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Determinants of Pakistan Stock Exchange (PSX) Index Under Optimal Conditions: A Factor Analysis Approach*

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Abstract

Macroeconomic variables play a significant role in the progress of the emerging economy. In this context this study attempts to obtain the optimal solution of Pakistan Stock Exchange (PSX) by utilizing M2, Gold Prices (per troy ounce) in US dollars, Real Effective Exchange Rate Index, 6 Month Treasury bill, Call Money Rate, Foreign Exchange, Net Foreign Assets, Food Price Inflation, Manufacturing Production and Balance of Trade as principal macroeconomic indicators. To achieve the optimal solution monthly data from June 2006 till November 2016 has been deployed. Exploratory factor analysis (EFA) has been used to extract the factors and these extracted factors were then used in a bi-objective goal programming model to obtain the optimal solution. Additionally, multiple regression analysis has been done to find the relationship of the extracted factors with PSX, and model reliability has been accessed through the coefficient of determination. Results retrieved from EFA suggest two significant factors at a threshold of 0.4. The multiple regression models formed from extracted factors explain more than 90% variation of the factors showing a reliable model. At the last stage, Bi-objective goal programming has been employed to get the optimal solution. It has been observed that extracted factors have successfully achieved the required goals and obtained satisfactory optimal solutions.

Keywords: Multicollinearity, Pakistan Stock Exchange, Factor Analysis, Bi-objective Goal Programming, Optimal Solution

JEL Classification Code: E41, E51, C61, C44

1. Introduction

Understanding the increasing importance of predictions regarding the equity markets of different countries, this paper attempts to find the optimal solution of the principal macroeconomic factors that may affect the Pakistan stock exchange (PSX). Since policy implementation for liberalization in 1991, the stock prices in Pakistan have increased. The efficient-market hypothesis is a hypothesis in financial economics that states that stock prices reflect all available information. As a result of this, stock prices may capture the future performances of corporate sectors. According to Goh et al. (2003), stock prices are fully reflecting the current position of macroeconomics variables. Thus, it can be used as a significant tool to access economic activities. Therefore, the relationship between stock prices and macroeconomic variables can help policymakers to design policies regarding various economic decisions. A well-performing stock market is a source of attraction for investors. Moreover, the stock market helps to regulate economic activity through growth and savings. According to Chen et al. (1986), the macroeconomic factors affect the stock market behavior of the country. Abundant literature has been found regarding the forecasting of equity markets and identification of effective determinants of macroeconomic variables to explore the relationship of stock market returns with macroeconomic variables. Thus, this study attempts to investigate the optimal solution for emerging economies by considering the factor analysis approach. The proposed research is novel in the context as it utilizes extracted factors under the framework of weighted goal programming.

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Pakistan stock market is considered to be a volatile market as asymmetric behavior is depicted in its returns. The volatile market seems to be at higher risk in certain cases (Nguyen & Nguyen, 2019). The market was formally divided into three exchanges namely Karachi, Lahore, and Islamabad stock exchanges till 2015. Among the three, Karachi stock Exchange (KSE) is considered the best performing market since its development. It has been an honor that KSE was globally rewarded by the title of best performing market in 2002. During 2003 and 2004 it was the third-largest equity market in the world. The market capitalization of KSE was 2.94 trillion in 2010 reflecting the rising trend of growth (Iqbal, 2008). Nowadays it is known as Pakistan Stock Exchange (PSX) as three exchanges have been merged. Under the umbrella of PSX, 11 different indexes are working. This study took KSE-100 index points as the top 100 financial and non-financial companies are listed there.

The study considered M2, Gold Prices (per troy ounce) in US dollars, Real Effective Exchange Rate Index, 6 Month Treasury bill, Call Money rate, Foreign Exchange, Net Foreign Assets, Food Price Inflation, Manufacturing Production and Balance of Trade. The data set has been retrieved from the IFS database. Understanding the importance of selected macro-economic variables, a brief explanation of each variable has been presented.

