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Periodic Replacement of a System Subject to Shocks under Random Operating Horizon

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랜덤한 운용시평하에서 충격 시스템의 보전방안

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Abstract

This paper presents a periodic replacement policy for a system subject to shocks when the system is operating for a finite random horizon. The system is subject to shocks during operation, and each shock causes downgrading of the system performance and makes it more expensive to run by the additional running cost. Shocks arrive according to a nonhomogeneous or a renewal process, and we develop periodic replacement policies under a finite random operating horizon. The optimum periodic replacement interval which minimizes the total operating cost during the horizon is found. Numerical examples are presented to demonstrate the results.

Keywords : Periodic Replacement, Random Operating Horizon, Shock Process

1. Introduction

When a system is put in service in industry or service facilities, it is common to become less effective as the operating time elapses. According to the random failure and deterioration of the system, the running cost of the system becomes large and it is often economical to replace the system with a new one in a periodic fashion. Also in a safety management point of view, performing timely maintenance activity is very important for the safety of employees.

One traditional way that handles this problem is to model those situations as an increasing failure rate with increasing maintenance cost. The optimum replacement time is found to balance the system replacement cost and the maintenance cost[1-3].

Another way to model those random phenomenon is to consider the system degradation as a result of shock to the system[4-7]. Many systems that are put in service in industry are usually exposed to various shocks such as extreme weathers or fluctuated voltages which result in a significant damage to system lifetime[7]. Also the word shock may be interpreted in a broad senses such as a failure of a part in a complex system[4]. Shocks arrive according to a random process and each shock increases the running cost by some magnitude. The system is replaced periodically as the system operating cost becomes high[4, 5]. In Sheu et al.[7], the minimal repairs upon failure are regarded as a nonhomogeneous Poisson shock process to system, and a generalized maintenance policy which incorporate the minimal repair, overhaul, and replacement is presented.

Boland and Proschan[4] proposed a periodic replacement policy for a system subject to shocks. Shocks arrive according to an nonhomogeneous Poisson process. The additional running cost caused by a shock is burdened to a normal running cost of

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a new system. The system is replaced at times T, $2T$, $3T$, ..., and the optimum T^* which minimizes the long run expected cost per unit of time is found.

Abdel-Hameed[5] treated the same problem as that of Boland and Proschan[4] except the arriving shock process. In Abdel-Hameed[5], shocks arrive according to a counting process whose jump size is of one unit magnitude. The derived expected operating cost per unit of time has the similar form as that of Boland and Proschan[4], and the condition for the existence of optimum periodic replacement interval is found.

Most previous researches on maintenance policies including the shock models assume that the system will be operating for an infinite time horizon. However the assumption nowadays has become less reasonable due to the recent rapidly developing industry technologies[9]. Newly developed systems or parts are releasing with enhanced lifetime, failure rate, and maintenance costs. Therefore, instead of assuming an infinite time horizon with repeated use of systems of the same kind, it is appropriate to assume that the system will be operating for a finite time horizon, and a new horizon will begin for the newly updated system.

The studies on the maintenance policies under a finite time horizon are rarely found in literature. Wells and Bryant[10] presented an age replacement policy for a part with phase-type lifetime distribution when the time horizon is probabilistic. Nakagawa and Mizutani[9] have dealt with a replacement policy when the time horizon is fixed in constant. Yun and Chung[11] found the optimum periodic replacement time for a system with Weibull lifetime distribution which will be operating for a random horizon time under minimal repair policy. Khatab et al.[12] extended the result of Yun and Chung[11] for the general lifetime distribution. Khatab et al.[13] also developed a modified block replacement policy when the length of operating time horizon is random. Yoo[14, 15] extended the minimal repair and block replacement policy of Khatab et al.[12, 13] to continue its operation beyond the end of horizon until the periodic replacement time comes. Yoo[16] also presented group replacement policies for a system operating for a random horizon. For a system subject to shocks, Boland and Proschan[4] found the total expected cost over a finite constant time horizon.

In this paper, periodic replacement policies for a system subject to shocks are presented when the system is operating for a random time horizon. Shocks arrive to the system according to some random process, and each shock increases the running cost of the system. After the maintenance policy of Boland and Proschan[4] and Abdel-Hameed[5], the system is replaced periodically T, $2T$, $3T$, ... until the end of operating horizon. We search for the optimum replacement interval T^* which minimizes the total operating cost of the system over the horizon.

