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Sensitivity Analysis of Dynamic Response by Change in Excitation Force and Cross-sectional Shape for Damped Vibration of Cantilever Beam

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가진력과 단면형상 변화에 따른 외팔보 감쇠 진동의 민감도 해석

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ABSTRACT

This paper describes the time rate of change of dynamic response of a cantilever beam inserted with a damping element, such as bonding, which is excited under a general force at various locations. A sensitivity analysis was performed in a finite element model to show that two types of second-order algebraic governing equations were used to predict the rate of change of dynamic displacement: one is related to the modal coordinate linked to a physical coordinate, and the other to the design parameter of the time rate of change of displacement. The sensitivity differential equation formulation includes more complicated terms compared with that of the undamped cantilever beam. The sensitivities of the dynamic response were observed by changing the location of the excitation force, displacement extraction, and cross-sectional area of the beam. The analytical results obtained by this suggested theory showed a relatively good agreement when compared with those obtained using the commercial finite element program. The suggested analysis procedure enables the prediction of the response sensitivity for any finite element model of the dynamic system.

Key Words : Sensitivity Analysis(민감도 분석), Dynamic Displacement Sensitivity(동적 변위 민감도), Damped Cantilever Beam Vibration(감쇠 외팔보 진동), Beam Finite Element(보 유한요소)

1. Introduction

Many structures and machines generate dynamic displacements using an exciting force; thus, it is vital to identify the characteristics of vibrations in the initial design. The analysis technique that is most frequently used now is finite element analysis; however, it is not easy to quantitatively determine the design direction for complex or large-scale structures. Therefore, unless a valid design tool is established, the design takes long time, or an

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excessive analysis cost is incurred owing to the errors introduced by the architect or their lack of experience.

Dynamic sensitivity analysis is performed to identify design factors that sharply change dynamic behaviors by determining the variable with a high or low contribution to target performance.[1] It is necessary to investigate how much the changes in mass, stiffness, exciting force, or shape in a dynamic system quantitatively contribute to the change in the vibration characteristics. A tool is needed to quantify how sensitive a change in specific design parameters is to dynamic behaviors in a dynamic system.[2] Sensitivity analysis in design represents the correlations between design parameters and responses. Most vibrating structures have a damping performance, but recent theoretical approach to sensitivity analysis has ignored damping and approximated the sensitivity analysis of modal variables.[3,4]

This study deals with vibrations to which an aperiodic external force is applied based on a modal analysis theory that includes general viscous damping and performs sensitivity analysis for dynamic displacements by applying finite elements to Euler-Bernoulli cantilever beams. If a commercial finite element program is used, it accompanies a somewhat complex process of calculating the changes in displacement in the existing and changed systems, including calculation of changes in members and derivation of the rate of change of responses in each step. The proposed analysis algorithm can be extended and applied to structures of regular shapes to facilitate the implementation of finite elements. For example, it can contribute to the research on the truss and grid-type structures with regularity of shape, foldable solar panels of artificial satellites applying them, and mechanical arrangement shapes of reinforcements for aircrafts, ships, and vehicles.[5]

This study sets the magnitude of external force, the position of external force, detection position, and cross-section shape as design parameters, observes the change rate of dynamic responses as design parameters, and then observes the rate of change of dynamic responses while changing the detection positions of dynamic displacements. The rate of change of dynamic responses is examined from the two equations of motion, that is, the differential algebraic equations that are expressed by the physical coordinates and coordinates representing ~~the~~ the rate of change of displacement by changing the design parameters. In other words, we use the equations of motion that include damping and include the rate of change from modal analysis. The responses of complex external force terms in a differential equation that represents the sensitivity, including the effect of damping, are used in the finite element model of cantilever beams. Furthermore, the change in cross-sectional shape in the forced damped vibration of the cantilever beam automatically reflects the change in mass and stiffness. Then, a series of analysis algorithms to calculate the quantitative rate of change responses by changing the design parameters are suggested.

