



수리계획법을 활용한 방재자원 배치 최적화: AED 배치 사례

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(2021년 5월 6일 접수, 2021년 6월 24일 수정, 2021년 6월 25일 채택)

Mathematical Programming and Optimization of the Resource Allocation and Deployment for Disaster Response : AED case study

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(Received May 6, 2021; Revised June 24, 2021; Accepted June 25, 2021)

요약

노년층뿐만 아니라 청년층에 속하는 성인들 사이에서도 병원 외 심장정지(OHCA)를 겪는 심장질환자의 수가 증가하고 있다. 자동심장충격기(AED)는 병원 외 심장정지 환자의 생존율을 개선하는 데 있어 중요하다. 심장정지 생존율은 제세동 시간에 대하여 지수적으로 감소(decline exponentially)하는 것으로 밝혀졌으나, 자동심장충격기의 최적 배치에 있어 심장정지 생존율의 이러한 특성을 반영한 국내 연구가 미미한 상황이다. 본 연구에서는 자동심장충격기의 최적 입지를 결정하기 위하여 exponential decay coverage 함수를 갖는 최대 gradual coverage 입지 모델에 대하여 고찰하였다. exponential decay coverage 함수는 심장정지 환자의 생존율에 대한 과다추정을 완화한다. 향후 시뮬레이션을 통하여 랜덤한 행인 위치 및 이동을 반영함으로써 행인의 심장정지 대응에 있어 창발적인(emergent) 특징을 식별할 수 있는 시설 입지 모델이 개발될 것으로 예측된다.

Abstract - The number of patients with cardiovascular diseases who experience an out-of-hospital cardiac arrest (OHCA) are increasing among young adults as well as the aged population. An automated external defibrillator (AED) is vital in improving survival rates of OHCA victims. Survival rates of OHCA were shown to decline exponentially in time to defibrillation, yet studies in Korea are uncommon that captures the properties of their survival rates in examining optimal locations of AEDs. In this study, we worked on the maximal gradual coverage location problem (MGCLP) with exponential decay coverage function to decide on their optimal locations. The exponential decay coverage function mitigates the drawback of over-estimating survival rates of OHCA patients. It is expected that a more sophisticated facility location problem will be developed to identify the “emergent” characteristics of pedestrians who responds to the OHCA occurrence by incorporating random pedestrian locations and movement through simulation.

Key words : out-of-hospital cardiac arrest, automated external defibrillator, decay coverage function, maximal gradual coverage location problem

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I. Introduction

The number of patients with cardiovascular diseases who experience an out-of-hospital cardiac arrest (OHCA) are increasing even among young adults as well as the aged population. Early defibrillation by an automated external defibrillator (AED) is vital in survival of OHCA victims since it has a chance to adjust the rhythm of the heart to a regular level. AEDs need to be located in a way that lay responders can instantly retrieve the equipment during emergency. Facility location problem framework is expected to help decision-makers to examine the optimal placement of AEDs. Precedented studies on a survival probability of OHCA have consented to acknowledge an exponentially declining rate of survival in time to defibrillation [1], [2], [3]. The effectiveness of locating AEDs depends significantly on well capturing such a change in survival probability. There was a study on locating AEDs to minimize the total distance between demand nodes and candidate facility sites, yet it departs from the nature of an exponentially declining rate of survival for OHCA victims [4].

II. Model formulation

2.1. Maximal gradual coverage location problem

In this study we propose to additionally locate AEDs based on the maximal gradual coverage location problem (MGCLP) [5], [6].

$$\max \sum_{i \in I} \sum_{j \in J} f(d_{ij}) y_{ij} \quad (1)$$

subject to

$$\sum_{j \in J} x_j \leq p \quad (2)$$

$$\sum_{j \in J} y_{ij} \leq 1 \text{ for all } i \in I \quad (3)$$

$$\sum_{i \in I} y_{ij} \leq mx_j \text{ for all } j \in J \quad (4)$$

$$x_j, y_{ij} \in \{0, 1\} \text{ for all } i \in I, j \in J \quad (5)$$

The objective in (1) is to maximize the total coverage of all demand nodes, which depends on the spatial proximity between demand nodes and candidate sites where