The contents of the paper are as follows. In section 2, a periodic replacement policy for a system subject to shocks which arrive according to a nonhomogeneous Poisson process is presented. Section 3 revisits the same problem as section 2 when shocks arrive according to a general renewal process. Numerical examples are presented to demonstrate the results in section 4, followed by conclusions in section 5.

2. Nonhomogeneous Poisson Shock Model

This section performs a cost analysis on a system subject to repeated shocks operating for a finite random horizon. The normal running cost of a new system is a per unit of time. Shocks arrive to the system according to a nonhomogeneous Poisson process. Each shock to the system increases the running cost by c_r per unit of time. The system is replaced periodically at times T, $2T$, $3T$, ..., and the cost of replacing the system by a new one is c_p, where $c_p > c_r$. The system will be put in service until the end of horizon, where the length of the horizon follows a probability distribution function. [Figure 1] depicts the k periodic replacements and the end of horizon at x.

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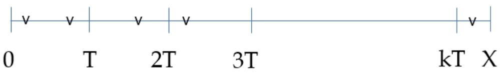

[Figure 1] Shocks arrive(v) randomly and system is replaced at intervals of T until the end of horizon(x)

The normal running cost of the new system during $[0, T)$ is simply aT. The expected additional running cost during $[0, T)$ when it is subject to nonhomogeneous Poisson shocks of intensity rate $\lambda(t)$ is[4] $c_r\int_0^T \Lambda(t)dt$, where $\Lambda(t)=\int_0^t \lambda(x)dx$. Therefore the expected cost during $[0, T)$ for system operation and replacement is

$$aT+c_r\int_0^T \Lambda(t)dt+c_p \tag{1}$$

Suppose that the operating horizon ends at x (refer to [Figure 1]). Then the total running cost during $[0, x)$ when there are k replacements is

$$\begin{aligned} & k\left[aT+c_r\int_0^T \Lambda(t)dt+c_p\right]+a(x-kT) \\ & \qquad +c\int_0^{x-kT}\Lambda(t)dt \\ & =ax+kc_p+kc_r\int_0^T \Lambda(t)dt+c_r\int_0^{x-kT}\Lambda(t)dt \end{aligned} \tag{2}$$

Let $F(t)$ be the distribution function of the horizon. Then the total expected running cost of the system subject to shocks with a finite random horizon and periodic replacement interval T is

$$\begin{aligned} TC(T) &= \sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}[ax+kc_p+kc_r\int_0^T \Lambda(t)dt \\ & \qquad +c_r\int_0^{x-kT}\Lambda(t)dt]dF(x) \\ &= A(T)+B(T)+C(T) \end{aligned} \tag{3}$$

where

$$A(T)=\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}(ax+kc_p)dF(x) \tag{4a}$$

$$B(T)=c_r\int_0^T \Lambda(t)dt\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}kdF(x) \tag{4b}$$

$$C(T)=c_r\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}\int_0^{x-kT}\Lambda(t)dtdF(x). \tag{4c}$$

Suppose that the length of horizon follows an exponential distribution, $F(x)=1-e^{-\theta x}$. Then, after the method of Khatab et al.[11],

$$\begin{aligned} A(T) &= \sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}(ax+kc_p)\theta e^{-\theta x}dx \\ &= a\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}x\theta e^{-\theta x}dx \\ & \qquad +\sum_{k=0}^{\infty}kc_p\int_{kT}^{(k+1)T}\theta e^{-\theta x}dx \\ &= \frac{a}{\theta}+\frac{c_p e^{-\theta T}}{1-e^{-\theta T}} \end{aligned} \tag{5a}$$

$$\begin{aligned} B(T) &= c_r\int_0^T \Lambda(t)dt\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}k\theta e^{-\theta x}dx \\ &= \frac{c_r e^{-\theta T}}{1-e^{-\theta T}}\int_0^T \Lambda(t)dt \end{aligned} \tag{5b}$$

and by putting $x-kT=y$ and applying partial integration,

$$\begin{aligned} C(T) &= c_r\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}\left[\int_0^{x-kT}\Lambda(t)dt\right]\theta e^{-\theta x}dx \\ &= c_r\sum_{k=0}^{\infty}\int_0^T\left[\int_0^y \Lambda(t)dt\right]\theta e^{-\theta(y+kT)}dy \\ &= \frac{c_r}{1-e^{-\theta T}}\int_0^T\left[\int_0^y \Lambda(t)dt\right]\theta e^{-\theta y}dy \\ &= \frac{c_r}{1-e^{-\theta T}}\int_0^T \Lambda(t)(e^{-\theta t}-e^{-\theta T})dt. \end{aligned} \tag{5c}$$

