Internal Model Control for Unstable Overactuated Systems with Time Delays

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Summary
In this paper, we have proposed a new internal model control structure (IMC). It is aimed at unstable overactuated multivariable systems whose transfer matrices are singular and unstable. The model inversion problem is essential to understand this structure. Indeed, the precision between the output of the process and the setpoint is linked to the quality of the inversion. This property is preserved in the presence of an additive disturbance at the output. This inversion approach proposed in this article can be applied to multivariable systems with no minimum phase or minimum phase shift with or without delays in their transfer matrices. It is proven by an example of simulation through which we have shown its good performance as a guarantee of stability, precision as well as rapidity of system responses despite the presence of external disturbances and we have tested this control structure in the frequency domain hence the robustness of the IMC.
Key words:
unstable overactuated system with time delays; internal model control; virtual outputs, stability; robustness.

1. Introduction

The control structure by internal model is known as robust, makes it possible to consider the effects of modeling errors and external disturbances. Its main advantages lie in the simplicity of the controller, its ease of implementation and the explicit adjustment of the robustness. As for the drawbacks of this structure, they mainly concern its applicability, reserved for stable open loop or stabilized systems, the controller designed in this case must also be stable and feasible [1], [2], [3], [4].

In this structure, the synthesis of the controller is reduced to a problem of constructing an inverse model of the system to be controlled [5]. In addition, the perfect inversion of the exact model is often impossible to achieve, in this case in the case of nonlinear systems and when the model can present sub-models with non-Minimum phase or presenting modeling delays or when the order of the numerator is lower than that of the denominator and in the case where the input number is greater than the output number [6], [7], [8].

The proposed internal model controller synthesis approach is based on a specific inversion exploited by [6] in the case of overactuated systems. The results obtained with this controller are very encouraging, which led to their extension to cases of overactuated systems with unstable dynamics which will be treated in this paper. The design of IMC for unstable multivariable systems is a two-step process. The first is to apply the pre-stabilization approach by stabilizing feedback to the unstable over-actuated system. The choice of this stabilization technique is because, in practice, knowledge of the complete state of the system is not always possible. Indeed, the state of the system is sometimes difficult to measure because of its variables which are not measurable or not accessible or because of the high price of the sensor allowing to measure them [9]. In this case, we only have system outputs. The second step is to apply the IMC approach to the stabilized overactuated system.

In this paper, we propose the parameterization of all proper stabilizing IMC for unstable overactuated systems such that the internal model controller and the internal model control are proper. A modified IMC structure has been proposed to apply to unstable systems without losing the advantages of IMC characteristics. In addition, we present an application of the result for controller design for overactuated system with time delays. The organization of the article is presented as follows. In Section II, a proposal for a structure by internal model control for unstable overactuated systems is presented. In Section III, an internal model control that have been apply to control unstable overactuated system, the IMC illustrate the effectiveness of the proposed controller. Finally, a conclusion and a future work are given in Section IV.

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2. Structure of the internal model control for unstable overactuated system

Overactuated systems having more inputs than outputs, hence their transfer matrices are singular. However, the IMC requires that the synthesis of its controller be equal to the direct inverse of the system model. However, direct inversion is impractical [10], [11]. To remedy this problem, we use the IMC structure presented in figure 1, inspired by the work of [6], [7] [12]. The solution proposed in this case consists in making the model M(s) square then in removing the excess outputs which will appear at the outputs of the system G(s).

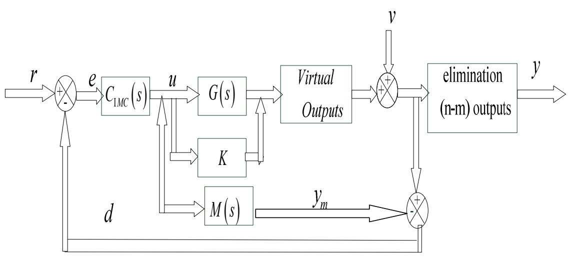

Fig. 1. Structure of the internal model control of unstable overactuated systems

where G(s) is the process transfer function, M(s) is the transfer function of the process model, $C_{IMC}(s)$ is the internal model controller, y is the controlled variable, r is the setpoint (reference), e is the error (offset), u is the control variable and v is the disturbance.

According to figure 1, the difference signal d between the output disturbed by the signal v of the process and that of the model is expressed by the following equation:

$$d=\left(G_{SVO}(s)-M(s)\right)u+v \tag{1}$$

where $G_{SVO}(s)$ represents the stabilized system G(s) having a square transfer matrix which is obtained using the technique of virtual outputs on the transfer matrix of G(s) which is singular [6], [12].

