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EXISTENCE OF POSITIVE SOLUTION FOR A SEMIPOSITONE SYSTEM WITH INTEGRAL BOUNDARY VALUES

Eunkyung Ko and Eun Kyoung Lee*

Abstract. We establish the existence of a positive solution to a semipositone system with integral boundary condition for the large value of the parameter involved in the system. We prove our results by using sub and super solution argument.

1. Introduction

In this paper, we study the existence of a positive radial solution to the following semipositone system with nonlocal boundary values on an exterior domain:

$$\begin{cases} -\Delta u = \lambda K_1(|x|)f_1(u(x), v(x)), & x \in \Omega_e, \\ -\Delta v = \lambda K_2(|x|)f_2(u(x), v(x)), & x \in \Omega_e, \\ u(x) \to 0, \quad v(x) \to 0, & \text{if } |x| \to \infty, \\ u(x) = \int_{\Omega_e} l_1(|y|)v(y)dy, & \text{if } |x| = r_0, \\ v(x) = \int_{\Omega_e} l_2(|y|)u(y)dy, & \text{if } |x| = r_0, \end{cases} \tag{1}$$

where $\Omega_e = \{x \in \mathbb{R}^N : |x| > r_0 \text{ for } r_0 > 0, N \geq 3\}$, λ is a positive parameter, $K_i \in C((r_0, \infty), (0, \infty))$ is such that $\int_{r_0}^{\infty} r^{\nu_i} K_i(r)dr < \infty$ for some $\nu_i > 1$, $f_i \in C(\mathbb{R}_+^2, \mathbb{R})$ and $l_i \in L^1(\Omega_e)$ is a nonnegative function satisfying $0 < w_N r_0^{N-2} \int_{r_0}^{\infty} r l_i(r)dr < 1$ for each $i = 1, 2$ when w_N is the surface area of the unit sphere in $\mathbb{R}^N$.

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*Corresponding author.

Such differential equations with an integral boundary condition arise in various areas of applied mathematics and physics like heat conduction, chemical engineering, underground water flow, thermo-elasticity and plasma phenomena. One may refer to [4], [11] - [14] and [16] for integral boundary value problems and the references therein.

Note that the change of variables $r = |x|$ and $t = (\frac{r}{r_0})^{2-N}$ transforms (1) into:

$$\begin{cases} -u''(t) = \lambda a_1(t) f_1(u(t), v(t)), \quad t \in (0,1), \\ -v''(t) = \lambda a_2(t) f_2(u(t), v(t)), \quad t \in (0,1), \\ u(0) = 0 = v(0), \\ u(1) = \int_0^1 g_1(s) v(s) ds, \\ v(1) = \int_0^1 g_2(s) u(s) ds, \end{cases} \tag{2}$$

with

$$\begin{aligned} a_i(t) &= \left(\frac{1}{N-2}\right)^2 r_0^2 t^{\frac{-2(N-1)}{N-2}} K_i(r_0 t^{\frac{-1}{N-2}}), \\ g_i(t) &= w_N \left(\frac{1}{N-2}\right) r_0^N t^{\frac{-2(N-1)}{N-2}} l_i(r_0 t^{\frac{-1}{N-2}}), \end{aligned}$$

where $a_i \in C((0,1),[0,\infty))$ is such that $\int_0^1 s^{\alpha_i}(1-s)^{\beta_i} a_i(s) ds < \infty$ for some $\alpha_i, \beta_i \in (0,1)$ and a nonnegative function $g_i \in L^1(0,1)$ is such that $0 < \int_0^1 s g_i(s) ds < 1$ for each $i = 1, 2$. We know that the existence of positive solutions for the system (2) guarantees the existence of positive radial solutions for (1). Hence we focus on the system (2) to investigate solutions for (1).

Nonlocal boundary value problems have been widely studied especially on a compact interval. The authors in [11] and [12] have established extensive works of nonlocal boundary value problems involving integral conditions. Some existence results are considered in [1], [2], [3], [8], [9] and [13] by applying the fixed point theorem, mixed monotone method, monotone iterative method and fixed point index under the condition either $f_i(0,0) = 0$ or $f(0,0) > 0$. In [15], the existence of positive solutions for a semipositone ($i.e. f_i(0,0) < 0$) differential system has been established when the boundary condition is local. To the best of our knowledge, the existence of a positive solution for the semipositone system with nonlocal boundary condition has not been treated. In this paper, we study the existence of a positive solution for a semipositone system with integral boundary conditions when a parameter involved in the system varies. We establish our result by sub and super solution argument.