By summing up (5a), (5b), and (5c), the total expected running cost under exponential time horizon is

$$TC(T)=\frac{a}{\theta}+\frac{c_p e^{-\theta T}+c_r\int_0^T \Lambda(t)e^{-\theta t}dt}{1-e^{-\theta T}} \tag{6}$$

Differentiating $TC(T)$ in (6) with respect to T and equating it to zero, we get the necessary condition for T being optimal

$$\begin{aligned} & [c_r\Lambda(T)-\theta c_p]e^{-\theta T}(1-e^{-\theta T}) \\ & \qquad =\left[c_p e^{-\theta T}+c_r\int_0^T \Lambda(t)e^{-\theta t}dt\right]\theta e^{-\theta T} \end{aligned} \tag{7}$$

or after some algebra,

$$L(T)\equiv\int_0^T \lambda(t)(1-e^{-\theta t})dt=\frac{\theta c_p}{c_r} \tag{8}$$

Substituting (7) for (6), the total cost when the system is periodically replaced by the interval T^*, which satisfies (8) becomes

$$TC(T^*) = \frac{a + c_r \Lambda(T^*)}{\theta} - c_p \qquad (9)$$

In (8), it is easily seen that

$$L(0) = 0 < \theta c_p / c_r$$

$$dL(T)/dt = \lambda(T)(1 - e^{-\theta T}) > 0$$

which implies that $L(T)$ is a strictly increasing function in T. Thus if

$$\lim_{T \to \infty} \int_0^T \lambda(t)(1 - e^{-\theta t}) dt > \frac{\theta c_p}{c_r} \qquad (10)$$

there exists a finite optimal periodic replacement interval T^* which minimizes $TC(T)$.

Case 1: Constant intensity rate

When the shocks arrive at a constant intensity rate $\lambda(t) = \lambda$, that is, when the shocks arrive according to a homogeneous Poisson process, $\Lambda(t) = \lambda t$ and the total cost (6) reduces to

$$TC(T) = \frac{1}{\theta}(a + \frac{\lambda c_r}{\theta}) + \frac{e^{-\theta T}}{1 - e^{-\theta T}}(c_p - \frac{\lambda T c_r}{\theta}) \qquad (12)$$

Since the condition (10) satisfies as

$$\lim_{T \to \infty} \int_0^T \lambda(1 - e^{-\theta t}) = \lim_{T \to \infty} \left[\lambda T - \frac{\lambda}{\theta}(e^{-\theta T} - 1) \right]$$

$$= \infty > \frac{\theta c_p}{c_r}$$

there exists a finite optimal periodic replacement interval T^* which satisfies (8) with the total cost during the horizon

$$TC(T^*) = \frac{1}{\theta}(a + c_r \lambda T^*) - c_p \qquad (13)$$

Case 2: Exponentially increasing intensity rate

Suppose that shocks arrive at an exponentially increasing intensity rate, $\lambda(t) = e^{\lambda t}, t > 0$. Then

$$\Lambda(t) = \int_0^T e^{\lambda x} dx = \frac{1}{\lambda}(e^{\lambda t} - 1)$$

and the total cost during the horizon (6) is

$$TC(T) = \frac{e^{-\theta T}}{1 - e^{-\theta T}} \left[c_p + \frac{c_r}{\lambda(\lambda - \theta)}(e^{\lambda T} - e^{\theta T}) \right]$$

$$+ \frac{1}{\theta}(a - \frac{c_r}{\lambda}) \qquad (14)$$

Also the condition (8) implies that

$$L(T) = e^{\lambda T} \left[\frac{\lambda(1 - e^{-\theta T}) - \theta}{\lambda(\lambda - \theta)} \right] + \frac{\theta}{\lambda(\lambda - \theta)}$$

$$= \frac{\theta c_p}{c_r} \qquad (15)$$

and it is evident to see that $L(\infty) = \infty > \theta c_p / c_r$.