2. Theory

2.1 Modal formulation of damped vibration

The equation of motion of forced vibration that applies an external force to a dynamic system in which damping exists and has n degrees of freedom is as follows

$$\boldsymbol{M}\ddot{\boldsymbol{x}}+\boldsymbol{C}\boldsymbol{x}+\boldsymbol{K}\boldsymbol{x}=\boldsymbol{f}(t) \quad (1)$$

Where $\boldsymbol{M}$ is a mass matrix, $\boldsymbol{K}$ a stiffness matrix, $\boldsymbol{C}$ a damping matrix, $\boldsymbol{f}$ an external force vector, $\boldsymbol{x}$ the displacement vector, and $(\dot{\ })$ the differentiation with respect to time t. If a linear system is assumed, the model displacement $\boldsymbol{u}$ that has generated the physical displacement $\boldsymbol{x}$, in consideration of the overlapping of modes, has the following relationship:

$$\boldsymbol{x}=\boldsymbol{\Phi u} \tag{2}$$

where $\boldsymbol{\Phi}$ is a modal matrix composed of n eigen vectors or modal vectors $\boldsymbol{\phi}_k(k=1,\cdots,n)$, and can be normalized as follows:

$$\boldsymbol{\Phi}^T\boldsymbol{M\Phi}=\begin{bmatrix}1 & & \\ & \ddots & \\ & & 1\end{bmatrix} \tag{3}$$

Using the eigenvalue λ_k or the square of angular natural frequency ω_k^2, this can be expressed as an eigenvalue problem as follows:

$$(\boldsymbol{K}-\lambda_k\boldsymbol{M})\boldsymbol{\phi}_k=\boldsymbol{0} \tag{4}$$

When Eq. (2) is substituted in Eq. (1) and this equation is premultiplied by $\boldsymbol{\Phi}^T$, the equation of motion for modal coordinate $\boldsymbol{u}$ is derived as follows:

$$\ddot{\boldsymbol{u}}(t)+\boldsymbol{\Gamma}\dot{\boldsymbol{u}}(t)+\boldsymbol{\Omega}^2\boldsymbol{u}(t)=\boldsymbol{Q}(t) \tag{5}$$

where the damping matrix Γ and eigenvalue matrix Ω^2 are as follows:

$$\boldsymbol{\Gamma}=\boldsymbol{\Phi}^T\boldsymbol{C\Phi}=\begin{bmatrix}2\omega_1\xi_1 & & \\ & \ddots & \\ & & 2\omega_n\xi_n\end{bmatrix} \tag{6.a}$$

$$\boldsymbol{\Omega}^2=\boldsymbol{\Phi}^T\boldsymbol{K\Phi}=\begin{bmatrix}\omega_1^2 & & \\ & \ddots & \\ & & \omega_n^2\end{bmatrix} \tag{6.b}$$

where ξ_k is the damping ratio. When Eq. (6.a,b) is substituted in Eq, (5), n uncoupled equations of motion are derived as follows:

$$\ddot{u}_k+2\omega_k\xi_k\dot{u}_k+\omega_k^2u_k=Q_k,\ (k=1,\cdots,n) \tag{7}$$

In Eq. (7), the external force term of the right side is expressed as follows:

$$Q_k=\sum_{i=1}^{n}\Phi_{ki}^T f_i(t) \tag{8}$$

2.2 Dynamic sensitivity and analysis process

An analysis process is required to quantify the change of dynamic response, that is, the rate of change of displacement, when the system design parameter $d_j(j=1,\cdots,p)$ is changed. To that end, when Eq. (2) is differentiated with respect to the design parameter d_j, the rate of change of displacement can be obtained as follows:

$$\frac{\partial\boldsymbol{x}}{\partial d_j}=\sum_{k=1}^{n}\left(\frac{\partial\boldsymbol{\phi}_k}{\partial d_j}u_k+\boldsymbol{\phi}_k\frac{\partial u_k}{\partial d_j}\right) \tag{9}$$

where the first term of the right side is the change rate of the modal vector for the design parameter and can be calculated by using the difference between the reference vector and the modified vector as the numerator and the difference δd_j between the reference design parameter d_o and the changed design parameter as the denominator. In other words, the rate of change of the modal vector and the eigenvalue using the finite difference method can be expressed as follows:

$$\frac{\partial\boldsymbol{\phi}_k}{\partial d_j}\approx\frac{\boldsymbol{\phi}(d_o+\delta d_j)-\boldsymbol{\phi}(d_o)}{\delta d_j} \tag{10}$$

$$\frac{\partial\lambda_k}{\partial d_j}\approx\frac{\lambda_k(d_o+\delta d_j)-\lambda_k(d_o)}{\delta d_j} \tag{11}$$

Regarding the second term of the right side in

Eq. (9), the following differential equation can be derived by differentiating Eq. (5) with respect to the design parameter d_j.