In this article, we assume the following hypotheses on f_i for $i = 1, 2$.

$(H1)$ $f_1(t, s)$ and $f_2(t, s)$ are quasimonotone increasing with respect to s and t, respectively, (i.e., $f_1(t, s_1) \le f_1(t, s_2)$ for $s_1 \le s_2$ and $f_2(t_1, s) \le f_2(t_2, s)$ for $t_1 \le t_2$).

$(H2)$ $f_i(0, 0) < 0$, for $i = 1, 2$.

$(H3)$ $\lim_{u+v\to\infty} f_i(u, v) = \infty$ and $f_{i,\infty} := \lim_{u+v\to\infty} \frac{f_i(u, v)}{u + v} = 0$, for $i = 1, 2$.

$(H4)$ $\underline{a_i} := \inf_{t\in(0,1)} a_i(t) > 0$ and there exist $d > 0$ and $\gamma \in (0, 1)$ such that

$$a_i(t) \le \frac{d}{t^\gamma} \text{ for } t \in (0, 1) \text{ and } i = 1, 2.$$

Now we state our main result precisely.

Theorem 1.1. *Assume that* $(H1) \sim (H4)$*. The problem* (2) *has at least one positive solution for* $\lambda >> 1$.

For the problem (1), we have the following corresponding result.

Corollary 1.2. *Assume that* $(H1) \sim (H3)$ *and*

$(H4')$ $\underline{K_i} := \inf_{r\in(r_0,\infty)} r^{2(N-2)} K_i(r) > 0$ *and there exist* $\tilde{d} > 0$ *and* $\eta \in (0, N - 2)$ *such that*

$$K_i(t) \le \frac{\tilde{d}}{r^{N+\eta}} \textit{ for } r \in (r_0, \infty) \textit{ and } i = 1, 2.$$

The problem (1) *has at least one positive radial solution for* $\lambda >> 1$.

The paper is organized as follows: In the next section we introduce the sub and super solution theorem. Section 3 is devoted to the proof of the main result, Theorem 1.1.

2. Prelimiaries

We introduce a theorem for sub and supersolutions to the system (2). First, we state the following definition of subsolution and supersolution of the system (2).

Definition 1. We say that (ψ_1, ψ_2) is a *subsolution* of problem (2) if $(\psi_1, \psi_2) \in C^2(0, 1) \times C^2(0, 1)$ with satisfying

$$\begin{cases} -\psi_1''(t) \le \lambda a_1(t) f_1(\psi_1(t), \psi_2(t)), \quad t \in (0, 1), \\ -\psi_2''(t) \le \lambda a_2(t) f_2(\psi_1(t), \psi_2(t)), \quad t \in (0, 1), \\ \psi_1(0) \le 0, \ \psi_2(0) \le 0, \\ \psi_1(1) \le \int_0^1 g_1(s)\psi_2(s)ds, \\ \psi_2(1) \le \int_0^1 g_2(s)\psi_1(s)ds. \end{cases}$$

We also say that (ζ_1, ζ_2) is a *supersolution* of problem (2) if $(\zeta_1, \zeta_2) \in C^2(0,1) \times C^2(0,1)$ with satisfying the reverse of the above inequalities.

Theorem 2.1. *Assume that* $(H1)$ *and there exist a subsolution* (ψ_1, ψ_2) *and a supersolution* (ζ_1, ζ_2) *of the problem* (2) *such that* $(\psi_1(t), \psi_2(t)) \le (\zeta_1(t), \zeta_2(t))$ *for all* $t \in [0,1]$. *Then* (2) *has at least one solution* (u, v) *such that*

$$(\psi_1(t), \psi_2(t)) \le (u(t), v(t)) \le (\zeta_1(t), \zeta_2(t)) \quad \text{for all } t \in [0,1].$$

Proof. See the Appendix in [8]. □

3. Proof of Theorem 1.1

Lemma 3.1. *Suppose that* $(H3)$ *holds. Let us define*

$$\tilde{f}_i(s,t) = \max_{(u,v)\in[0,s]\times[0,t]} f_i(u,v) \quad \text{for each } (s,t) \in [0,\infty) \times [0,\infty).$$