Therefore when shocks arrive at an exponentially increasing intensity rate, there exists a finite optimal periodic replacement interval T^* which satisfies (15) with the total cost during the horizon

$$TC(T^*) = \frac{1}{\theta} \left[a + \frac{c_r}{\lambda}(e^{\lambda T^*} - 1) \right] - c_p \qquad (16)$$

Case 3: Exponentially decreasing intensity rate

When the shocks occur at an exponentially decreasing intensity rate, $\lambda(t) = e^{-\lambda t}, \lambda, t > 0$, then $\Lambda(t) = (1 - e^{-\lambda t})/\lambda$ and the total cost during the horizon (6) becomes

$$TC(T) = \frac{e^{-\theta T}}{1 - e^{-\theta T}} \left[c_p + \frac{c_r}{\lambda(\lambda + \theta)}(e^{-\lambda T} - e^{\theta T}) \right]$$

$$+ \frac{1}{\theta}(a + \frac{c_r}{\lambda}) \qquad (17)$$

After some algebra, (8) reduces to

$$L(T) = \frac{1 - e^{-\lambda T}}{\lambda} - \frac{1 - e^{-(\lambda + \theta) T}}{\lambda + \theta} = \frac{\theta c_p}{c_r}$$

Thus if

$$L(\infty) = \frac{1}{\lambda} - \frac{1}{\lambda + \theta} > \frac{\theta c_p}{c_r},$$

or if $\lambda(\lambda + \theta) < c_r / c_p$, there exists a finite optimal replacement interval T^* with the total cost during the horizon

$$TC(T^*) = \frac{1}{\theta} \left[a + \frac{c_r}{\lambda}(1 - e^{-\lambda T^*}) \right] - c_p \qquad (18)$$

If $\lambda(\lambda + \theta) \geq c_r / c_p$, then $T^* \to \infty$, that is, the optimal policy is not to replace the system until the end of horizon.

3. Renewal Process Shock Model

In this section, the problem presented in section 2 is revisited when the shocks arrive according to a renewal process. Let $G(t), t > 0$ be the distribution function of the inter-arrival time of renewal shock process. Abdel-Hameed[5] has shown that the expected total cost of running the system per period is given by

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$$aT+c_r\int_0^T M(t)dt+c_p \qquad (19)$$

where $M(t)$, which is called as renewal function, is the expected number of shocks in [0,t), $M(t)=\sum_{n=0}^{\infty}G^{(n)}(t)$, and $G^{(n)}(t)$ is the n-fold convolution of $G(t)$ with itself[16]. The renewal function is also defined as $M(t)=\int_0^t m(x)dx$, where $m(t)$ is the renewal density of the renewal shock process[16].

As in section 2, suppose that the length of horizon follows an exponential distribution, $F(x)=1-e^{-\theta x}$. Since the expected total cost of running the system per period (19) is the same form as that of the nonhomogeneous Poisson shock model (1), the expected total cost during the horizon can be obtained similarly as

$$TC_1(T)=\frac{a}{\theta}+\frac{c_pe^{-\theta T}+c_r\int_0^T M(t)e^{-\theta t}dt}{1-e^{-\theta T}} \qquad (20)$$

Following the method of section 1, the necessary condition for T being optimal is

$$L_1(T)\equiv\int_0^T m(t)(1-e^{-\theta t})dt=\frac{\theta c_p}{c_r} \qquad (21)$$

and the total cost when the system is periodically replaced by the interval T^*, which satisfies (21) is

$$TC_1(T^*)=\frac{a+c_rM(T^*)}{\theta}-c_p \qquad (22)$$

It is easily seen that from (21),

$$L_1(0)=0<\frac{\theta c_p}{c_r}$$

$$dL_1(T)/dt=m(T)(1-e^{-\theta T})>0$$

which implies that $L_1(T)$ is strictly increasing in $T>0$. Thus if

$$\lim_{T\to\infty}\int_0^T m(t)(1-e^{-\theta t})dt>\frac{\theta c_p}{c_r}$$

there exists a finite optimal periodic replacement interval T^* which minimizes $TC_1(T)$.

