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The bounds for fully saturated porous material

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Abstract The elasticity tensor for water may be employed to model the fully saturated porous material. Mostly water is assumed to be incompressible with a bulk modulus, however, the upper and lower bounds of off-diagonal components of the elasticity tensor of porous materials filled with water are violated when the bulk modulus is relatively high. In many cases, the generalized Hill inequality describes the general bounds of Voigt and Reuss for eigenvalues, but the bounds for the component of elasticity tensor are more realistic because the principal axis of eigenvalues of two phases, matrix and water, are not coincident. Thus in this paper, for anisotropic material containing pores filled with water, the bounds for the component of elasticity tensor are expressed by the rule of mixture and the upper and lower bounds of fully saturated porous materials are violated for low porosity and high bulk modulus of water.

Key Words : Hill inequality, Voigt and Reuss bounds, Anisotropy, Elasticity tensor

1. Introduction

A simplest method to estimate the effective moduli of composite materials is Voigt-Reuss bounds [1-2]. The Voigt bound indicates the upper bound and the Reuss bound is the lower bound. Generally these bounds are constructed for the eigenvalues of the elasticity tensor (or compliance tensor) [3-7], but it is more reasonable to use the rule of mixture for the elasticity tensor (or compliance tensor) when the geometrical structure of porous materials is not simple and the principal axis of matrix is not coincident with that of water.

Mostly the eigenvalues are used for applying the rule of mixture, but the principal axis of matrix and water are not coincident. Thus the matrix is filled with water. It is much better to assume that the axis of solid matrix is coincident with that of water. Then we can conclude the rule of mixture in elasticity tensor (or compliance tensor) is more reasonable than that employed in eigenvalues or elastic modulus that is a scalar.

The objective of this study is to estimate the fluid effect to solid biological tissues to find out the cell to cell signaling due to the fluid, especially for the solid matrix (or bone matrix) structure is very complicated for estimating with considering bone cells attached to the solid bone matrix.

2. Method : Hill Inequality

Using the strain energy and Voigt-Reuss bounds [3-7], we can construct the

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bounds for eigenvalues. First, the Hooke`s law is given by

$$T_{ij} = C_{ijkm}E_{km} \quad (1)$$

where C_{ijkm} is the elasticity tensor in three dimensions, T_{ij} are the stress tensor, and E_{km} are the strain tensor. If we convert it for six dimensions [8,9], the equation (1) is written as

$$\hat{T} = \hat{C}\hat{E} \quad (2)$$

where $\hat{C}$ is the six dimensional elasticity tensor (or can be expressed as a matrix), $\hat{T}$ is the six dimensional stress vector, and $\hat{E}$ is the six dimensional strain vector.

Note that the elasticity tensor (or matrix) $\hat{C}$ is the inverse of the compliance tensor (or matrix) $\hat{S}$. The inverse of the Hooke`s law (eq. 1) is $\hat{E} = \hat{S}\hat{T}$. For eigenvalue problems, the eigenvalues of the matrix $\hat{C}$ (or $\hat{S}$) should satisfy the following equations,

$$(\hat{C} - \Lambda\hat{1})\hat{N} = 0 \text{ or } (\hat{S} - (1/\Lambda)\hat{1})\hat{N} = 0 \quad (4)$$

Here the eigenvector $\hat{N}$ shows the orientation of the eigenvalue Λ. The strain energy Σ is obtained by multiplying the strain tensor to the stress tensor,

$$\Sigma = \frac{1}{2}\hat{E}^T\hat{C}\hat{E} = \frac{1}{2}\hat{T}^T\hat{S}\hat{T} \quad (5)$$

By employing the principles of minimum potential energy and minimum complementary energy [4], the strain energy Σ establishes the following inequalities,

$$\hat{E} \cdot \hat{C}^{eff} \cdot \hat{E} \le \hat{E} \cdot \hat{C}^{V} \cdot \hat{E} \text{ and} \quad (6)$$
$$\hat{T} \cdot \hat{S}^{eff} \cdot \hat{T} \le \hat{T} \cdot \hat{S}^{R} \cdot \hat{T}$$

Then the above inequalities must hold the inequalities for eigenvalues shown in (4),

$$\Lambda^{eff} \le \Lambda^{V}, \frac{1}{\Lambda^{eff}} \le \frac{1}{\Lambda^{R}} \quad (7)$$

or

$$\Lambda^{R} \le \Lambda^{eff} \le \Lambda^{V} \quad (8)$$

The equation (8) is the generalized Hill inequality for eigenvalues of elastic materials.