Then the followings are true: for each $i = 1, 2$,

(*i*) $f_i(u,v) \le \tilde{f}_i(u,v)$ *for all* $(u,v) \in [0,\infty) \times [0,\infty)$,
(*ii*) $\tilde{f}_i$ *is nondecreasing (i.e.,* $\tilde{f}_i(s_1,t_1) \le \tilde{f}_i(s_2,t_2)$ *for* $(s_1,t_1) \le (s_2,t_2)$*),*
(*iii*) $\lim_{u+v\to\infty} \tilde{f}_i(u,v) = \infty$ *and*
(*iv*) $\tilde{f}_{i,\infty} := \lim_{u+v\to\infty} \dfrac{\tilde{f}_i(u,v)}{u+v} = 0.$

Proof. It is obvious that $f_i(u,v) \le \tilde{f}_i(u,v)$ for each $(u,v) \in [0,\infty) \times [0,\infty)$. Let (s_1,t_1), $(s_2,t_2) \in [0,\infty) \times [0,\infty)$ be with $(s_1,t_1) \le (s_2,t_2)$. As $[0,s_1] \times [0,t_1] \subset [0,s_2] \times [0,t_2]$, we know $\tilde{f}_i(s_1,t_1) \le \tilde{f}_i(s_2,t_2)$. Thus, $\tilde{f}$ is nondecreasing for each $i = 1,2$. Next, since $\lim_{u+v\to\infty} f_i(u,v) = \infty$ and $f_i(u,v) \le \tilde{f}_i(u,v)$ for all $(u,v) \in [0,\infty) \times [0,\infty)$, we have $\lim_{u+v\to\infty} \tilde{f}_i(u,v) = \infty$. Now, it remains to show that $\tilde{f}_{i,\infty} = \lim_{u+v\to\infty} \dfrac{\tilde{f}_i(u,v)}{u+v} = 0$. From $(H3)$, for given $\epsilon > 0$, there exists $K > 0$ such that

$$\frac{f_i(u,v)}{u+v} < \epsilon \text{ for } u+v > K. \tag{3}$$

We take a set $D := \{(s,t) \in [0,\infty)\times[0,\infty) \mid s+t \le K\}$ and let $M_i := \max_{(s,t)\in D} f_i(s,t)$, and then we define $h_i : [0,\infty)\times[0,\infty) \to [0,\infty)$ by $h_i(s,t) = \max\{\epsilon(s+t),\ M_i\}$. Now, we claim that

$$h_i(s,t) \ge \tilde{f}_i(s,t) \quad \text{for all} \quad (s,t) \in [0,\infty) \times [0,\infty).$$

Indeed, if $(s,t) \in D$, we find that

$$h_i(s,t) \ge M_i = \max_{(u,v)\in D} f_i(u,v) \ge \max_{(u,v)\in[0,s]\times[0,t]} f_i(u,v) = \tilde{f}_i(s,t)$$

since $[0,s]\times[0,t]\subset D$ from $(s,t)\in D$. If $(s,t)\in D^c\cap[0,\infty)\times[0,\infty)$, we obtain

$$\begin{aligned}
h_i(s,t) &= \max\{\epsilon(s+t),\ M_i\} = \max\{\max_{(u,v)\in[0,s]\times[0,t]}\epsilon(u+v),\ M_i\}\\
&\geq \max\{\max_{(u,v)\in D^c\cap([0,s]\times[0,t])}\epsilon(u+v),\ M_i\}\\
&> \max\{\max_{(u,v)\in D^c\cap([0,s]\times[0,t])} f_i(u,v),\ \max_{(u,v)\in D} f_i(u,v)\}\\
&\geq \max\{\max_{(u,v)\in D^c\cap([0,s]\times[0,t])} f_i(u,v),\ \max_{(u,v)\in D\cap([0,s]\times[0,t])} f_i(u,v)\}\\
&= \tilde{f}_i(s,t),
\end{aligned}$$

where we used (3) in the second inequality. Choosing $N>0$ such that $\epsilon N > \max\{M_1,M_2\}$, it follows that for $u+v>N$,

$$\frac{\tilde{f}_i(u,v)}{u+v}\leq\frac{h_i(u,v)}{u+v}=\frac{\epsilon(u+v)}{u+v}=\epsilon.$$

□

Lemma 3.2. *Suppose that* $(H2)$, $(H3)$ *and* $(H4)$ *hold. Then there exists a subsolution* (ψ_1,ψ_2) *of the problem* (2) *for* $\lambda >> 1$.