Real effective exchange rate measures the status of any country currency with respect to the group of major currencies. The trading abilities of any country can also be analyzed by accessing its real effective exchange rate. Thus, this variable plays an important role in economic decision-making for developing countries. T-bills are considered short-term money market instruments issued by the government through primary and secondary financial markets. These bills are issued to generate the funds for a short period of time and they are utilized by the government. Balance of trade is the difference between imports and exports of any country. To get revenue, a country should increase its exports and try to minimize the imports. Gold prices have been considered the safest investment avenue for the last few years. The metal market is getting attention from investors after the global financial crisis. The downfall of stock prices leads the investors toward gold investment. The gold market may provide an attractive and alternative investment horizon for investors. The variation in the consumer price index is simply the indicator of inflation for any country.

The cost of living increases due to the high rate of inflation. The increasing rate of inflation results in an in the demand for market instruments as the demand for market instruments reduces the volume of the equity market. Thus inflation plays an important role in the development of the equity market in any country, Foreign exchange rate plays an important role for developing economies like Pakistan as the devaluation of the currency in international markets leads toward expensive imports. The increasing import causes lower profits and the stock prices of the local companies. Thus to gain investor confidence towards the equity market, currency decline must control against the US dollar. M2 has been taken as an indicator of the money supply of Pakistan. Money supply and stock prices are considered to be directly proportional to each other. On the other hand increase in money supply results in a rise in the rate of inflation (Ngoc, 2020). Manufacturing products can be used to access the growth rate of any country. The increase in manufacturing production results in an increase in the industrial sector that provides profit and flourishes the industrial companies. Due to this reason, share prices of local companies rise. Thus manufacturing production measures the overall economic activity of the country. According to the World Bank, net foreign assets is the sum of foreign assets held by monetary authorities and deposit money banks, less their foreign liabilities. Since the interest rate is considered as a value or cost (Mahmoud, 2018), call money rate is regarded as a proxy of the interest rate at which short-term funds are borrowed and lent in the money market. It provides short-term liquidity to financial markets.

This is one of the unique studies in the scenario of an emerging economy like Pakistan as it first employed exploratory factor analysis to extract the reliable factors afterwards weighted goal programming model is utilized to find the optimal value of the PSX.

2. Literature Review

In the earlier studies various econometric and financial methods have been used to predict or forecast decisions regarding stock markets. We will give a brief review of some related literature here.

Nishat and Shaheen (2004) attempted to find the long-run relationship between the Karachi stock exchange with the inflation rate, output growth, money supply, and interest rate. They took quarterly time series from 1973–2002. They employed a vector error correction model to the said variables after identifying the co-integration among the selected variables. They also explained the effect of macro-economic variables by utilizing the impulse response and variance decomposition techniques. They found that all variables are cointegrated and two equilibrium relationships are established among these variables. Furthermore, they concluded that there is a causal relationship between the Karachi stock market and the economy.

Ray (2012) explored the effect of macroeconomic variables on the stock prices in India. He took annual time series of consumer price index, gold prices, industrial production index, oil prices, foreign exchange rate, money supply, call money rate as a proxy of interest rate, gross fixed

capital formation, foreign direct investment, wholesale price index, the balance of trade and foreign exchange reserves for India. He confirmed the asymmetry behavior of the selected variables and performed a unit root test to identify the stationarity properties of all variables. The multivariate Granger causality test has been employed to observe the effect of macroeconomic variables on stock returns. Results reveal that there is no causal association between stock prices and interest rate, stock market, and industrial production.

Muhammad et al. (2009) took the quarterly data set of the foreign exchange rate, foreign exchange reserves, gross fixed capital formation, money supply, interest rate, industrial production, and wholesale price index. They took monthly time series of said variables spanning from 1987 to 2007. They employed AR (2) and MA (1) by taking the Karachi stock exchange as the dependent variable. They conclude that the exchange rate and exchange reserve highly affect the stock prices.

Ouma and Murio (2014) studied the impact of macro-economic variables on the stock exchange of Kenya. They used monthly data set spanning from January 2003 till December2013. Using the Arbitrage Pricing Theory and Capital Asset Pricing Model they found a significant relationship between stock market returns and macroeconomic variables. Money supply, exchange rates, and inflation are found to be significant.