Suppose that the inter-arrival time of shocks follows an Erlang distribution of order two,

$$g(t)=\alpha^2te^{-\alpha t},$$

where $g(t)=dG(t)/dt$ and α is the rate parameter. The renewal density and the renewal function are known respectively as[17]

$$m(t)=\frac{\alpha}{2}(1-e^{-2\alpha t}) \qquad (23a)$$

$$M(t)=\frac{\alpha}{2}\left[t-\frac{1}{2\alpha}(1-e^{-2\alpha t})\right] \qquad (23b)$$

After tedious algebra, we get

$$\int_0^T M(t)e^{-\theta t}dt=\frac{1-e^{-(2\alpha+\theta)T}}{4(2\alpha+\theta)}+\frac{2\alpha-\theta}{4\theta^2}(1-e^{-\theta T})-\frac{\alpha Te^{-\theta T}}{2\theta} \qquad (24)$$

and substituting (24) for (20), the expected total cost during the horizon is expressed as

$$TC_1(T)=\frac{a}{\theta}+\frac{(2\alpha-\theta)c_r}{4\theta^2}+\frac{1}{e^{\theta T}-1}\times\left[c_p-\frac{\alpha Tc_r}{2\theta}+\frac{(e^{\theta T}-e^{-2\alpha T})c_r}{4(2\alpha+\theta)}\right] \qquad (25)$$

Substituting (23a) for (21), we get

$$L_1(T)=\frac{\alpha}{2}[T+\frac{1-e^{-(2\alpha+\theta)T}}{2\alpha+\theta}-\frac{1-e^{-2\alpha T}}{2\alpha}-\frac{1-e^{-\theta T}}{\theta}]=\frac{\theta c_p}{c_r} \qquad (26)$$

From (26), it is obvious that

$$\lim_{T\to\infty}L_1(T)=\infty>\frac{\theta c_p}{c_r}$$

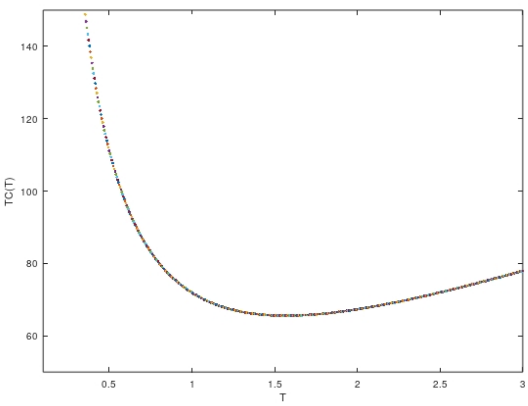
and there exists a finite optimal T^* which minimizes $TC_1(T)$.

4. Numerical Examples

In this section, some numerical examples which illustrate the results of section 3 are presented. Suppose that the length of the operating horizon of a system is randomly determined by an exponential distribution with parameter $\theta=0.2$, which means that the horizon time is expected to last $1/\theta=5$ years. Shocks arrive to the system according to some random process, and the additional operating cost $c_r=3$ per unit of time is burdened each time a shock comes. The normal running cost is $a=1$ per unit of time. The replacement of the system by a new one costs $c_p=10$.

4.1 Constant Intensity Rate

Suppose that shocks arrive according to homogeneous Poisson process with an intensity rate of $\lambda = 3$. The total cost function (12) is evaluated for the above parameters using the open mathematical software GNU Octave. The optimal replacement period is obtained as $T^* = 1.57$ year with the expected total cost $TC(T^*) = 65.59$. [Figure 2] illustrates the behaviour of the cost function $TC(T)$ and the existence of the finite optimum replacement interval. <Table 1> displays the optimal replacement intervals T^* and the corresponding total costs $TC(T^*)$ for various intensity rates of λ. The table shows that as the intensity rate increases, that is, as the shock occurs frequently, the optimal replacement interval becomes shorter, whereas the total cost becomes larger.



[Figure 2] The shape of $TC(T)$ for homogeneous Poisson shocks with intensity rate $\lambda = 3$

<Table 1> The optimal replacement intervals with the total cost for constant intensity rate

λ	1	2	3	4	5	6
T^*	2.82	1.94	1.57	1.35	1.20	1.09
TC^*	37.37	53.32	65.59	75.94	85.07	93.32

4.2 Exponentially Increasing Intensity rate

Suppose that shocks comes to the system according to a nonhomogeneous Poisson process with an increasing intensity rate $\lambda(t) = e^{\lambda t}$. Again, the cost function (14) is computed to obtain the optimal periodic replacement intervals with the expected total costs for several valus of λ, which is summarized in <Table 2>. Like <Table 1>, the optimum replacement interval gets shorter whereas the total cost becomes larger as the value of λ increases.