Proof. Consider the following boundary value problem

$$\begin{cases} -\phi''(t)=\lambda\phi(t), & t\in(0,1),\\ \phi(0)=0=\phi(1). \end{cases} \tag{4}$$

Let $\phi_1\in C^2[0,1]$ be the eigenfunction corresponding to the first eigenvalue λ_1 of (4) such that $\phi_1(t)>0$; $t\in(0,1)$. Then, there exists $d_1>0$ such that

$$0<\phi_1(t)\leq d_1t(1-t)\ \text{for}\ t\in(0,1). \tag{5}$$

Let $\sigma\in(1,2-\gamma)$, $\epsilon>0$, $m>0$ and $\mu>0$ be such that

$$-m>[\lambda_1\sigma\phi_1^2-\sigma(\sigma-1)|\phi_1'|^2]\ \text{ in } (0,\epsilon]\cup[1-\epsilon,1)\ \text{and} \tag{6}$$

$$\phi_1>\mu\ \text{ in } (\epsilon,1-\epsilon). \tag{7}$$

This is possible since $\phi_1=0$ and $|\phi_1'|>0$ at $t=0,1$.

Let us denote $\underline{f_i}:=\min_{(s,t)\in[0,\infty)\times[0,\infty)} f_i(s,t)$. Clearly, $\underline{f_i}<0$ for each $i=1,2$. Now we define $(\psi_1,\psi_2)=(\lambda k_0\phi_1^\sigma,\lambda k_0\phi_1^\sigma)$, where $k_0>0$ is chosen so that $-k_0<\frac{d_1^{2-\sigma}d}{m}\min\{\underline{f_1},\underline{f_2}\}$. Then, we have $-\psi_i'=-\lambda k_0\sigma\phi_1^{\sigma-1}\phi_1'$, which yields

$$-\psi_i''=-\lambda k_0\sigma(\sigma-1)\phi_1^{\sigma-2}|\phi_1'|^2-\lambda k_0\sigma\phi_1^{\sigma-1}\phi_1''=\lambda\left[\lambda_1k_0\sigma\phi_1^\sigma-k_0\sigma(\sigma-1)\frac{|\phi_1'|^2}{\phi_1^{2-\sigma}}\right]$$

as $\phi_1'' = -\lambda_1\phi_1$. For $t \in (0, \epsilon]$, it follows

$$\begin{aligned}
-\psi_i'' &= \lambda\left[\lambda_1 k_0\sigma\phi_1^\sigma - k_0\sigma(\sigma-1)\frac{|\phi_1'|^2}{\phi_1^{2-\sigma}}\right] \\
&= \frac{\lambda k_0}{\phi_1^{2-\sigma}}\left[\lambda_1\sigma\phi_1^2 - \sigma(\sigma-1)|\phi_1'|^2\right] \\
&\le -\frac{\lambda k_0 m}{\phi_1^{2-\sigma}} \le \frac{-\lambda k_0 m}{d_1^{2-\sigma}t^{2-\sigma}(1-t)^{2-\sigma}} \\
&\le \frac{-\lambda k_0 m}{d_1^{2-\sigma}t^\gamma} \le \frac{-\lambda k_0 a_i(t) m}{d_1^{2-\sigma}d} \\
&\le \lambda a_i(t)\min\{\underline{f_1}, \underline{f_2}\} \\
&\le \lambda a_i(t) f_i(\psi_1, \psi_2),
\end{aligned}$$

using (5), (6) and the condition ($H4$). A similar argument holds for $t \in [1-\epsilon, 1)$.

Let $t \in (\epsilon, 1-\epsilon)$. As (7) and $\lim_{u+v\to\infty} f_i(u,v) = \infty$, it holds

$$f_i(\lambda k_0\phi_1^\sigma(t), \lambda k_0\phi_1^\sigma(t)) \ge \frac{1}{\underline{a_i}}\lambda_1 k_0\sigma\phi_1^\sigma(t) \quad \text{for } \lambda >> 1.$$

Thus, for such a $\lambda >> 1$, we obtain

$$\begin{aligned}
-\psi_i'' &= \lambda\left[\lambda_1 k_0\sigma\phi_1^\sigma - k_0\sigma(\sigma-1)\frac{|\phi_1'|^2}{\phi_1^{2-\sigma}}\right] \\
&\le \lambda\lambda_1 k_0\sigma\phi_1^\sigma(t) \\
&\le \lambda\underline{a_i} f_i(\lambda k_0\phi_1^\sigma(t), \lambda k_0\phi_1^\sigma(t)) \\
&\le \lambda a_i(t) f_i(\psi_1, \psi_2).
\end{aligned}$$