The control signal u is described as a function of the reference signal r and the disturbance signal v by this equation:

$$u(s)=\frac{C_{IMC}(s)}{I_m+C_{IMC}(s)\left(G_{SVO}(s)-M(s)\right)}r-\frac{C_{IMC}(s)}{I_m+C_{IMC}(s)\left(G_{SVO}(s)-M(s)\right)}v \tag{2}$$

The output vector expression is given there as follows:

$$y(s)=\frac{C_{IMC}(s)G(s)}{I_m+C_{IMC}(s)\left(G_{SVO}(s)-M(s)\right)}r+\frac{I_m-C_{IMC}(s)M(s)}{I_m+C_{IMC}(s)\left(G_{SVO}(s)-M(s)\right)}v \tag{3}$$

According to [6], when the plant G(s) is stable, the process model M(s) is stable, the controller $C_{IMC}(s)$ is stable then we can apply the IMC structure. However, is it possible to use the IMC structure given in figure 1 for unstable systems ? the answer is the following: we can use this structure on condition of stabilizing the systems with controllers.

Chen et al. give a solution to this problem and propose the parameterization of all proper stabilizing IMC for minimum-phase unstable system such that the IMC and the internal model controller are proper and showed that stabilizing controllers for SISO minimum-phase unstable system can be represented by IMC structure [13]. However, this method cannot apply for unstable overactuated system.

We propose in this paper, in the case of unstable overactuated system, a local loop pre-stabilization is necessary before applying the IMC [14]. The problem of stabilizing linear multivariable systems is solved by Lyapunov quadratic approaches using LMI tools. For this, the proposed structure for over-actuated multivariable continuous systems must be modified based on a stabilization technique using a stabilizing status feedback control by Lyapunov approach, shown in figure 2.

According to figure 2, the transfer matrix of the overactuated system G(s) is of dimension *(nxm)*, knowing *(n <m)* where *n* is the number of outputs and *m* is the number of inputs, it is described by the following expression:

$$G(s)=\begin{pmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1m}(s) \\ G_{21}(s) & G_{22}(s) & \dots & G_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1}(s) & G_{n2}(s) & \dots & G_{nm}(s) \end{pmatrix} \tag{4}$$

and the process model M(s) is given as follows:

$$M(s)=\begin{pmatrix} M_{11}(s) & M_{12}(s) & \cdots & M_{1m}(s) \\ M_{21}(s) & M_{22}(s) & \dots & M_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ M_{n1}(s) & M_{n2}(s) & \dots & M_{nm}(s) \end{pmatrix} \tag{5}$$

The matrix represented by the block named Virtual Outputs (VO) which aims to have an invertible square transfer M(s) matrix in order to perform the inversion operation to obtain the internal model controller. This transfer matrix $M_+(s)$ *((m-n), m)* is made up of first-order systems with fast dynamics and unit static gain or constants, it is given as follows [6], [7]:

$$M_{+}(s)=\begin{pmatrix} M_{n+1,1}(s) & M_{n+1,2}(s) & \cdots & M_{n+1,m}(s) \\ M_{n+2,1}(s) & M_{n+2,2}(s) & \cdots & M_{n+2,m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ M_{m1}(s) & M_{m2}(s) & \cdots & M_{mm}(s) \end{pmatrix} \quad (6)$$

The block called "elimination (m-n) outputs" its role is to remove excess (m-n) outputs using logical operators, as shown in the following figure 2 [7]:

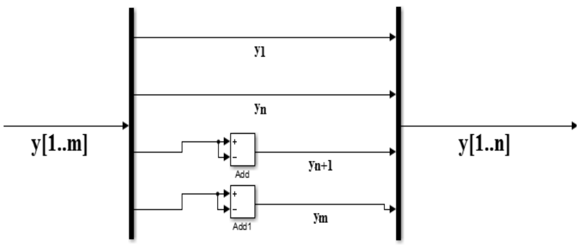

Fig. 2. Internal architecture of the elimination block (n-m) outputs

From figure 2, K is the process feedback gain, stabilizes the unstable process G(s). This state feedback looping establishes a control law u(t), its equation is defined as follows [15]:

$$u(t)=Kx(t) \quad (7)$$

The gain equation K is defined as follows [16], [17]:

$$K=RP^{-1} \quad (8)$$

knowing that P is a symmetric matrix. It is positive definite and a matrix R solutions of Linear Matrix Inequality (LMI) is describe by this equation [18], [19]:

$$AP+PA^{T}+BR+R^{T}B^{T}<0 \quad (9)$$

where $A\in\Re^{m\times n}$ and $B\in\Re^{n\times m}$ denote respectively the state and command matrices of the system to be controlled. The structure of the controller $C_{IMC}(s)$ is shown in figure 3 and its expression is given by equation (5) [2], [3]:

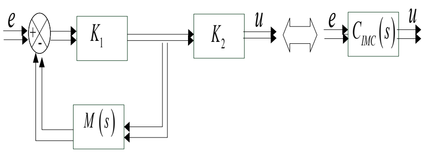

Fig. 3. Generalized structure of the IMC controller

$$C_{IMC}(s)=\frac{K_1K_2}{\mathrm{I_m}+K_1M(s)} \quad (10)$$

With $K_1=a\,\mathrm{I_m}, a\in R^{+}$ allowing to invert the process model M(s) and to ensure the stability of the controller. The value of a must be chosen sufficiently large, which allows the controller $C_{IMC}(s)$ to be approximated by the inverse of the model [6], [7].

The gain matrix K_2 makes it possible to compensate for the static errors of the system, its expression is described by the following equation:

$$K_2=\frac{\mathrm{I_m}+K_1M(0)}{K_1M(0)} \quad (11)$$

where *M (0)* is the matrix of static gains of the process model M(s).

3. Simulation Results

The system studied is an example from the literature and described by the transfer matrix given by equation (9), this system having three inputs and two outputs:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}=G(s)\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}=\begin{pmatrix} \frac{1e^{-0.8s}}{(2s+1)(4s-1)} & \frac{0.5e^{-0.8s}}{(4s-1)} & \frac{1e^{-s}}{(2s+1)(4s-1)} \\ \frac{3e^{-0.1s}}{(4s-1)} & \frac{1.5e^{-0.2s}}{(4s-1)} & \frac{1e^{-2s}}{(4s-1)} \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \quad (12)$$

The system G(s) is unstable, it admits four positive eigenvalues among these six eigenvalues $(\lambda_i)_{1\le i\le 6}$, which are as follows;

$\lambda_1=-1.33, \lambda_2=0.36, \lambda_3=0.25, \lambda_4=-1.36, \lambda_5=?.36, \lambda_6=?.25$

Using the LMI technique, the P and R matrices for stabilizing the feedback control are described by the following equations:

$$P=\begin{bmatrix} 3.97 & -1.58 & -1.39 & 0.08 & -0.11 & 0.92 \\ -1.58 & 2.96 & -1.90 & -0.11 & -0.24 & 1.27 \\ -1.39 & -1.90 & 5.43 & -0.46 & -0.63 & 0.26 \\ 0.08 & -0.11 & -0.46 & 3.74 & -1.27 & 0.31 \\ -0.11 & -0.24 & -0.63 & -1.27 & 3.62 & 0.42 \\ 0.92 & 1.27 & 0.20 & 0.31 & 0.42 & 5.60 \end{bmatrix} \quad (13)$$

$$R=\begin{bmatrix} 5.03 & -10.67 & 38.06 & -9.79 & 0.97 & 38.97 \\ -71.54 & 14.33 & -14.43 & -3.93 & 4.77 & -98.96 \\ 11.02 & 0.97 & 2.73 & 3.15 & -13.27 & 15.55 \end{bmatrix} \quad (14)$$

The state feedback gain matrix K is defined as follows:

$$K=\begin{bmatrix} -19.31 & -30.45 & -11.34 & -9.45 & -9.93 & 18.77 \\ -73.49 & -79.98 & -54.80 & -18.46 & -24.64 & 17.61 \\ 10.82 & 12.10 & 7.78 & 2.05 & -0.14 & -2.16 \end{bmatrix} \quad (15)$$

Three simulation scenarios are considered: the first consists in studying the nominal case without the presence of external disturbances, the second concerns the study of the robustness of the IMC with respect to external disturbances and the third corresponds to the study of the robustness of the IMC in the frequency domain.

- *Scenario 1: Nominal case*

In this scenario, we will consider the class of linear overactuated systems stabilized by LMI in the nominal case (in the absence of external disturbances). The values of the gains K_1 and K_2 are respectively given as follows:

$$K_1 = 0.1\, I_3 \quad , \quad K_2 = \begin{bmatrix} -30.03 & -16.02 & -3.22 \\ 103.51 & -201.41 & 13.19 \\ 31.04 & 3.23 & -0.65 \end{bmatrix}$$

Figures 4 shows the simulation results obtained from the closed-loop study system without IMC. The dynamics of the outputs $y_1(t)$ and $y_2(t)$ are stabilized, however unsatisfactory performance is obtained, on the one hand in transient state of slight oscillations at start-up, on the other hand, at steady state the outputs are not precise and they do not follow the instructions $r_1(t)$ and $r_2(t)$.