Farooq and Keung (2004) explored the relationship between stock prices and exchange rate by decomposing Karachi Stock Exchange Index into three sub-sectors. The results indicated the absence of log run cointegration between Pakistan stock exchange and exchange rate. Thus, they performed a granger non-causality test and concluded that there is a shot-run causality present between the general stock index and the exchange rate of Pakistan.

Mortimer (2012) used factor analysis in his paper to segment primary male grocery shoppers based on store and product quality assessments. He used three stages methodological approach for his research. Factor analysis with varimax rotation was applied to extract the factors. 8 factors were retained enabling the identification of specific male shopper types using cluster analysis. He explored a new shopper type which is one of the unique and not found earlier in research.

Makka and Sockel (2001) had investigated the underlying dimensions of the motivation and retention constructs in their article. They applied confirmatory factor analysis on a sample of 118 individual groups of professionals to analyze complex IS constructs and to investigate whether job satisfaction, perception of management on career development, loyalty, burnout, and turnover intent are indicator variables or not to the latent constructs of motivation and retention. The study concluded that job satisfaction and perception of management on the development of career are significant factors for the motivation construct, loyalty, and burnout while turnover intent is the indicators for the retention construct.

Dan (2013) employed a weighted goal programming method to allocate the budget of an institute of higher learning. They took the data set of a Nigerian university. They constructed five goals that need to be satisfied. First to raise personnel cost, second to reduce the overhead cost and third to raise capital expenditure, fourth raise revenue, and fifth is to reduce the total budget. The study concluded that the optimal value of the objective function is not zero, thus all goals are not satisfied. Since the deviational variables respective to achieve goals are found to be zero and non-achieved goals are not equal to zero, the final value of the objective function was 4.24.

The above literature highlighted the importance of macroeconomic variables to predict the stock market index of various countries. It has been observed that numerous studies focused on the econometrics and time-series approach to predict the stock market returns while this study applied a novel approach as it first extracts principal factors from several macroeconomic variables by employing EFA then utilizing the factors in form of weighted goals, formulate bi objective goal programming model to obtain the optimal solution for PSX.

3. Methodology

3.1. Exploratory Factor Analysis (EFA)

Exploratory factor analysis is a statistical technique that is used to reduce data to a smaller set of summary variables and to explore the underlying theoretical structure of the phenomena. It is used to identify the structure of the relationship between the variable and the respondent. The factor analysis can be expressed in terms of a model with assumptions;

$$(\underset{\sim}{X} - \underset{\sim}{\mu}) = \gamma f + \in$$

Where $\underset{\sim}{X} = (x_1, x_2, \ldots, x_p)$, $\underset{\sim}{f} = (f_1, f_2, \ldots, f_m)'$ and γ are the factor loadings.

Assuming a random sample of size m $x_1, x_2, \ldots, x_m$ and p number of variables ($m < p$) belongs to a multivariate normal distribution with mean μ and variance-covariance matrix Σ. Similarly $(\underset{\sim}{X} - \underset{\sim}{\mu})$ follows multivariate normal distribution having mean 0 and variance-covariance matrix Σ. Moreover, we assume $E(\in_i) = 0$ and the factors and errors are assumed to be uncorrelated.

Usually, the scree plot is used to determine the number of factors to retain in exploratory factor analysis or principal components to keep in principal component analysis. The idea of scree plot has been given by Cattell (1966). A scree plot is a line plot of the eigenvalues of factors or principal

components in analysis. The factors with high eigenvalues are relatively few as compared to factors with low eigenvalues. The number of extracted factors can be judged from the point where the slope of the line changes. Kaiser (1960) suggested that all factors having eigenvalues greater than 1 should be taken, as eigenvalues capture the amount of variation suggested by the factors. One characteristic of factor analysis is that it can estimate a variable's score on a factor based on their scores for their constituent variables. Factor scores are composite measures that can be computed for each subject on each factor. Factors are the variables that experimenters control during an experiment to determine their effect on the response variable. The main use of this technique is to reduce a data set to a more meaningful manner.

