Method And Mathematical Algorithm For Finding The Quasi-Optimal Purpose Plan

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Summary

A method and a mathematical algorithm for finding a quasi-optimal assignment plan with rectangular efficiency matrices are proposed. The developed algorithm can significantly reduce the time and computer memory consumption for its implementation in comparison with optimal methods.

Key words:
radio electronic means, mathematical algorithm, optimal methods, Quasi-Optimal Purpose Plan.

1. Introduction

Formulation of the problem. In [14], [16] the problem of finding the optimal assignment is formulated as follows: find a set of assignment parameters $\{x_{ij}\}$, i=1,2,…,m; j=1,2,…,n, maximizing performance indicator (PI)

$$M(X) = \sum_{j=1}^{n}\sum_{i=1}^{m} Q_{ij}x_{ij}\ , \qquad (1)$$

and satisfying the constraints

$$\sum_{j=1}^{n} x_{ij} = 1\ ; \qquad i = 1,2,...,m\ , \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} = 1\ ; \qquad j = 1,2,...,n\ , \qquad (3)$$

where $Q_{ij} = B_j P_{ij}, i = 1,2,...,m; j = 1,2,...,n;$

B_j – importance j work, $B_j>0$;
P_{ij} – performance efficiency j work i performer, $P_{ij}\geq 0$;
m ,n – the number of performers and works, respectively.

To find the optimal destination plan, as a rule, use the Hungarian method [1-10, 14], [11-16]. Using the Hungarian method, an optimal appointment plan is obtained that minimizes PI (1) with square efficiency matrix (EM) $\|Q_{ij}\|$, (m=n). Therefore, the maximization problem is previously transformed into a minimization problem; if a m≠n, then the efficiency matrix is expanded to square. For this purpose, fictitious work or fictitious performers may be introduced, for whom $Q_{ij} = 0$.

The algorithm for finding optimal designs is complex [15], [16]. Their implementation on a computer in real time requires a significant investment of time and memory.

The purpose of the article is to develop simpler to implement, but effective methods and algorithms for solving the assignment problem with rectangular efficiency matrices.

The problem statement of finding a quasi-optimal (close in efficiency to optimal) assignment plan is formulated as follows:

find a set of destination parameters $\{x_{ij}\}$, maximizing PI (objective function (OF)) (1), and satisfying the following systems of constraints:

Manuscript received February 5, 2021
Manuscript revised February 20, 2021
https://doi.org/10.22937/IJCSNS.2021.21.2.10

at m=n: (2),(3),(4);
at m<n: (2),(4),

$$\sum_{i=1}^{m} x_{ij} \le 1\,; \qquad j = 1,2,...,n\,, \tag{4}$$

where m>n: (3),

$$\sum_{j=1}^{n} x_{ij} \le 1\,; \qquad i = 1,2,...,m\,, \tag{5}$$

When describing the method and algorithm, we will use the individual results presented by the authors in [15], in relation to the assignment problem.

2. Theoretical Consideration

Method and algorithm for finding a quasi-optimal assignment plan. When finding an assignment plan using the proposed method, a preliminary stage and no more than (m-1) consecutively repeated iterations are performed. Let us describe the content of operations for the cases m=n; m<n, and then we will show the features of solving the problem for m>n.

At the preliminary stage, we find the maximum elements of the rows of the efficiency matrix.

$$Q_{ik} = \max_{1 \le j \le n} Q_{ij}; i = 1,2,...,m\,, \tag{6}$$

where k - the number of the column to which the maximum element belongs i strings.

The elements

$$Q_{ik} \ge Q_{ij}; j = 1,2,...n; i = 1,2,...,m; j \ne k; k \in N, \tag{7}$$

where N – set of column numbers.

For clarity, mark the maximum line elements with the sign (*).If the number of columns with elements Q^*_{ij}, which we denote "b", will be equal to the threshold value

$$b = b_п\,, \tag{8}$$

where

$$b_п = \min(m, n)\,, \tag{9}$$

then the destination parameters, the coordinates of which correspond to the coordinates Q^*_{ij}, will be equal to one. For destination parameters, constraints (2), (3), (4) are satisfied. Therefore, for m≤n the solution in this case was found at the preliminary stage, and the found destination plan $x^*_{ij} = 1$ will be optimal [2], which can be confirmed by equivalent transformations. Found assignment plan maximizes PI (1), since condition (7) is satisfied. If the number of columns "b" with elements Q^*_{ij} less than the threshold value (8) (for m≤n magnitude $b_п = m$ (9)), then some columns will contain multiple elements Q^*_{ij}, individual columns will be left without such elements. Therefore, it is necessary to perform no more than (m-1) iterations, at which the maximum elements of the rows are replaced or equal to them (the condition $Q_{ik} = Q_{ij}, k \ne j$ (7)), or the closest string elements.