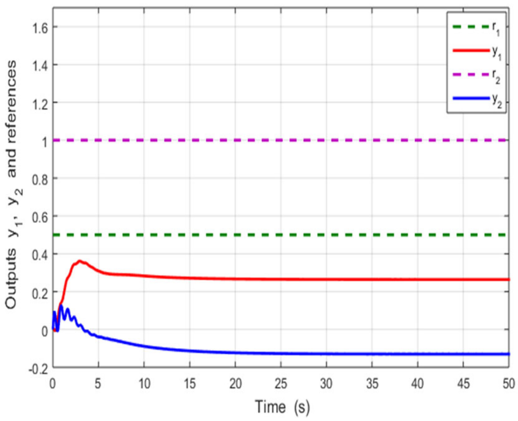

Fig. 4. : Evolution of the responses $y_1(t)$ and $y_2(t)$ of the stabilized closed loop system

Let us now consider the problem of following the trajectories of the system, we apply the internal model control using the controller $C_{IMC}(s)$. The numerical simulations of the different dynamics of the system are shown in figure 5, the desired set points $r_1(t)=0.5$ and $r_2(t)=1$, are followed by the outputs $y_1(t)$ and $y_2(t)$ with zero error in steady state.

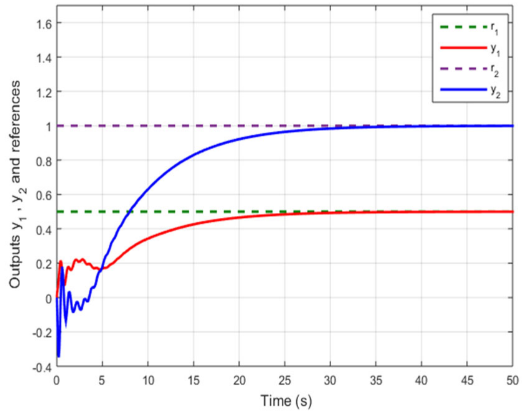

Fig. 5. Evolution of the responses $y_1(t)$ and $y_2(t)$ with IMC

Figure 6 illustrates the evolution of the control signals $u_1(t)$ and $u_2(t)$. We notice that the signals converge towards negative finite values as soon as the outputs of the system $y_1(t)$ and $y_2(t)$ follow their reference instructions.

Weak peaks at the level of control signals which appear at the initial instants due to the low value of has chosen it is equal to 0.1. If a increases the more the peaks increase. This is because the system at startup behaves like an open loop system subjected to a control vector $C_{IMC}(s)$ which is equal to a r(s).

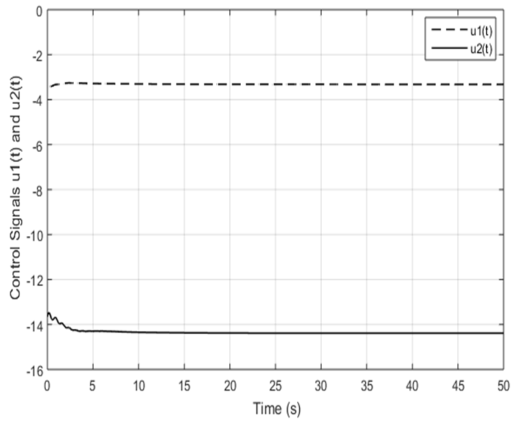

Fig. 6. Evolution of control signals $u_1(t)$ and $u_2(t)$

- *Scenario 2: Robustness study with respect to external disturbances*

The objective of this scenario is to test the robustness of the proposed internal model controller with respect to external disturbances.

We apply directly to the outputs of the system three disturbance signals of the echelon type of amplitude 0.05 from the instant 50s. The results of simulations of figures 7 and 8, show that the disturbances were quickly rejected proving the robustness of the internal model control with respect to the external disturbances of overactuated linear systems.