3.2. Goal Programming

The idea of goal programming was formally introduced by Charnes and Cooper (1961). They claimed that the idea of goal programming was originated from the efforts of Charnes and Cooper (1952). Later on, Contini (1968) proposed a stochastic goal programming model, Keown (1978) incorporated the chance-constrained for stochastic goal program in non-linear programming framework, and Kalpic et al. (1995) employed a linear goal programming technique for planning objectives. Nowadays goal programming is appearing as a useful technique among policymakers.

In goal programming, the decision-maker is the person who is responsible for the decision to the undertaking problem. Decision variables are all those factors over which decision maker has control and these decision variables must be non-negative in the optimization problem. For a proposed problem, the goal programming paradigm evaluates the solution quality. The problem may have a single criterion or multi-criterion. The decision-maker sets the objective which needs to be minimized or maximized. The goals must be set by the decision-maker according to the particular target level. The weights are assigned to positive and negative deviational variables according to the priority levels. The sum of these deviational variables is combined to form a linear goal program. The decision-maker needs to minimize these unwanted deviational variables.

The basic idea behind the formulation of goal programming is to define and assign different goals to each objective function in such a manner that the solution minimizes the sum of deviations of that particular objective function. Let assume; $y_1, y_2, \ldots, y_p$ are the decision variables.

S be the number of objectives need to be minimized.

G_s goals of the objective functions.

α_{js} are the coefficients of decision variables.

Practically the achievement of all goals is not always attainable, thus we define an objective function like;

Minimize

$$Z = \sum_{s=1}^{s} \left| \left(\sum_{i=1}^{p} \alpha_{is} y_p - G_s \right) \right| \tag{1}$$

Introducing deviational variables in terms of negative and positive parts to find the solution to the above objective function.

$$D_i = \left(\sum_{i=1}^{p} \alpha_{is} y_p - G_s \right) \tag{2}$$

Thus (1) can be shown as

Minimize

$$Z = \sum_{s=1}^{s} \left| (D_i) \right| \tag{3}$$

We can also assign different weights to make some goals important than the others, thus a weighted goal program can be written as;

Minimize $$Z = \sum_{s=1}^{s} (W_s^+ S_s + W_s^- p_s)$$

Under the constraints $$\sum_{i=1}^{p} \alpha_{is} y_p - (S_s - p_s) = G_s$$

The analysis has been done on LINGO 17.0 version.

4. Estimation Results

This section explains the estimated results obtained by EFA and weighted goal programming.

4.1. Data Description and Summary Statistics

We took monthly data of Pakistan stock exchange along with different macro-economic variables of Pakistan spanning from June 2006 till November 2016. Table 1 presents the summary statistics of all the under study macro-economic variables. To determine the statistical behavior of variables, we determine the mean, median, maximum, minimum, and standard deviation of the variables. The standard deviation of Balance of trade, Foreign exchange rate, and M2 is quite high showing the high amount of variation or dispersion is associated with these variables.

Table 2 present the correlations between the selected macro-economic variables with the PSX and individual collinearity diagnostic measures. The correlation between Balance of trade, Foreign exchange, Gold Prices, Manufacturing Production, and Real Effective Exchange Rate with the PSX exhibits a positive sign. While remaining variables have a negative relationship with the PSX.

Table 1: Summary Statistics of Macro-Economic Variables of Pakistan

Variables	Mean	Median	Maximum	Minimum	Std. Dev
Real Effective Exchange Rate	104.444	101.975	125.31	90.3	9.084
6 month treasury bill rate	10.139	9.92205	14.01	5.8214	2.326
Balance of Trade	141671.1	138390	289622	51450	54849.18
Gold Prices	1176.543	1210.62	1772.14	585.78	323.925
Consumer Price Index	9.805	8.83	25.33	1.32	5.406
Foreign Exchange Rate	11427.33	11278	19874	3532.17	3585.575
Money Supply (M2)	7130.188	6546.465	13157.3	3366.71	2767.114
Manufacturing Production	106.840	104.2975	143.926	88.7171	11.771
Net Foreign Assets	208.946	216.14	317.789	68.4705	53.297
Call Money Rate	9.893	9.6667	20.03	5.451	2.404