To find the coordinates of an element Q_{i0j0}, which will replace Q_{ik}, use the value δ_{i0j0}, calculated according to the following rule:

$$\delta_{i0j0} = \min_{k \in L_2}\{\min_{i \in R_k}\{\min_{l \in L_0}\{Q_{ik} - Q_{il}\}\}\}\,, \tag{10}$$

where Q_{ik} - maximum element i strings (6), located in k-м column set L_2;

l – set column numbers L_0;

L_2 – many columns containing more than one item Q_{ik}, $k \in L_2$ (number of elements in k-м column denote q_k, columns set L_2 mark (+));

R_k – the set of row numbers, the maximum elements of which are in the columns of the set L_2 ($k \in L_2$) (row set row set R_k mark (+));

L_0 – set of columns without maximum row elements (columns of set L_0 mark (-));

i_0, j_0 – item coordinates Q_{ij} closest in size to the maximum element i_0 strings.

The elements Q_{i0j0} highlight with signs: (*)at $\delta_{i0j0}=0$, (⊗)at $\delta_{i0j0} \ne 0$. Element highlighted ⊗, will be called a quasi-maximal element i_0 strings.

Operation (10) is performed only for those columns containing at least two elements Q^*_{ij} (L_2), if there are column numbers of the set L_0.

At the preliminary stage or during the execution of iterations from the distribution process, we will exclude the columns (and, accordingly, rows), in which there is one element marked with (*) or (⊗).Such columns form the set L_1.

If a $b=b_п$ at m≤n, then the iteration process ends. In this case, the set L_2 will be empty, and the number of columns with maximum and quasi-maximum elements will become m. At m=n, L_0 will be empty, and at m<n lots of L_0 can contain multiple columns. Using the coordinates of the maximum and quasi-maximum elements, we determine the coordinates of the destination plan $x^*_{ij}=1$, i=1,2,…,m, $j\in L_1$.

At m>n magnitude b_π=n . If b=n, then there will be several elements in separate columns Q^*_{ij}. Therefore, all restrictions (3) will not be met. It is necessary to stop the execution of iterations and achieve the fulfillment of constraints (3) at the final stage of the formation of the assignment plan. For all columns of the set L_2 from the maximum elements of the rows, select the largest value

$$Q^*_{i0k}=\max_{i\in R_k} Q^*_{ik}; k\in L_2 . \quad (11)$$

For all other elements of these columns we erase the selection marks, form an assignment plan, if necessary, form a set of "free" work performers and a set of work for which performers are not assigned.

At m>n striving for a better appointment plan through expansion to square and use the method in full at m=n does not lead to a positive effect, since the results are identical[5].

Using the description of the algorithm to illustrate the method for m<n and m>n, let's give examples.

Find a quasi-optimal assignment plan that maximizes PI (1), satisfying constraints (2), (3), (4) if the efficiency matrix has the form

$$Q=\begin{Vmatrix} 0{,}55 & 0{,}75 & 0{,}80 & 0{,}70 & 0{,}10 \\ 0{,}30 & 0{,}35 & 0{,}40 & 0 & 0{,}20 \\ 0{,}20 & 0{,}50 & 0{,}55 & 0 & 0{,}10 \\ 0{,}30 & 0{,}15 & 0{,}60 & 0{,}40 & 0{,}15 \end{Vmatrix} . \quad (12)$$

Decision. After completing the preliminary stage, we get

$$Q=\begin{array}{c} \begin{array}{ccccc} - & - & + & - & - \end{array} \\ \begin{Vmatrix} 0{,}55 & 0{,}75 & 0{,}80^* & 0{,}70 & 0{,}10 \\ 0{,}30 & 0{,}35 & 0{,}40^* & 0 & 0{,}20 \\ 0{,}20 & 0{,}50 & 0{,}55^* & 0 & 0{,}10 \\ 0{,}30 & 0{,}15 & 0{,}60^* & 0{,}40 & 0{,}15 \end{Vmatrix} \end{array} \begin{array}{c} \\ + \\ + \\ + \\ + \end{array} ; \quad (13)$$

L_0={1;2;4;5}; L_1=0; L_2={3}; R_3={1;2;3;4}; b_π=min{4;5}=4;b=1.

Because $b<b_\pi$, then after performing three iterations (blocks 3;4;…;12), get δ_{12}=0,05;$Q^{\otimes}_{12}$=0,75; δ_{21}=0,10;$Q^{\otimes}_{21}$=0,30; δ_{44}=0,20;$Q^{\otimes}_{44}$=0,40;L_1={1;2;3;4}; L_0={5};L_2=0;b=4;b=b_π.

