of the CAI system, artifacts can be generated when the image is reconstructed by using the conventional cross-correlation (CC) method. In this work, we propose an effective method for more accurate reconstruction in CAI considering the source distribution of partially-coded field-of-view (PCFOV) in the reconstruction in attempt to overcome this drawback. We employed an iterative algorithm based on compressed-sensing (CS) and compared the reconstruction quality with that of the CC algorithm. Both algorithms were implemented and performed a systematic Monte Carlo simulation to demonstrate the possibility of the proposed method. The reconstructed image qualities were quantitatively evaluated in sense of the root mean square error (RMSE) and the peak signal-to-noise ratio (PSNR). Our simulation results indicate that the proposed method provides more accurate location information of the simulated gamma source than the CC-based method.

© 2020 Korean Nuclear Society, Published by Elsevier Korea LLC. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

1. Introduction

Accurate localization of radioactive materials widely used for medical and industrial purposes is an important technology in various fields such as nuclear safety, nuclear defense, homeland security, etc. [1,2]. Coded-aperture imaging (CAI), one of the techniques developed for this purpose, was introduced in the fields of x-ray cameras and space telescopes [3–5]. This system can visualize the source distribution using the detector pattern from incident radiation, and performs spectroscopy for incoming radiation. From the obtained gamma-ray spectrum, the identification of the type of nuclide can be obtained, and only the nuclide-specific pattern can be acquired to obtain a nuclide-specific image. Along with Compton camera, the CAI system is also used as gamma-ray imaging equipment for nuclear power plants, decommissioning, and decontamination as well as for medical imaging and industrial applications [6–9]. The CAI system is characterized by high-sensitivity to low-energy sources but with a small field-of-view (FOV) and low image contrast compared to Compton camera. Despite these shortcomings, the CAI-based gamma ray imaging system is still attractive since it can be used for broad energy range and has advantage of excellent angular resolution.

In a CAI-based gamma ray imager, radiations released from radioactive materials pass through a mask, which forms multiple overlapping mask shadows on the detector surface, and impinge on the position-sensitive detector array to form radiation signals from each pixel. The image of the source distribution (i.e., source directions) is reconstructed typically by using the conventional cross-correlation (CC) reconstruction method [10]. However, at this time, if the location of the gamma source to be retrieved is outside the fully-coded field-of-view (FCFOV) region of the CAI system, artifacts may be generated in the reconstructed image. Here, the field-of-view (FOV) being reconstructed is limited only to FCFOV and does not include the whole partially-coded field-of-view (PCFOV) region. The CC-based image reconstruction process,
which does not consider radiations from the source directions that are incompletely modulated by the mask into image reconstruction, inherently encompasses coding noise associated with this partial encoding effect. Fig. 1 shows a geometric representation of the extension of FOV in a CAI system: [A] FCFOV, [B] PCFOV, and [C] outside-FOV for a mosaicked mask geometry.

There have been some attempts to exempt the gamma-ray imaging system from the common limitations of partial encoding. Stand-off radiation imaging system (SORIS) employed a nonplanar mask to obtain a wide view of about 180°, but it compromised the angular resolution as a trade-off [11]. A near-4π gamma-ray imaging device comprising multiple three-dimensional (3D) position-sensitive pixellated detectors with combination of Compton and CAI techniques was also introduced; however, its detection efficiency is known to be poor [12].

In order to overcome these drawbacks, recent research suggests that a large horizontal FOV can be obtained by combining multiple coded-aperture systems into a single hexagonal device, but the approach cannot be applied to the vertical and diagonal directions [13].

In this work, an effective imaging method was proposed for more accurate reconstruction in CAI, where source distributions in both PCFOV and FCFOV are considered. In addition, an iterative reconstruction algorithm based on compressed-sensing (CS) [14–17] was compared with the CC algorithm. The CS method is a state-of-the-art mathematical theory to solve inverse problems, and it exploits the sparsity of the image with high accuracy. In the following sections, the CS-based scheme for CAI reconstruction and the simulation conditions are briefly described, and simulation results will be presented.

---

**Fig. 1.** A schematic configuration of CAI-based gamma-ray imager geometry used in the simulation (not to scale). With this geometry, FCFOV and PCFOV were calculated as 304.5 cm × 304.5 cm and 974.4 cm × 974.4 cm, respectively.

---

**Fig. 2.** (a) The MURA pattern (rank 11, centered mosaicked by 2 × 2) and (b) the CAI geometry of actual dimension used in the simulation.
2. Material and method

Using the Monte Carlo N-Particle eXtended (MCNPX)-Polimi code system [18], an optimization study for the tungsten-based coded-aperture design consisting of centered-mosaic modified uniformly redundant array (MURA) patterns was performed by simulating the particle transport [10]. Fig. 1 shows the schematic configuration of a CAI-based gamma-ray imaging system used in the simulation (not to scale). The mask implemented in this system employs 11-rank centered-mosaic MURA patterns expanded to 21 × 21 pixels. Furthermore, the mask of system is thick enough (2 cm), so that the energy-dependency of attenuation is insignificant until about 3 MeV.