Table 2: Correlation Between PSE and Macro-Economic Variables with Collinearity Diagnostics

Variables	Correlations	Tolerance	VIF
Call money rate	–0.69092	0.141	7.087
6 month treasury bill rate	–0.7283	0.093	10.698
Balance of Trade	0.699207	0.246	4.066
Consumer Price Index	–0.568842	0.322	3.101
Foreign Exchange Rate	0.354555	0.235	4.252
Gold Prices	0.108601	0.463	2.16
Money Supply (M2)	0.881744	0.056	17.893
Manufacturing Production	0.61297	0.543	1.841
Net foreign assets	–0.437799	0.377	2.649
Real Effective Exchange Rate	0.896578	0.075	13.394

The variance inflation factor values show multicollinearity problems among the explanatory variables. To overcome the problem of multicollinearity factor analysis has been used.

Figure 1 presents the time series plot of the macro-economic variables under study. Each variable has plotted against time. The left panel of the figure presents the time series plot for the real effective exchange rate, Balance of trade, Inflation, money supply, and net foreign assets. While the right panel presents the time series plot of the remaining variables.

The real effective exchange rate shows an upward trend with respect to time. The sharp decline in 2008 and middle of 2009 and 2010 can easily be evident from the graph. After 2013, the real effective exchange rate increases till the end of 2016. The balance of trade of Pakistan during the whole time period shows random behavior as the imports and exports are not balanced. The consumer price index shows a sharp decline after the Asian financial crisis of 2008. Its trend behavior is downward indicating that purchasing power of consumers is increasing and this trend remains the same till the end of 2016. M2 shows an upward trend throughout the time period while net foreign assets show random behavior. Treasury bill rate shows sharp decline after the Asian financial crisis of 2008. Gold prices also show an upward trend from the start of time period till 2011. After 2012, its prices come down in the international market and become stable till the end of 2016. The foreign exchange shows a cyclical behavior and manufacturing production exhibits periodic movements that may be seasonal in Pakistan respectively. Call money rate shows decline after a sharp increase around 2008. This downward movement remains intact till the end of 2016.