3. The Voigt and Reuss bounds for fully saturated porous material.

For anisotropic materials, we can simply consider the rule of mixture for components of elasticity tensor $\hat{C}$ because the tensor contains not only mechanical properties but also the orientation of each mechanical properties. For example, employing the rule of mixture for the elastic modulus E does not make the axis of two phase composites be coincident, but the rule of mixture employing to the elasticity tensor makes the axis of two phase composites be coincident. The expression for rule of mixture in two phase composite is given by

$$\hat{C}^{eff} = (1-\phi)\hat{C}_1 + \phi\hat{C}_2. \quad (9)$$

Then the bounds for Voigt and Reuss bounds are

$$((1-\phi)\hat{S}_1 + \phi\hat{S}_2)^{-1} \le \hat{C}^{eff} \le (1-\phi)\hat{C}_1 + \phi\hat{C}_2 \quad (10)$$

We can examine the results numerically for bone matrix, which is a composite of collagen and mineral crystals, containing bone fluid inside pores: The technical elastic constants of bone matrix are $E_1 = 15.6 GPa$, $E_2 = 17.8 GPa$, $E_3 = 22.2 GPa$, $G_{12} = 6.9 GPa$, $G_{13} = 6.9 GPa$, $G_{23} 8.1 GPa$, $v_{12} = 0.337$, $v_{13} = 0.231$, $v_{21} = 0.384$, $v_{23} = 0.241$, $v_{31} = 0.330$ and $v_{32} = 0.302$ [10].

The elasticity tensor for bone fluid is given by [11].

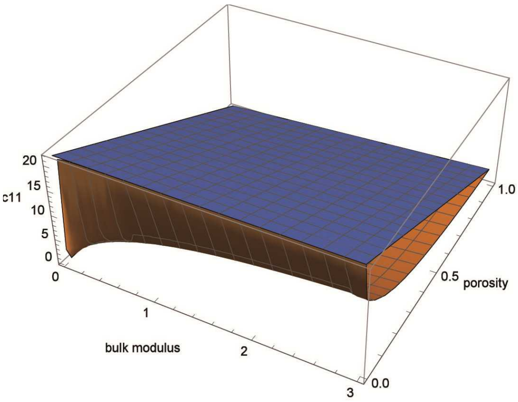

Fig. 1. The Voigt bound (blue) and Reuss bound (red) for c11 of elasticity tensor

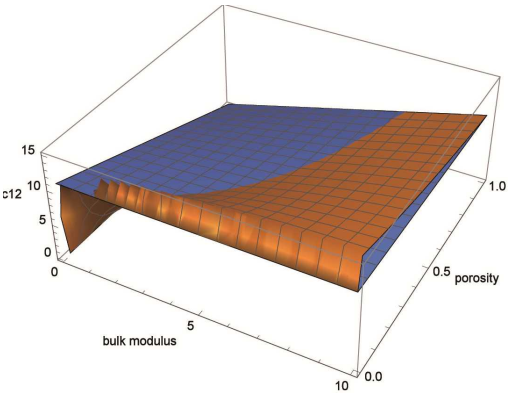

Fig. 2. The Voigt bound (blue) and Reuss bound (red) for c12 of elasticity tensor

$$\hat{C} = K_w \begin{pmatrix} 1.00025 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1.00025 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1.00025 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0001 \end{pmatrix}, \quad (11)$$

where K_w is the bulk modulus of water. By employing the given mechanical properties, we construct the upper and lower bounds (eq. 10) with the bulk modulus of water K_w to be unknown.

4. Discussion and Conclusion

When the elasticity tensor of water is established, the water is assumed to be an incompressible elastic material. Water has a bulk modulus, which is the reciprocal of the compressibility.

Figure 1 shows the bounds of c_{11} of the elasticity tensor and Figure 2 show the bounds of c_{12} of the elasticity tensor, which is off-diagonal component. c_{11} satisfies the upper and lower bounds, but c_{12} violates the bounds when the bulk modulus of water K_w is high and the porosity is low because the assumption constructing the elasticity tensor of water is incompressible elastic material. Figures 1 and 2 will give you a guidance to use how much bulk modulus and porosity are needed for satisfying the upper and lower bounds for fully saturated porous materials.

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