Matrix (13) takes the form

$$Q=\begin{Vmatrix} 0{,}55 & 0{,}75^{\otimes} & 0{,}80 & 0{,}70 & 0{,}10 \\ 0{,}30^{\otimes} & 0{,}35 & 0{,}40 & 0 & 0{,}20 \\ 0{,}20 & 0{,}50 & 0{,}55^* & 0 & 0{,}10 \\ 0{,}30 & 0{,}15 & 0{,}60 & 0{,}40^{\otimes} & 0{,}15 \end{Vmatrix} . \quad (14)$$

By the coordinates of the matrix elements (14), marked by signs (*) and (⊗),define a quasi-optimal assignment plan x^*_{12}=1; x^*_{21}=1; x^*_{33}=1; x^*_{44}=1; and calculate the value PI (1)

$M(x_{k0})$=0,75+0,30+0,55+0,40=2,00.

Because m<n, then performers will not be assigned to the fifth job.

Using the optimal methods [2], [3], we get

$$X_o=\begin{Vmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{Vmatrix} .$$

PI (1) for an optimal assignment plan will be $M(X_o)$=0,70+0,30+0,50+0,60=2,10.

The PI value for a quasi-optimal assignment plan is slightly less than for an optimal assignment plan.

Example. Solve the problem if m>n and has the form

$$Q=\begin{Vmatrix} 9 & 6 & 5 & 8 \\ 4 & 8 & 6 & 2 \\ 6 & 7 & 9 & 4 \\ 2 & 7 & 3 & 1 \\ 1 & 1 & 1 & 1 \end{Vmatrix} . \quad (15)$$

In this case, the assignment plan must satisfy constraints (3), (4), (5).

Decision. After completing the preliminary stage, we get

$$Q=\begin{array}{c} \begin{array}{cccc} + & + & & - \end{array} \\ \begin{Vmatrix} 9^* & 6 & 5 & 8 \\ 4 & 8^* & 6 & 2 \\ 6 & 7 & 9^* & 4 \\ 2 & 7^* & 3 & 1 \\ 1^* & 1 & 1 & 1 \end{Vmatrix} \end{array} \begin{array}{c} \\ + \\ + \\ \\ + \\ + \end{array} ; \quad (16)$$

L_0={4}; L_1={3}; L_2={1;2}; R_1={1;5}; R_2={2;4}; b_π=min(5;4)=4;b=3.

Because $b<b_{\text{п}}$, then after performing one iteration (blocks 3;4;…;12), get $\delta_{54}=0; Q^*_{54}=1$; $L_1=\{1;3;4\}$; $L_0=0; L_2=\{2\}$.
Matrix (16) takes the form

$$Q=\begin{Vmatrix} 9^* & 6 & 5 & 8 \\ 4 & 8^* & 6 & 2 \\ 6 & 7 & 9^* & 4 \\ 2 & 7^* & 3 & 1 \\ 1 & 1 & 1 & 1^* \end{Vmatrix}. \qquad (17)$$

Because $b=b_{\text{п}}$, $m>n$, then we find the largest element among the maximum elements of the second column. (17) will take the form:

$$Q=\begin{Vmatrix} 9^* & 6 & 5 & 8 \\ 4 & 8^* & 6 & 2 \\ 6 & 7 & 9^* & 4 \\ 2 & 7 & 3 & 1 \\ 1 & 1 & 1 & 1^* \end{Vmatrix}, \qquad (18)$$

Quasi-optimal assignment plan corresponding (18) $x^*_{11}=1$; $x^*_{22}=1$; $x^*_{33}=1$; $x^*_{54}=1$; the fourth performer is not assigned to any of the jobs. The PI value will be
$M(x_{k0})=9+8+9+1=27$.
Using the optimal method, we get

$$X_o=\begin{Vmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Vmatrix}.$$

The PI value for the optimal assignment plan will be $M(X_o)=8+4+9+7=28$.
As before, the condition $M(X_{k0})<M(X_o)$.
Thus, PI values for quasi-optimal and optimal assignment plans, as a rule, will differ slightly from each other. In some cases, they will match.

Conclusions

The algorithm is distinguished by the simplicity of finding the elements of the efficiency matrix that are closest in magnitude to the maximum element of the row. It consists of a preliminary stage, a limited number of sequentially repeated iterations, and the final stage of forming an assignment plan. In the proposed method, the maximization problem is not preliminarily transformed into a minimization problem, and the rectangular efficiency matrix is not expanded to a square one. The algorithm can be used in the allocation of funds if the mathematical formulation of the problem coincides with that stated in the article.

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