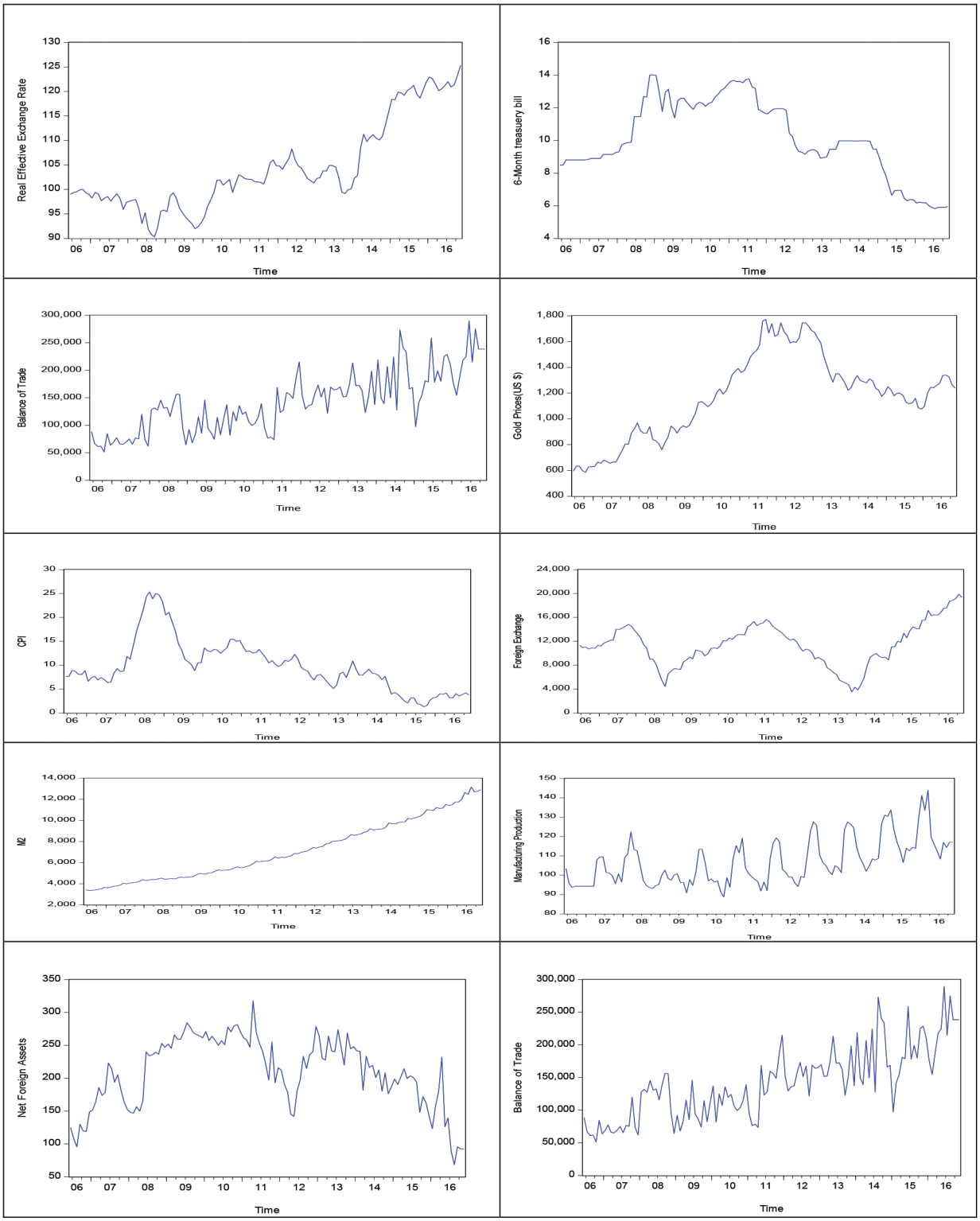

Figure 1: Time Series Plot of Macro-Economic Variables

4.2. Exploratory Factor Analysis (EFA)

To find the macro-economic factors, we employed the technique of factor analysis. Since it has been noted that Variance Inflation factors (VIFs) of several macro-economic variables indicate the problem of multicollinearity, factor analysis is the most appropriate method. Moreover, we also find the measure of sample adequacy i.e. Kaiser (1974) and Bartlett's test to test whether the original correlation matrix is an identity matrix or not. Table 3 present the results of both tests. Kaiser (1974) suggests that the value 0.785 is a good value and we can move forward for factor analysis as the sample size is adequate. Bartlett's test also shows a highly significant value i.e. $p < 0.001$and therefore factor analysis is appropriate for this case.

Table 3 presents the results for extracted unrotated and rotated factors. It has been noted from the estimated results that most of the variables are highly loaded on the first factor before rotation. After applying the Varimax rotation the structure of factors gets changed and easy to interpret. Roll and Ross (1980) highlighted that generally three to five factors are extracted to identify the macroeconomic relationship. We extracted two factors for our data set. The first factor comprises four variables namely 6 months treasury bill rate, Call money rate, net foreign assets, and inflation. Similarly, the second factor comprises M2, gold prices, and balance of trade. Moreover, a real effective exchange rate has not been added as it takes a value greater than 0.5 for both factors and assuming that along with two factors it significantly affecting the PSX.