$$\frac{\partial \ddot{u}_k}{\partial d_j}+2\omega_k\xi_k\frac{\partial \dot{u}_k}{\partial d_j}+\omega_k^2\frac{\partial u_k}{\partial d_j} = \frac{\partial Q_k}{\partial d_j}-2\frac{\partial \omega_k}{\partial d_j}\xi_k\dot{u}_k-\frac{\partial \omega_k^2}{\partial d_j}u_k \tag{12}$$

In addition, Eq. (14) can be derived by introducing the following parametric differential equation (13.a,b,c) and using Eq. (8).

$$q_k=\frac{\partial u_k}{\partial d_j},\ \dot{q}_k=\frac{\partial \dot{u}_k}{\partial d_j},\ \ddot{q}_k=\frac{\partial \ddot{u}_k}{\partial d_j} \tag{13.a,b,c}$$

$$\ddot{q}_k+2\omega_k\xi_k\dot{q}_k+\omega_k^2q_k=\frac{\partial \boldsymbol{\phi}_k^T}{\partial d_j}\boldsymbol{f}+\boldsymbol{\phi}_k^T\frac{\partial \boldsymbol{f}}{\partial d_j}-2\frac{\partial \omega_k}{\partial d_j}\xi_k\dot{u}_k-\frac{\partial \omega_k^2}{\partial d_j}u_k \tag{14}$$

In this formulation of the differential equation for the rate of change, it should be noted that the left sides in Eqs. (5) and (14) have the same form, but the right sides are different. Before a design parameter is changed, it has the initial condition of $q_k=\dot{q}_k=0$ at $t=0$.

The analysis flowchart is shown in Fig. 1 by summarizing the analysis process for determining the change rate of displacement due to a change of the design parameter. First, the eigenvalue λ_k and modal vector $\boldsymbol{\phi}_k$ are determined from the Eq. (4) of the modal analysis. From Eqs. (10) and (11), the rate of change $\partial\boldsymbol{\phi}_k/\partial d_j$ and $\partial\lambda_k/\partial d_j$ of modal vector and eigenvalue for the design parameters are calculated, respectively. Here, the damping ratio ξ_k obtained experimentally or theoretically is applied. Then u_k is obtained from the uncoupled modal equation of motion (7) and the solution q_k of the differential equation for the change rate is determined by calculating the four terms in the right side of Eq. (14). The convolution integral can be used to determine the solutions u_k and $q_k$of each differential equation. Here, the forms of u_k and q_k are expressed very differently depending on the form of the applied external force. Finally, the rate of change of dynamic responses $\partial \boldsymbol{x}/\partial d_j$ can be calculated by calculating the terms on the right side of Eq. (9).

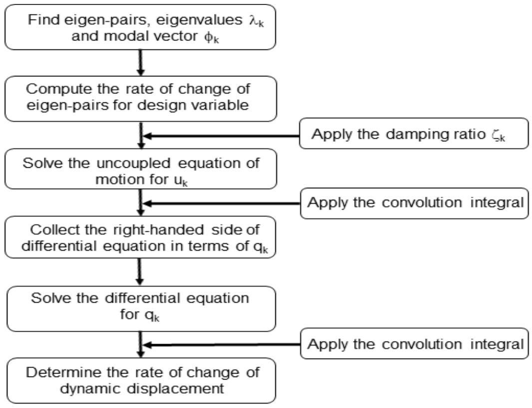

Fig. 1 A flow chart for algorithm for predictions on the rate of change of dynamic displacement

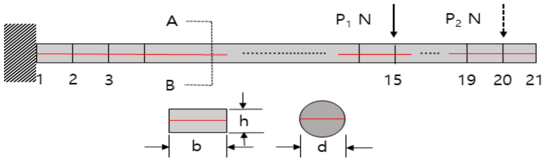

Fig. 2 A finite element model for the cantilever beam

3. Numerical Analysis

In this study, the total length of the beam $\ell=1.4m$, the length of finite element $\ell_e=0.07m$, the density of material $\rho=7{,}850kg/m^3$, the modulus of elasticity of the material $E=2\times10^{11}N/m^2$, and the damping ratio $\xi_k=0.1$ were used. As shown in Fig. 2, the cantilever beam was divided into 20 finite elements, and the nodes were numbered from 1 to 21. For example, the external forces P_1 and P_2 at the nodal positions of two finite elements of the cantilever beam were applied as follows:

$$f_k(t)=\begin{cases}P_1,\ P_2\ [N], & (0\le t\le 3\,\mathrm{sec})\\ 0\qquad [N], & (t>3\,\mathrm{sec})\end{cases} \tag{15}$$

For the change of design parameters, the cross-sectional area of the cantilever was selected. At first, it was a rectangular cross section with an area of $20mm\times 5mm(\text{width}(b)\times\text{thickness}(h))$. This was changed to a circular cross section with the diameter(ϕ) of 5 mm. In addition, various external forces were applied independently from different nodes.