To achieve a large FCOV for gamma-ray imaging, one of the simplest ways would be using a large detector. However, this method has limitations in physical fabrication of the device and the cost issue. Another method is to take an image once, rotate the mask by 90°, and then acquire the image again to neutralize the inherent partial-encoding artifacts. However, this way requires an additional step which can be time-consuming and has technical disadvantage in that the mask must be rotated every time. Another approach is to copy and expand the mask to a 2n-1 × 2n-1 array, called ‘centered mosaic’ MURA, to obtain FCOV, which can overcome the physical and cost limitations of the methods mentioned above [19].

With consideration of the mask design parameters, the detector system is composed of a 11 × 11 array of CsI(Tl) scintillators [20,21]. Here, the mask and the detector sizes are 8.7 × 8.7 × 2.0 cm³ and 4.6 × 4.6 × 2.0 cm³, respectively. In order to reduce the sampling and the mechanical artifacts due to the shape of the mask pieces, the object-to-mask distance (OMD) was set to 311.4 cm and the mask-to-detector distance (MDD) was set to 2.3 cm (focal length) with magnification of 1, so that the shadow of the mask pixels having a finite thickness matched the detector pixel size [22].

Fig. 3. Some examples of the simulated simple MURA pattern shadows cast on the detector surface from the selected source points: (a, d, and g) the source points of (19, 5), (19, 12), and (19, 19), (b, e, and h) the corresponding MURA shadows, and (c, f, and i) the 2D perspective plots of the MURA shadows.
Fig. 4. The two source distributions used in the simulation (a,c) and their MURA shadows (b,d): one contains three point sources only within FCFOV (CASE 1) and the other contains four point sources, one in PCFOV and three within FCFOV (CASE 2).

Fig. 5. The source distributions for: (a) CASE 1 and (d) CASE 2, (b, e) the resultant CC-reconstructed images of each case, and (c, f) their enlarged images of the dotted box A at the center.
this geometry, the FCFOV and PCFOV were calculated as 304.5 cm \times 304.5 cm and 974.4 cm \times 974.4 cm, respectively, and the image pixel sizes associated with FCFOV and PCFOV were 27.68 cm and 26.3 cm, respectively. As the magnification factor of the system is kept the same with the collimator mask of enough thickness, the reconstructed image is not susceptible to OMD variation and is only affected by the source location in the projected object plane. The spatial resolution of reconstructed images, here, depends on the mask rank in CC method and on the system matrix size in iterative methods (CS, MLEM, SART, etc.). Fig. 2 shows the MURA pattern and the CAI geometry of actual dimension used in the simulation.

Fig. 3 shows some examples of the simulated simple MURA pattern shadows cast on the detector surface from the selected source points. Each simulation was conducted for 10^{9} particle histories of 1 MeV photon located at the source points of (19, 5), (19, 12), and (19, 19) – Fig. 3(a, d, and g), respectively. Background radiations such as cosmic rays and natural backgrounds were not modeled or implemented in this simulation work. Fig. 3(b, e, and h) show the corresponding MURA shadows, and Fig. 3(c, f, and i) show the two-dimensional (2D) perspective plots of the MURA shadows. The MURA shadow (b) generated from the source point of (19, 5) located in PCFOV was incompletely coded, whereas the shadows (h) from the source points in FCFOV were completely coded.

Fig. 4 shows the two kinds of source distribution scenarios assumed in the simulation (a, c) and their MURA shadows (b, d): one contains three point sources only within FCFOV (CASE 1) and the other contains four point sources, one in PCFOV and three within FCFOV (CASE 2). For the CS-based image reconstruction, system matrices of the same field-of-view size with PCFOV were acquired by the MCNP-Polimi simulation. CS-based iterative process for image reconstruction in CAI is described in Ref. [17], and the mathematical conditions for the description of the CS-based image reconstruction scheme for the CAI system are described in Ref. [19]. Based on the configuration, a CS-based reconstruction algorithm for CAI was implemented on the MCNP-polimi results using Matlab™ 9.0 programming software.

Fig. 6. The intensity profile measured along the line segments \( BC \) indicated in Fig. 5(b) and (e), the CC-reconstructed images for CASE 1 and the same for CASE 2. The corresponding intensity profile of the original source distribution is also indicated for the reference.

Fig. 7. (a) The source distribution for CASE 2, (b) the resultant CS-reconstructed image with \( A_{FCPE-FOV} \) and (c, e) its enlarged images inside the boxes A and B. (d) The enlarged image of the CS-reconstructed image with \( A_{FC-FOV} \) is also indicated for comparison.
The number of iterations was set to 3000 in the CS-based algorithm. Although the CS-based algorithm implemented in this work has not yet been accelerated by the graphics processing unit (GPU), the reconstruction times for the given simulation conditions were less than 10 s on a conventional workstation (OS: Windows 10, CPU: 2.13 GHz, RAM: 16 GB). The conventional CC-based algorithm was also implemented for the comparison of reconstructed image quality.