Taking the threshold of positive 0.4, the study eliminated factor loadings less than 0.4, and if the same variable holds in both factors we consider a high loaded variable in the particular factor. After extracting the meaningful variables in terms of factor, we applied the method of least squares and run the following regression models;

$$f_1 = \alpha_0 + \alpha_1 6 \text{ month tresury bill} + \alpha_2 \text{ Call money rate} + \alpha_3 \text{ Net foreign assets} + \alpha_4 \text{ Consumer Price Index} + \varepsilon_0 \quad (4)$$

$$f_2 = \beta_0 + \beta_1 \text{ Money supply} + \beta_2 \text{ Gold prices} + \beta_3 \text{ Balance of trade} + \varepsilon_1 \quad (5)$$

Where f_1 and f_2 are the factor scores and act as dependent variables while right-hand side variables are explanatory macroeconomic variables extracted from high factor loadings. For the feasibility of interpretation, we named our factors. Since factor 1 comprises variables that might reflect the basis of prices, we named it as a price-based factor while factor 2 is based on assets comprising variables, thus we named it as an asset-based factor. Since the goal programming model allows us to comprise different goals into a single objective function, we estimate the following model;

$$\text{PSX} = \gamma_0 + \gamma_1 f_1 + \gamma_2 f_2 + \gamma_3 \text{ REER} + \varepsilon_2 \quad (6)$$

Where PSX is Pakistan stock exchange Index, f_1 and f_2 are the mined factors, and REER presents the real effective exchange rate of Pakistan. Table 4 shows the estimated results of the above least square models. Model (4) was found to be significant for all variables collectively representing the first factor. Moreover, 93.3% of the variation in the dependent variable has been explained by the model. These estimates will further be used to obtain the optimum value for the first factor which acts as a first prescribed goal (needs to be achieved). Similarly, estimates of the Model (5) are also significant at a 5% level of significance and overall 95% of the variation has been explained from the second factor. Hence it will be utilized as the second goal.

The estimated model (6) shows that factor 1 is negatively related to the PSX while factor 2 possesses a positive relationship with PSX. The real effective exchange rate shows a positive and significant effect on the PSX. This model will further be used to make the bi-objective goal programming model by taking both factors as goals to find the optimal value of the PSX. We develop the stated problem in terms of bi-objective goal programming in the next section.

Table 3: Extracted Factors Through Varimax Factor Rotation and Sample Adequacy Test Statistic (*P*-value)

Variables	f_1	f_2
Money	–	0.860
Gold Prices	–	0.824
Real Effective Exchange Rate	–	–
6-months treasury bill rate	0.913	–
Call money rate	0.856	–
Foreign Exchange Rate	–	–
Net foreign assets	0.830	–
Consumer Price Index	0.741	–
Manufacturing Production	–	–
Balance of Trade	–	0.809
KMO	0.785	
Bartlett's	1210.402 (0.00)	

4.3. Goal Programming Model

In goal programming formulation, we need to add negative and positive deviation variables. The motive behind the idea of goal programming is to reduce the unwanted deviation variables subject to prescribed goals (two goals)

Table 4: Least Square Estimates for Model (4), (5), and (6)

Model	Dependent Variable	Constant	Coefficients	*P*-value	R^2
1	f_1	–4.269	0.160*	0.000	0.933
			0.083*	0.001	
			0.008*	0.000	
			0.019*	0.008	
2	f_2	–3.489	0.004*	0.000	0.958
			0.002*	0.000	
			0.000003*	0.000	
3	PSE	–54965.367	1101.374	0.109	0.809
			499.965	0.482	
			675.564*	0.000	

Note: *Shows the significance at a 5% level of significance.

i.e. the optimal value of price-based factor and the optimum value of the asset-based factor. Before constructing the linear programming model we named all variables in terms of decision variables (x_j).

x_1: 6-months treasury bill rate, x_2: Call money rate, x_3: Net foreign assets, x_4: Consumer price index, x_5: M2, x_6: Gold prices, x_7: Balance of trade.