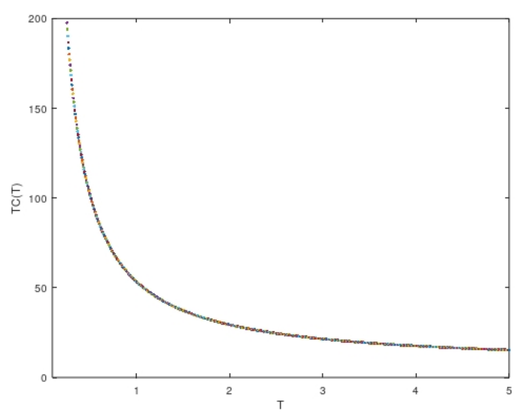
<Table 2> The optimal replacement intervals with the total cost for increasing intensity rate $\lambda(t) = e^{\lambda t}$

λ	1	2	3	4	5	6
T^*	1.57	1.16	0.95	0.81	0.71	0.63
TC^*	52.06	64.25	75.23	85.45	95.12	104.37

4.3 Exponentially Decreasing Intensity Rate

When shocks arrive according to a nonhomogeneous Poisson process with a decreasing intensity rate $\lambda(t) = e^{-\lambda t}$, the existence of a finite optimal replacement interval depends on the value of $\lambda(\lambda+\theta)$. If $\lambda(\lambda+\theta) < c_r/c_p$, i.e., if $\lambda(\lambda+0.2) < 0.3$, there exists a finite optimum T^*. Otherwise, the optimum policy is not to replace the system at all until the end of horizon, i.e., $T^* \rightarrow \infty$.

Suppose $\lambda = 3$ so that $\lambda(\lambda+0.2) > 0.3$. [Figure 3] verifies that the cost function (17) is strictly decreasing in $T > 0$, and the optimal policy is not to replace the system, i.e., $T^* \rightarrow \infty$. On the other hands, when $\lambda = 0.01$, we have $\lambda(\lambda+0.2) < 0.3$; the optimum replacement interval is $T^* = 2.85$ year with the expected total cost $TC(T^*) = 37.20$



[Figure 3] Shape of strictly decreasing total cost for nonhomogeneous Poisson shocks with decreasing intensity rate $\lambda(t) = e^{-3t}$

4.4 Renewal Shock Process

To illustrate the last example, suppose that the inter-arrival time of shocks follows an Erlang distribution $g(t) = \alpha^2 te^{-\alpha t}$, where α is a rate parameter. Again using the software GNU Octave, the optimal replacement intervals and the corresponding total costs are computed for several values of rate parameter α. The results are summarized in <Table 3>. The pattern of the change is similar to the previous results; as the value of rate parameter increases, the optimum replacement interval gets shorter whereas the total cost becomes large.

<Table 3> The optimal replacement intervals with the total cost for renewal shock process

α	1	2	3	4	5	6
T^*	4.24	2.85	2.28	1.95	1.73	1.57
TC^*	23.02	34.03	42.60	49.85	56.26	62.07

5. Conclusion

This paper suggested a periodic replacement policy for a system subject to shocks which will be operating for a finite random horizon. Shocks arrive to a system according to a random process, and each shock causes some additional running cost. The system is replaced periodically to balance the increasing running cost and the replacement cost.

For each homogeneous and nonhomogeneous Poisson shock process with increasing and decreasing intensity rate, the expected total cost during the operating horizon was obtained assuming exponential horizon time distribution. An analysis for renewal shock process was also performed when the inter-arrival time of shocks follows an Erlang distribution. Numerical demonstrations to obtain the optimum replacement intervals were presented for Poisson and renewal shock processes.

Most previous researches on the maintenance policies assume that the system will be operating infinitely, replacing old ones with new ones. However, in modern times with rapidly changing industrial technologies, it is more reasonable to assume that the horizon time is finite as presented in this paper. In this study, the length of horizon was assumed to follow a specific exponential distribution. Further research is needed to overcome this limitations.

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