AEDs will be placed. Constraint (2) indicates that there are at most p facility sites where AEDs can be placed. Constraint (3) ensures that demand node i is serviced by a facility at j only when the facility is opened at j . Constraint (4) states that each demand node can be serviced by at most one facility. (5) is the binary integer constraint for the following conditions; $x_j = 1$ only if the facility is located at site j and $y_{ij} = 1$ only if the demand node i is serviced by a facility opened at j .

2.2. Decay coverage function

In the MGCLP, the decay coverage function is formulated to describe the gradual change of coverage over the distance, d_{ij} opposed to service coverage falling from one to zero beyond the threshold as follows;

$$f(d_{ij}) = \begin{cases} 1 & d_{ij} \leq R_1 \\ [0, 1] & d_{ij} \in [R_1, R_2] \\ 0 & R_2 \end{cases} \quad (6)$$

where R_1 is defined as the minimal coverage threshold ensuring that a demand node is fully covered and R_2 as the maximal coverage threshold beyond which the demand node is not covered at all. For instance, the decay coverage function can be represented linearly as in equation (7) and **Fig. 1**.

$$f(d_{ij}) = 1 - \frac{d_{ij}}{R} \quad (7)$$

Equation (7) where $R_1 = 0$ and $R_2 = R > 0$ indicates that a demand node is fully covered by a facility only if $d_{ij} = 0$, partially covered yet falls linearly up to the threshold R , beyond which a demand node is not covered at all. The observation in [7] is that the MGCLP, if its decay coverage function is linear, becomes identical to the classical P-median location problem with the goal of minimizing the sum of total distances.

Linearly declining coverage is not consistent with the empirical results related to OHCA victims that their survival rates turned out to be exponentially decreasing in time to defibrillation. Linear decay coverage function suffers from over-estimating survival rates of OHCA patients rather than the counterparts in reality, causing

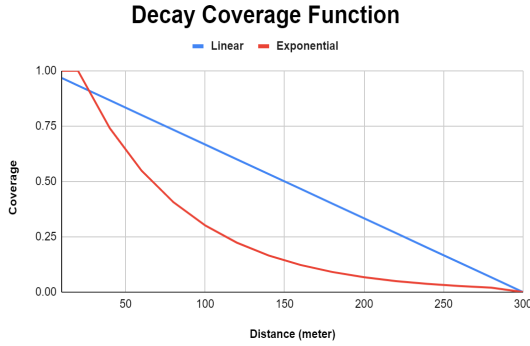


Fig. 1. Types of decay coverage functions.

the decision on location facilities to be made suboptimally.

In order to better capture the change of survival rates of OHCA victims, [8] proposed the exponential decay coverage function. This type of coverage function is that the transition from instant defibrillation to moderate delays has more significant impacts on the quality of life than the counterpart from moderate delays to serious ones.

Specifying the parameter $R_1 = 20$ with unit of meter is based on “line of sight” within which a pedestrian can instantaneously spot an AED. In order to set up R_2 properly, one need to make assumptions because the locations of lay responders are unknown. **Table 1** encapsulates an entire picture of how lay responders finally comes to OHCA victims with an AED and deliver shocks to them. For example, 2:18 is the time expended in transition from the call-to-dispatch center to the alert process. In the table, the time spent to perform the fifth process is the actual time the lay responder retrieves an AED. Such a time-frame is crucial in the distance threshold ensuring that the AED should be located to perform defibrillation in time. Considering the series of these processes, [8] set up $R_2 = 300$.