3. Results and discussion

Fig. 5 shows the source distributions for: (a) CASE 1 and (d) CASE 2, (b, e) the resultant CC-reconstructed images of each case, and (c, f) their enlarged images of the box A at the center. Note that, backward projection results in the CC-reconstructed image are accompanied by overall noise artifacts spread over the large area of the image. It is associated with the intrinsic nature of the CC reconstruction method, which creates sidelobe artifacts of finite intensity leaking out of the peak region. In case of the CC-based method, the physical thickness and transmission of the mask are not taken into account for the reconstruction, and the shape of the impulse response function is represented as a triangular shape.

In this paper, methods to remove this noise artifacts were not applied. As indicated in Fig. 5(c), the two point sources located both sides, except for the one at the center, were not well localized (see the dotted circle marked by D). In addition, when the source distribution was extended to PCFOV, as shown in CASE 2, the CC-reconstructed image suffered from background noise due to the insufficient mask data (also see the area marked by the arrow E).

Fig. 6 shows the intensity profiles measured along the line segments, BC, as shown in Fig. 5(b), for the CC-reconstructed images for CASE 1 and the same for CASE 2. The corresponding profile of the original source distributions is also indicated for the reference. Fig. 7 shows (a) the source distribution for CASE 2, (b) the resultant CS-reconstructed image with a predetermined system matrix considering the source distribution both in PCFOV and FCFOV (AFCPC-FOV), and (c, e) its enlarged images of the dotted boxes A and B. The enlarged image of the CS-reconstructed image with a system matrix considering the source distributions only in FCFOV (AFC-FOV) is also indicated for comparison, as shown in Fig. 7(d).

In comparison with Fig. 5(e) and (f), the CS-reconstructed images with AFCPC-FOV, shown in Fig. 7(b) and (e), present much lower noise than that of the CC-reconstructed image as they almost completely reconstruct the original source distribution both in FCFOV and PCFOV. In addition, the image quality of the CS-based reconstruction result with AFC-FOV turned out to be much better than that with AFCPC-FOV (Fig. 7(d) and (e), which shows the area marked by the dotted square). This indicates that the proposed CS-based reconstruction method using AFCPC-FOV provides more accurate location information of the source distribution than the CC-based method can, thus, demonstrating its reliability. Fig. 8 shows the intensity profiles measured along the line segments, CD, shown in Fig. 7(b), for the CS-reconstructed images with AFC-FOV and AFCPC-FOV. The corresponding profile of the original source distributions is also indicated for the reference.

The root mean square error (RMSE) and the peak signal-to-noise ratio (PSNR) of the reconstructed images were also calculated, which are defined as:

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_{ori} - x_{recon})^2},
\]

Fig. 8. The intensity profiles measured along the line segments, CD, as shown in Fig. 7(b) for the CS-reconstructed images with AFC-FOV and AFCPC-FOV. The corresponding intensity profile of the original source distributions is also indicated for the reference.

Fig. 9. (a) RMSE and (b) PSNR values calculated from the enlarged reconstructed images shown in Figs. 5 and 7.
**Fig. 10.** Complete sets of images for the simulated thyroid gland: (a) the source distribution, (b) the MURA shadow image, (c) the CC-reconstructed image and (d) the CS-reconstructed image with $A_{FCPC-FOV}$.

**Fig. 11.** The enlarged images of the FCFOV areas of Fig. 10(c) and (d).
where $x_{ori}$ and $x_{recon}$ are the original and the reconstructed source distributions, respectively \([23]\). The RMSE measures the error of the reconstructed image from the original source distribution, and the PSNR measures the ratio between the maximum value of the original source distribution and the reconstructed images shown in Fig. 11.

\[
PSNR = 10 \cdot \log_{10} \left[ \frac{\max(x_{ori})^2}{\frac{1}{N} \sum_{i=1}^{N} (x_{ori} - x_{recon})^2} \right],
\]

(7)

Fig. 12. The intensity profiles measured along the line segments, $BC$, for the both reconstructed images shown in Fig. 11.

PSNR values for the CS-reconstructed image with $A_{PC-FOV}$ were about 0.0011 and 107.1, about 0.03 times smaller and 1.30 times larger than that with $A_{FC-FOV}$, respectively. In addition, the proposed CS-based method is also very promising for the clustered source distribution, presenting superior reconstruction quality, compared to the conventional CC-based method. The proposed method will be applicable to the development of an advanced CAI system as an augmentation tool.

**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Acknowledgement**

This work was partly supported by the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Ministry of Trade, Industry & Energy (MOTIE) of the Republic of Korea (20181520302230), by the Nuclear Safety Research Program through the Korea Foundation of Nuclear Safety (KoFONS) using the financial resource granted by the Nuclear Safety and Security Commission of the Republic of Korea (No. 1903011-0119-CG100), and by National Research Foundation of the Republic of Korea (NRF-2018M3A7B4070992).

**References**