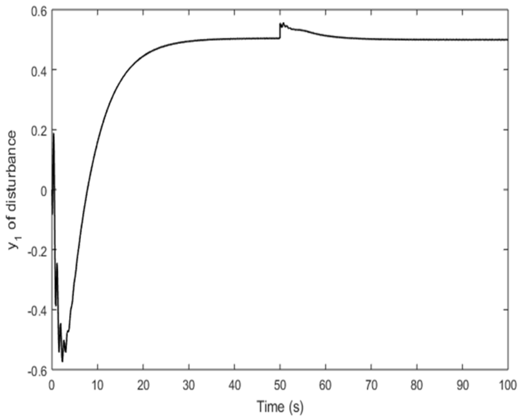

Fig. 7. Response of output $y_1(t)$ in the presence of disturbance

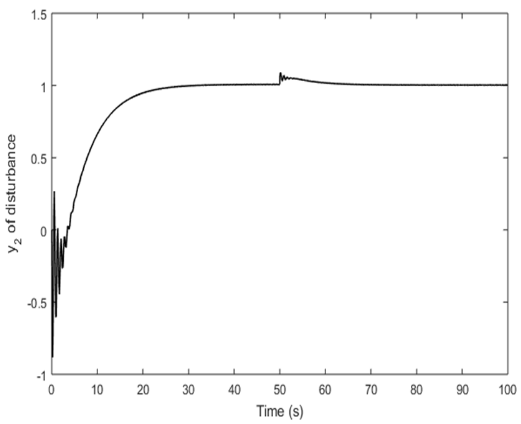

Fig. 8. Response of output $y_2(t)$ in the presence of disturbance

- *Scenario 3: Study of the Robustness of IMC in the frequency domain*

In this scenario, we propose to test the robustness of the internal model control. We apply two sinusoidal references signals $r_1(t)$ and $r_2(t)$ which are described by the following equation:

$$r_1(t) = 0.5\sin(2\pi f t) \quad where \quad f = 0.01 Hz$$
$$r_2(t) = \sin(2\pi f t) \tag{15}$$

Figures 9 and 10 show the evolutions of the sinusoidal responses of the system. We notice that the outputs $y_1(t)$ and $y_2(t)$, follow their references $r_1(t)$ and $r_2(t)$ in amplitudes and in frequencies which makes it possible to note that the IMC satisfies in this case the assigned objectives.

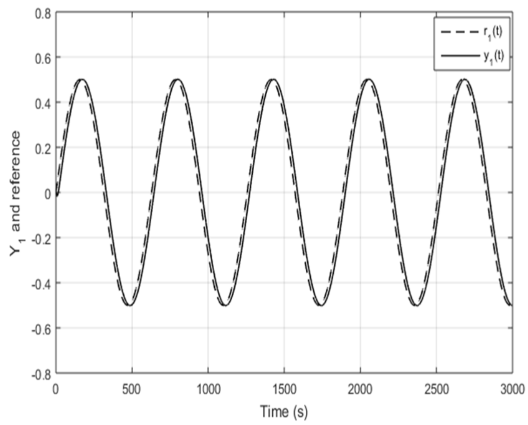

Fig. 9. Evolution of $y_1(t)$ for a sine set point with a frequency of 0.01Hz

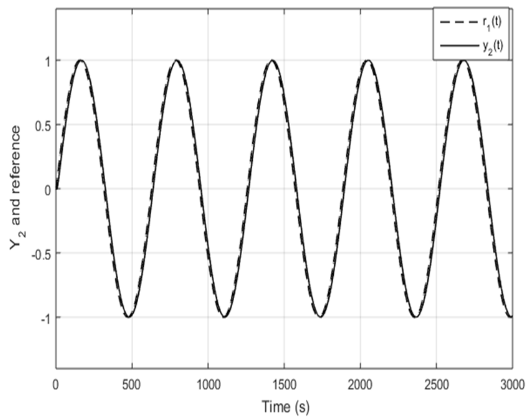

Fig. 10. Evolution of $y_2(t)$ for a sine set point with a frequency of 0.01Hz

From figures 9 and 10, one can notice the capacity of the proposed IMC structure to ensure the reconstitution of the reference signals.

4. Conclusion

For internal model control, inverting the model of a process to be controlled is the most difficult step to achieve. In this paper, we propose a new method of performing the inverse of the models to design a new structure of IMC for unstable overactuated systems whose transfer matrices are singular.

In this paper we have developed a new control approach by internal model for unstable overactuated systems whose transfer matrices are singular.

The new structure of IMC is proven by a simulation example through which we have shown its satisfactory performance as a guarantee of stability, precision as well as rapidity of system responses despite the presence of external disturbances and sinusoidal reference signals. hence the robustness of IMC.

In perspective, we plan to improve the developed inversion technique, in particular the influence of the criterion on robustness and servo performance. It would also be beneficial to develop the transient behavior according to the criterion used and to develop inversion strategies of fuzzy models of nonlinear overactuated multivariable systems.

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