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# The piezoelectricity of trabecular bone in cancellous bone wave propagation

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**Abstract** The orientation of trabeculae and porosity determine the wave propagation in cancellous bone. Wave propagation, as well as charge density and piezoelectricity, stimulate bone remodeling. Also, Charged ions in the fluid affect wave propagation in cancellous bone. But the trabecular struts' piezoelectricity does not change the waveform of cancellous bone. However, the underlying mechanism is unknown yet why trabecula struts' piezoelectricity does not change wave propagation through cancellous bone. Thus, we derived the governing equation indicating that trabecular struts' piezoelectric properties show that those do not affect wave propagation in cancellous bone.

**Key Words** : Cancellous bone, Poroelasticity, Ultrasound, Charge density, Piezoelectricity

## 1. Introduction

Charged ions in bone fluid influence the wave propagation in cancellous bone, and charged ions may be the primary stimuli in the bone remodeling process [1]. Also, the trabecular struts' piezoelectricity may provide another stimulus in the bone remodeling process. However, the underlying mechanism of how trabecular struts' piezoelectric properties affect wave propagation is not known in trabecular bone. The waveforms of ultrasound through trabecular bones show that piezoelectricity has almost no effect in cancellous bone wave propagation [2] But he does not explain why the piezoelectricity can be negligible in wave propagation of cancellous bones. Thus, we formulate the governing poroelastic equation, including charge density and piezoelectricity in order to explain why the trabecular piezoelectricity can be ignored in cancellous bone wave propagation.

## 2. Poroelasticity theory including charge density and piezoelectricity

The governing equations for the poroelasticity theory [3, 4] including both charge density and trabecular struts' piezoelectricity are given by [1, 3, 5]

$$\rho_{11}\ddot{u}+\rho_{12}\ddot{U}-Nu_{i,jj}-(A+N+K)u_{j,ji} \\ -(Q+K_2)U_{j,ji}+b(\dot{u}_i-\dot{U}_i)+q_s\Phi_{,i}=0 \quad (1)$$

$$\rho_{22}\ddot{U}_i+\rho_{12}\ddot{u}_i-((R+K_4)U_{j,j}+(Q+K_3)u_{j,j})_{,i} \\ -b(\dot{u}_i-\dot{U}_i)+q_s\Phi_{,i}=0 \quad (2)$$

$$\left\{q_f(RU_{j,j}+Qu_{j,j})-\varepsilon b[\dot{\Phi}+(\phi G/\varepsilon)]\Phi\right\}_{,ii}=0 \quad (3)$$

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where $N$, $R$, $Q$, and $A$ are poroelastic parameters. $u$ and $U$ are fluid and solid displacements. The variable $b$ is defined by $\phi^2\mu/K$. $\mu$ is the fluid viscosity, K is the permeability of bone fluid, $\varepsilon$ is the permittivity, and $\phi$ is the porosity. And, G is the proportional constant between electric current and electrical potential. We define $P \equiv A+2N$ the poroelastic parameters, P, Q, and R, [6, 7].

$$P=\frac{\phi(K_s/K_f-1)K_b+\phi^2K_s+(1-2\phi)(K_s-K_b)}{(1-\phi-K_b/K_s+\phi K_s/K_f)}+\frac{4G}{3}, \quad (4)$$

$$Q=\frac{(1-\phi-K_b/K_s)\phi K_s}{1-\phi-K_b/K_s+\phi K_s/K_f}, \quad (5)$$

and

$$R=\frac{K_s\phi^2}{(1-\phi-K_b/K_s+\phi K_s/K_f)}, \quad (6)$$

where $K_i$ is the bulk modulus of $i$, where $i$ is for solid ($s$), fluid ($f$), and bone ($b$). The bulk modulus $K_s$ is for solid bone matrix and $K_f$ is for bone fluid. The elastic modulus of solid bone material is 18.9 GPa. and the shear modulus of the solid bone material is calculated as 7.94 GPa from isotropic assumption. The poroelastic parameters P, Q, and R were numerically calculated with the bulk modulus of fluid (or water), 2.3 GPa, and the piezoelectric constants in the governing equations, can be obtained by

$K_1=e_p\cdot r_p+\zeta\cdot b_p$,

$K_2=e_p\cdot t_p+\zeta\cdot z_p$ , $K_3=\zeta_b\cdot r_p+e_s\cdot b_p$,

$K_4=\zeta_b\cdot(e_s/A_0-\xi_s\cdot(\zeta_b-e_s\cdot\xi/A_0)/(A_0^2-\xi\cdot\xi_s))$,
$+e_s\cdot A_0\cdot(\zeta_b-e_s\cdot\xi/A_0)/(A_0^2-\xi\cdot\xi_s)$

where

$r_p=\xi/A_0-(\xi_s/(A_0^2-\xi\cdot\xi_s))\cdot(e_p-\zeta\cdot\xi/A_0)$

$t_p=e_s/A_0-(\xi_s/(A_0^2-\xi\cdot\xi_s))\cdot(\zeta_b-\xi\cdot e_s/A_0)$

$b_p=(A_0/(A_0^2-\xi\cdot\xi_s))\cdot(e_p-\zeta\cdot\xi/A_0)$

$z_p=(A_0/(A_0^2-\xi\cdot\xi_s))\cdot(\zeta_b-\zeta_b\cdot\xi/A_0)$.