When the above-mentioned rectangular external force Q_k is applied to Eq. (5), the dynamic displacement $u_k(t)$ can be determined using the convolution integral as follows:

$$u_k(t)=\frac{-\xi_k\omega_k Q_k}{\{(\xi_k\omega_k)^2+\omega_k^2\omega_{dk}\}}\times\left\{e^{-\xi_k\omega_k t}\left(\sin\omega_{dk}t+\frac{\omega_{dk}}{\xi_k\omega_k}\cos\omega_{dk}t\right)-\frac{\omega_{dk}}{\xi_k\omega_k}\right\} \tag{16}$$

The solution of the first term in the right side of Eq. (14) is as follows:

$$q_{1k}(t)=\frac{-\xi_k\omega_k}{\{(\xi_k\omega_k)^2+\omega_k^2\omega_{dk}\}}\frac{\partial\phi_k^T}{\partial d_j}f_k\times\left\{e^{-\xi_k\omega_k t}\left(\sin\omega_{dk}t+\frac{\omega_{dk}}{\xi_k\omega_k}\cos\omega_{dk}t\right)-\frac{\omega_{dk}}{\xi_k\omega_k}\right\} \tag{17}$$

where ω_{dk} is the damping natural frequency. In this study, only the cross-sectional area of the beam is changed for the initially determined external force; thus, there is no change in external force in the second term of Eq. (14). As a result, $q_{2k}(t)$ disappears and the third term $q_{3k}(t)$ and the fourth term $q_{4k}(t)$ are expressed as Eqs. (18) and (19), respectively:

$$q_{3k}(t)=\frac{\partial\sqrt{\lambda_k}}{\partial d_j}\frac{\xi_k\omega_k Q_k}{\{(\xi_k\omega_k)^2+\omega_k^2\omega_{dk}\}}\left(\xi_k\omega_k+\frac{\omega_{dk}^2}{\xi_k\omega_k}\right)\times\frac{1}{\omega_{dk}}e^{-\xi_k\omega_k t}(t\cos\omega_{dk}t-\sin\omega_{dk}t) \tag{18}$$

$$q_{4k}(t)=\frac{\partial\lambda_k}{\partial d_j}\frac{\xi_k\omega_k Q_k}{\{(\xi_k\omega_k)^2+\omega_k^2\omega_{dk}\}}\times\left[-\frac{1}{2\omega_{dk}}e^{-\xi_k\omega_k t}(t\cos\omega_{dk}t-\sin\omega_{dk}t)+\frac{1}{2\xi_k\omega_k}e^{-\xi_k\omega_k t}t\sin\omega_{dk}t+\frac{1}{(\xi_k\omega_k)^2+\omega_{dk}^2}\left\{e^{-\xi_k\omega_k t}\left(\sin\omega_{dk}t+\frac{\omega_{dk}}{\xi_k\omega_k}\cos\omega_{dk}t-\frac{\omega_{dk}}{\xi_k\omega_k}\right)\right\}\right] \tag{19}$$

For design parameters, the material, member shape, external force condition, and constraint can be considered, but in this study, the member shape and external force condition were selected. The external force was applied at nodes 3 and 4, which are close to the fixed end of the cantilever beam, and for the detection of displacement, nodes 10 and 12 were selected, where a significant change of displacement to the free end appears. To observe the precision of the derived theoretical value, it was compared with the value derived using a commercial finite element program.[6] Figs. 3 and 4 show the change of displacement for a change in cross-sectional area when the diameter of the circular cross-section of the cantilever beam was changed to $\phi=5mm$. In Fig. 3, an external force of $5N$ was applied to node 4, and the change in displacement was observed at node 10. In Fig. 4, an external force of $3N$ was applied to node 3 and the change in displacement was observed at node 12.