Then the decay coverage function is formulated below and represented in **Fig. 1** by comparison with the linear function.

$$f(d_{ij}) = \begin{cases} 1 & d_{ij} \leq 20 \\ \exp(-0.015(d_{ij} - 20)) & d_{ij} \in [20, 300] \\ 0 & 300 \end{cases} \quad (8)$$

Coverage under an exponential decreasing pattern is higher than the counterpart under a linear decaying pat-

Table 1. Time intervals from call to dispatch center through shock delivery with an AED

Number	Process	Time(minute, second)
1	Call to dispatch center	2:18
2	Alert	0:30
3	Leaving to AED	0:24
4	Arrival near AED	0:30
5	Leaving to victim	1:08
6	Arrival at victim	1:07
7	Shock	0:23
Total		6:00

tern until approximately 26 meters, but beyond vice versa.

III. Numerical Experiment

3.1. Overview

This section provides an entire description of applying the MGCLP into an illustrative example. The workflow of our study is presented in **Table 2**.

The flow of study can be summarized as follows: The GAMS (General Algebraic Modeling System) is applied to examine optimal placement of AEDs. Two types of facility location problems, the P-median and the MGCLP will be solved to achieve the purpose mentioned previously. Linear decay coverage function is derived from the p-median problem and the exponential decay coverage function is defined based on the literature on survival rates of OHCA victims.

3.2. Collecting data

Our numerical experiment requires a group of the following data; the set of demand nodes I and the set of locations where AEDs are pre-installed. We set the city of Gwacheon as an illustrative example. First a small size of this city whose population is 72,088 and whose area is 35.86 km^2 makes it manageable to perform numerical experiment, which is still more realistic than solving a facility location problem in a virtual two-dimensional space as in [6]. Second, the city is concentrated around the main streets and geographically

Table 2. Workflow

1. Raw data (addresses / places)	
(1) Demand node (points with floating population) (2) Facility site (pre-installed AED locations plus candidate sites for additional AED placement)	
2. Geo-coding (addresses / places)	
(1) Demand node (points with floating population) (2) Facility site (pre-installed AED locations plus candidate sites for additional AED placement)	
3. Decay coverage function ⇒ Survival rates of OHCA's	
Linear decay coverage function	Exponential decay coverage function
4. Facility location problem	
P-median location problem	Maximal gradual coverage location problem
(1) Optimal sites of AEDs / (2) Optimal assignment of demand nodes to AEDs (3) Coverage	

isolated in that it is connected to adjacent cities such as Seoul (toward North) and Anyang (toward Southwest) only via the main streets. The total 19 locations of pre-installed AEDs in Gwacheon were collected from the National Emergency Medical Center (NEMC). The locations with frequent movement of floating population were used as a proxy for demand nodes because the locations where OHCA occurred were random and are hard to probabilistically associate the occurrence of past OHCA's in a certain place with their occurrence in the future. SK Telecom Big Data Hub provides the list of places which were amongst the highest ranking in search from January 2019 to February 2021 on the T-map, the commonly-used mobility platform providing navigation service based on both real-time traffic information and street pattern information. Then the total 86 lists of places were obtained. And we extracted 18 candidate locations where AEDs will be additionally placed on the intersections around the places which were searched frequently on T-map platform within the boundary of the city. Addresses of both demand nodes and AEDs which have been acquired previously are qualitative, thus they need to be processed quantitatively. Geo-coding helps one to identify the geographical coordinates (latitude plus longitude) of points and to calculate the distances between any pair of two

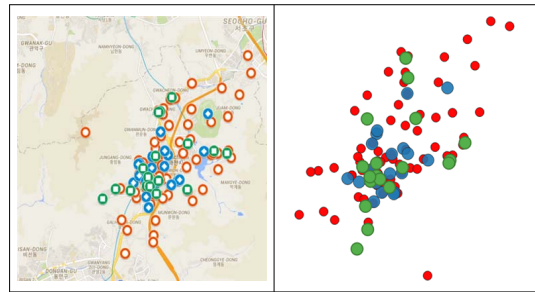


Fig. 2. Demand nodes and automated external defibrillators on the map of the city of Gwacheon (left) and on a two-dimensional space (right) where red, blue and green points are demand nodes, pre-installed AEDs and candidate sites where AEDs will be additionally installed.

different points via QGIS software. Finally the geo-coded locations were projected on the map of Gwacheon and an two-dimensional space as in Fig. 2 below.