The range of decision variables is fixed according to the maximum and minimum of the variables (shown in Table 1). The general goal programming model of the stated weighted goals can be presented as:

$$\text{Min } Z = W_1 * p_1 + W_2 * p_2$$

Subject to:

$$f_1 + d_1 - p_1 = f_1 * \text{ goal constraint formulated by estimates obtained by model} \quad (4)$$

$$f_2 + d_2 - p_2 = f_2 * \text{ goal constraint formulated by estimates obtained by model} \quad (5)$$

$x_1 \geqq 5.8214; x_1 \leqq 14.0100; x_2 \geqq 5.4510;$
$x_2 \leqq 20.0300; x_3 \geqq 68.4705; x_3 \leqq 317.7890;$
$x_4 \geqq 1.3200; x_4 \leqq 25.3300; x_5 \geqq 3366.71;$
$x_5 \leqq 13157.30; x_6 \geqq 585.7800; x_6 \leqq$
$1772.1400; x_7 \geqq 51450.0; x_7 \leqq 289622.0.$

Where d_1, d_2, p_1, and p_2 are the negative and positive deviational variables of the j^{th} goal respectively. f_1 * and f_2 * are the optimum price-based and asset-based factors.

In the goal programming model the first step is to find f_1 * = –2.2210 and f_2 * = –1.8105 by minimizing both factors with their respective subjects to constraints. In the second step, we run the bi-objective goal programming model by allocating the equal weights i.e. 0.5 and got the results. Table 5 presents the final stage results;

Table 5: Estimated results for the optimal value of PSX

Variables	Value
x_1: 6-months treasury bill rate	6.00
x_2: Call money rate	6.00
x_3: Net foreign assets	69.00
x_5: Consumer price index	2.00
x_6: M2	3367.00
x_7: Gold prices	586.00
x_8: Balance of trade	51450.00
D1	0.00
D2	0.00
P1	0.00
P2	0.00

Since the proposed objective function in terms of weight is equal to zero, the solution satisfies our prescribed goals. The values of d_1 and d_2 are zero showing that factor 1 (assigned as goal 1) and factor 2 (assigned as goal 2) are achieved through our proposed model. Putting the values of the above results in our model, we got that the PSX achieves an optimal value of 31028.59 for the analyzed time series when x_1: 6 months treasury bill rate is 6.00, x_2: Call money rate is 6.00, x_3: Net foreign assets is 69.00, x_4: Consumer price index is 2.00, x_5: M2 is 3367.00, x_6: Gold prices is 586.00 and x_7: Balance

of trade is 51450.00. It shows that the variables extracted through factor analysis have positively influenced the PSX as their values are close to their maximum value whereas the other variables might close to their minimum values.

5. Conclusion and Recommendations

This study attempts to find the optimal value of the Pakistan stock exchange (PSX) by utilizing M2, Gold Prices (per troy ounce) in US dollars, Real Effective Exchange Rate Index, 6 Month Treasury bill, Call Money Rate, Foreign Exchange, Net Foreign Assets, Food Price Inflation, Manufacturing Production and Balance of Trade as principal macroeconomic indicators. Exploratory factor analysis (EFA) has been used to extract the factors and these extracted factors were then used in a bi-objective goal programming model to obtain the optimal solution. The researchers extracted price-based factors and asset-based factors by taking the variables collectively. Using the multiple regression they obtained the model which later was used to find the optimal value of PSX. They formulated the bi-objective linear goal program as they had more than one objective needs to be minimized. Goal programming allows us to make goals on an ordered or priority basis; there are many advantages of goal programming to apply for financial decision-making over linear programming. In linear programming, one may either maximize or minimize the objective function subject to fixed constraints but goal programming permits deviations from prescribed goals provided the given constraints are minimized. Therefore in this study, after satisfying the required constraints assumptions, we get the optimal value of the PSX. The proposed modeling framework can be used to access other financial returns to make decisions related to investment. Emerging economies possess charismatic features and open new avenues for investors to get high returns. The mathematical and operational programming including machine learning provides such formulation which is the need of today's globalized world.

The optimal value of stock exchange indexes for any country can be a question for investors who want to invest there. This study aims to find the answer to this question and find the optimal value subject to the fixed goal of minimizing the price-based factor and asset-based factor through deviational variables.

This study can be extended by taking more macro-economic variables of Pakistan as the emerging economy has provided more investment horizons for investors. Moreover, by taking a financial or econometric model, more factors can be explored and the optimal conditions of the PSX can be found. Furthermore, by taking structural breaks into account, this study can also be further extended through the nonlinear framework. Moreover, hybrid operation research programming might be the new horizon in financial research.

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