Both cases show similar changing trends of displacement, and the displacement change increased as the node was closer to the free end, and the magnitude of change decreased with time. This is in contrast to the undamped case where the maximum and minimum values always behave within a constant range.[3] Furthermore, a difference from the theoretical value occurs at the minimum and maximum points of the responses. In the case of Fig. 4, the phase appears slightly shifted. This difference appears to be caused because the rate of change of the eigenvector for the change in cross-sectional area and the rate of change of the eigenvalue in Eqs. (10) and (11) of the analysis algorithm were evaluated using the forward difference method.

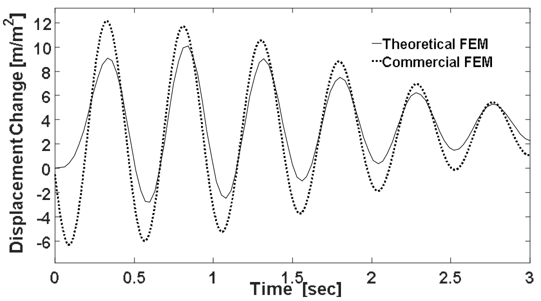

Fig. 3 Comparisons of the displacement change at node 10 by applying 5N at node 4 and by changing rectangular area to circular diameter 5mm

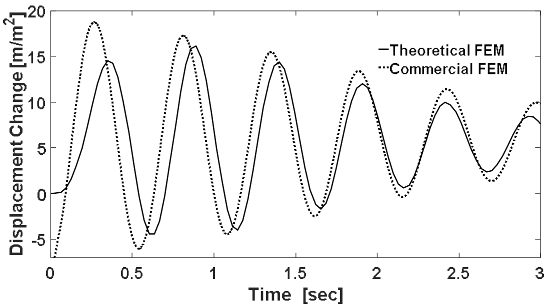

Fig. 4 Comparisons of the displacement change at node 12 by applying 3N at node 3 and by changing rectangular area to circular diameter 5mm

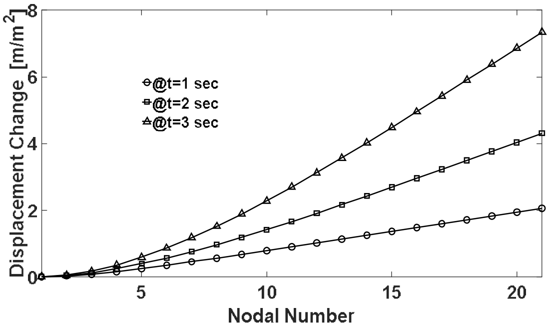

Fig. 5 The displacement change at all nodes by applying 5N at node 4, and by changing rectangular area to circular diameter 5mm

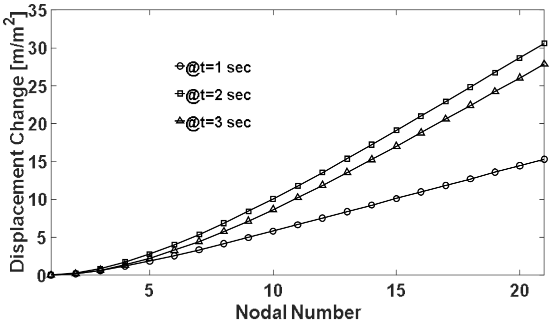

Fig. 6 The displacement change at all nodes by applying 3N at node 3, and by changing rectangular area to circular diameter 5mm

Figs. 5 and 6 show the observations of displacement changes at all nodes when the elapsed times were 1, 2, and 3 seconds in the same input and output conditions as those in Figs. 3 and 4 when the diameter of the circular cross section was changed to the same value of $\phi = 5mm$.

4. Conclusions

This study formulated and analyzed the forced vibration with damping for cantilever beams to predict the change rate of dynamic responses by changing the design parameters and obtained the following conclusions.

(1) A second-order differential algebraic equation that expresses the sensitivity of dynamic displacements with damping was used through the change in the cross-sectional shape for the change of design parameters. Consequently, an analysis process to identify the sensitivity of displacement was suggested.
(2) The rate of change of the dynamic displacement at some or all nodes were observed regarding the detection positions of dynamic displacements by setting various external force application positions and magnitudes of external force. It was observed that the rate of change increased and then decreased as the detection position moved closer to the free end.
(3) A phase shift and differences in maximum and minimum values were observed when compared with the result obtained from a commercial finite element program. This suggests that an improvement of the evaluation method for rates of change of eigenvalue and eigenvector is required.
(4) This study laid the foundation for identifying the sensitivity of dynamic response in a bonded structure which there exist iconnection parts accompanied by a damping function.

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