Also, it is easy to see $\psi_i(0) = \lambda k_0\phi_1^\sigma(0) = 0$ and

$$\begin{aligned}
\psi_1(1) &= \lambda k_0\phi_1^\sigma(1) = 0 \le \int_0^1 g_1(s)\psi_2(s)ds \\
\psi_2(1) &= \lambda k_0\phi_1^\sigma(1) = 0 \le \int_0^1 g_2(s)\psi_1(s)ds
\end{aligned}$$

since g_i and ψ_i are nonnegative functions. Thus, (ψ_1, ψ_2) is a subsolution of the problem (2) for $\lambda >> 1$. □

Lemma 3.3. *Assume* ($H3$). *For each* $\lambda > 0$, *there exists a positive real number* $M(\lambda) >> 1$ *such that* $(M(\lambda)e_1, M(\lambda)e_2)$ *is a supersolution of problem* (2), *where*

(e_1, e_2) *is the unique positive solution of*

$$\begin{cases} -e_1''(t) = a_1(t), & t \in (0,1), \\ -e_2''(t) = a_2(t), & t \in (0,1), \\ e_1(0) = 0 = e_2(0), \\ e_1(1) = \int_0^1 g_1(s)e_2(s)ds, \\ e_2(1) = \int_0^1 g_2(s)e_1(s)ds, \end{cases}$$

and the exact form of (e_1, e_2) *is in* [10].

Proof. From Lemma 3.1, we recall that $f_i(u,v) \le \tilde{f}_i(u,v)$ for all $(u,v) \in [0,\infty)\times[0,\infty)$, $\tilde{f}_i$ are nondecreasing, $\lim_{u+v\to\infty} \tilde{f}_i(u,v) = \infty$ and $\tilde{f}_{i,\infty} = \lim_{u+v\to\infty} \frac{\tilde{f}_i(u,v)}{u+v} = 0$, for $i = 1,2$. Thus we can choose $M(\lambda) >> 1$ such that

$$\frac{1}{\lambda(||e_1||_\infty + ||e_2||_\infty)} \ge \frac{\tilde{f}_i(M(\lambda)||e_1||_\infty, M(\lambda)||e_2||_\infty)}{M(\lambda)(||e_1||_\infty + ||e_2||_\infty)}.$$

Now we define $(\zeta_1, \zeta_2) = (M(\lambda)e_1, M(\lambda)e_2)$. Then it follows

$$\begin{aligned} -\zeta_i''(t) = -M(\lambda)e_i''(t) &= M(\lambda)a_i(t) \\ &\ge \lambda a_i(t)\tilde{f}_i(M(\lambda)||e_1||_\infty, M(\lambda)||e_2||_\infty) \\ &\ge \lambda a_i(t)\tilde{f}_i(M(\lambda)e_1(t), M(\lambda)e_2(t)) \\ &\ge \lambda a_i(t)f_i(\zeta_1(t), \zeta_2(t)) \end{aligned}$$

Also, it is easy to see that $\zeta_i(0) = M(\lambda)e_i(0) = 0$ and

$$\begin{aligned} \zeta_1(1) &= M(\lambda)e_1(1) = M(\lambda)\int_0^1 g_1(s)e_2(s)ds = \int_0^1 g_1(s)\zeta_2(s)ds \\ \zeta_2(1) &= M(\lambda)e_2(1) = M(\lambda)\int_0^1 g_2(s)e_1(s)ds = \int_0^1 g_2(s)\zeta_1(s)ds. \end{aligned}$$

Thus, $(M(\lambda)e_1, M(\lambda)e_2)$ is a supersolution of problem (2). □

Proof of Theorem 1.1

Proof. For $\lambda >> 1$, by Lemma 3.2, there exists a subsolution (ψ_1, ψ_2) of problem (2) and we can choose $M(\lambda) >> 1$ such that $(\zeta_1, \zeta_2) = (M(\lambda)e_1, M(\lambda)e_2)$ is a supersolution of (2) and $(\psi_1, \psi_2) \le (M(\lambda)e_1, M(\lambda)e_2)$ from Lemma 3.3. Therefore, Theorem 2.1 concludes that (2) has at least one positive solution for $\lambda >> 1$. □

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Eunkyung Ko
Major in Mathematics, College of Natural Science, Keimyung University, Daegu 42601, South Korea
E-mail address: ekko@kmu.ac.kr

Eun Kyoung Lee
Department of Mathematics Education, Pusan National University, Busan, South Korea
E-mail address: eklee@pusan.ac.kr