3.3. Optimization problem

We solve the MGCLP with the decay coverage function formulated in section 2.2. In addition, the P-median problem in [4] will be solved with the same dataset used in the MGCLP to present the extent of coverage over-estimated.

IV. Result

After solving the two optimization problems regarding optimal placement of AEDs in the city of Gwacheon, comparison of results will be given in Fig. 3.

Only 19 locations of pre-installed AEDs provided demand nodes with coverage of 13.3890 (15.56%). In the MGCLP, coverage increased to 23.1612 (26.93%) when 19 optimal locations were selected among the total 37 facility sites (locations of pre-installed AEDs plus candidate facility sites). The extent of coverage reached the highest as 27.9293 (32.47%) when the total 32 AEDs were allowed to be located. In the P-median location problem, 35.2780 out of the total 86 demand nodes was covered when there are the maximum 19 facility sites where AEDs can be located. And service coverage achieved the highest as 41.5932 (48.3642%) given 31

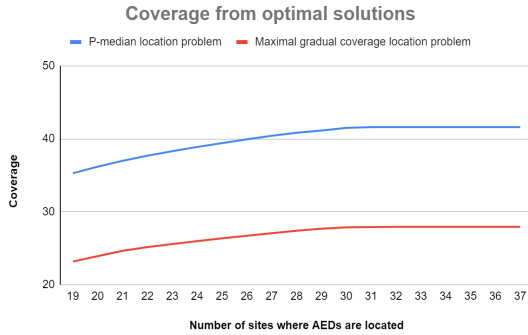


Fig. 3. Comparison of maximal gradual coverage location problem (red) and P-median location problem (blue).

candidate sites which can be placed. Given the assumed linearity of the decay coverage function, coverage was over-estimated by approximately 50% on average.

V. Conclusion

We solved the MGCLP with the decay coverage function based upon the exponentially declining survival rates of OHCAs. As an example of the city of Gwacheon, optimal locations of AEDs, on average, yielded coverage almost twice as high in our framework as the current placement of AEDs did. Coverage has tripled when we solve the MGCLP with linear decay coverage function equivalent to the P-median location problem. Past studies on OHCAs commonly acknowledged that their survival rates tend to be exponentially declining in time to defibrillation rather than linearly declining. Hence one needs to identify the changing patterns of survival rates of OHCAs in order to assess the effectiveness of the proposed optimal locations of facilities with higher accuracy. There are still opportunities for further improvement in our study. First, the distances were measured based on the Euclidean metric measuring a straight line between two points. In real-world problems, the true road distance between a pair of points is affected by curvature, travel time depending on road network. We need to take the distance metric providing flexibility such as the Minkowski metric into consideration to assess the performance of facility location problems in improving service coverage with rigor. Furthermore, our problem is both deterministic and static. In reality, pedestrians are more likely to cease-

lessly travel from one destination to another destination rather than to be stationed in one place. Simulation is expected to potentially enable researchers to identify the “emergent” characteristics of pedestrians in a three-dimensional environment in by incorporating the stochasticity of pedestrian locations, the occurrence of OHCAs or other factors.

감사의 글

본 연구는 산업통상자원부 지능정보기술 기반 효과적 외국인 노동자 산업안전 교육 서비스 개발사업 (20012292)과 산업통상자원부 스마트 디지털 엔지니어링 전문인력양성사업 (P0008475- G02P04570001901)의 연구비지원에 의해 수행되었습니다. 이에 감사드립니다.

사용기호

I : a set of demand nodes

J : a set of candidate facility sites

m : the number of demand nodes

$i \in I$: a demand node

$j \in J$: a candidate facility site

d_{ij} : the distance between a demand node i and a facility site j

$f(d_{ij})$: decay coverage as a function of the distance

p : the maximum number of facility sites where AEDs can be placed

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