The piezoelectric constants for cubic symmetry (isotropic material) are given by [5]

$\xi=16$; $\xi_s=0.038$; $A_0=0.018$; .
$e_p=0.038$; $e_s=0$; $\zeta=0.001$;
$\zeta_b=0$

Thedetailedstepsforcalculatingthegoverningequationsareintheliterature[1].

## 3. Results and discussion

Trabecular struts' piezoelectricity does not influence wave propagation in trabecula. In equations (1), (2), and (3), Biot parameters, P, Q, and R are much greater than piezoelectric parameters $K_1$ , $K_2$ , $K_3$, and $K_4$. The numerical results are almost identical to previous results in Yoon [1]. Thus we do not include the numerical results in this paper. The piezoelectricity is not a significant factor in wave propagation. This result is identical to Hosokawa [2] in that the difference of waveforms propagating through the bone without and with piezoelectricity is negligible. We can conclude that the wave propagation in the cancellous bone can ignore the trabecular struts' piezoelectric properties for the diagnosing purpose in cancellous bone.

## Appendix

The governing equation of the previous study including only charge density [1, 3] is given by

$$\rho_{11}\ddot{u}+\rho_{12}\ddot{U}-Nu_{i,jj}-(A+N)u_{j,ji}\,, \\ -(Q)U_{j,ji}+b(\dot{u}_i-\dot{U}_i)+q_s\Phi_{,i}=0 \tag{7}$$

$$\rho_{22}\ddot{U}_i+\rho_{12}\ddot{u}_i-((R)U_{j,j}+(Q)u_{j,j})_{,i}, \\ -b(\dot{u}_i-\dot{U}_i)+q_s\Phi_{,i}=0 \tag{8}$$

$$\left\{q_f(RU_{j,j}+Qu_{j,j})-\varepsilon b[\dot{\Phi}+(\phi G/\varepsilon)]\Phi\right\}_{,ii}=0. \tag{9}$$

The piezoelectric parameters $K_1$, $K_2$, $K_3$ , and $K_4$ are added to the above governing equations to establish the governing equations, including both charge densities and piezoelectricity.

The constitutive equations for porous piezoelectric materials are [5]

$$\sigma_s = C\cdot\varepsilon_s + m\varepsilon_f - e\cdot E_s - \varsigma\cdot E_f, \tag{10}$$

$$\sigma_f = m\cdot\varepsilon_s + S\varepsilon_f - \hat{\varsigma}\cdot E_s - e_f\cdot E_f, \tag{11}$$

$$D_s = e\cdot\varepsilon_s + \varsigma\varepsilon_f + \xi E_s + A\cdot E_f, \tag{12}$$

$$D_f = \varsigma\cdot\varepsilon_s + e_f\varepsilon_f + A\cdot E_s + \xi_f\cdot E_f, \tag{13}$$

where $\sigma_s(\sigma_f)$ and $\varepsilon_s(\varepsilon_f)$ are the stress and strain tensors on the solid and fluid. $E_s(E_f)$ And $D_s(D_f)$ are electric field and electric displacement vectors. $C$ is the elasticity tensor. The elastic constant S is the pressure built in the fluid. $e_s(e_f)$ and $\xi_s(\xi_f)$ are piezoelectric and dielectric tensors of solid and fluid. $m$ ; $\varsigma$, $\hat{\varsigma}$ ; and are $A$ electric, piezoelectric, and dielectric properties of given porous materials. The equation of motion and Gauss equation are employed and form the general poroelastic governing equation, which includes the piezoelectric constants,

$$\rho_{11}\ddot{u}+\rho_{12}\ddot{U}-Nu_{i,jj}-(A+N+K_1)u_{j,ji} \\ -(Q+K_2)U_{j,ji}+b(\dot{u}_i-\dot{U}_i)=0 \tag{14}$$

$$\rho_{22}\ddot{U}_i+\rho_{12}\ddot{u}_i-((R+K_4)U_{j,j} \\ +(Q+K_3)u_{j,j})_{,i}-b(\dot{u}_i-\dot{U}_i)=0 \tag{15}$$

Then we include the charge density using the equations described in previous literature [1, 3], which you can find those in equations (7-9). We obtain the governing equations for porous materials, including charge density and piezoelectricity, given in equations (1-3).

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