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MIMO Channel Capacity Maximization Using Periodic Circulant Discrete Noise Distribution Signal

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Abstract

Multiple Input Multiple Output (MIMO) is one of the important wireless communication technologies. This paper proposes MIMO system capacity enhancement by using convolution of periodic circulating vector signals. This signal represents statistical dependencies between transmission signal with discrete noise and receiver signal with the linear shifting of MIMO channel capacity by positive extents. We examine the channel capacity, outage probability and SNR of MIMO receiver by adding log determinant signal with validated in terms of numerical simulation.

Keywords : MIMO, Periodic Circulant Signal, Channel Capacity, Outage Probability.

1. Introduction

Multiple Input Multiple Output (MIMO) system has attracted significant interests among researchers and developers of new generation wireless systems because of its potentials of achieving high spectral efficiency, by multiplexing multiple users on the same time-frequency resources. MIMO system is successfully investigated and deployed because of the rapidly increasing demand of multimedia challenges tremendously with high data rate and reliability. Telecommunication industry put MIMO technology as one of the most promising research area of wireless communication.[1]

MIMO system estimates channel capacity as a linear increase of spectral efficiency by utilizing diversity and spatial multiplexing technique. Capacity of MIMO systems utilizes random matrix theory which depends on number of transmitting (N_T) and receiving antenna (N_R).[10]

The key advantage of the stationary random process is ergodic process which gives the independent mean value of sequence. Recent works on the digital signal processing area have proposed extracting useful signals using Monte Carlo equation calculating the sum of probable bits as the coefficient terms which are in scalar form. The application of Monte Carlo method is very useful on today's applications of digital signal networks.[4] From a random digital variable which has probability mass function is greater than zero, it can extract an expected function by Monte Carlo equation. The expected function will generate a new signal with same probability. Actually discrete sum of function helps to generate expected signals without restriction with equal probability.[5]

In this paper, we add a periodic circulant signal in the receivers to improve channel capacity and the data rate of MIMO. Using Monte Carlo equation, signal is generated with statistically dependence of transmitting and noise discrete signal. The proposed concept helps to enhance the capacity of MIMO system by recovering loss signals in the receiver side, shown in a tree structure Figure 1.[6,10]

This paper is organized as follows. In the section 2 existing MIMO channel capacity is reported. System model with periodic circulant MIMO channel capacity and mathematical model are described in section 3. Simulation results comparing channel capacity rate between the existing and proposed system with SNR

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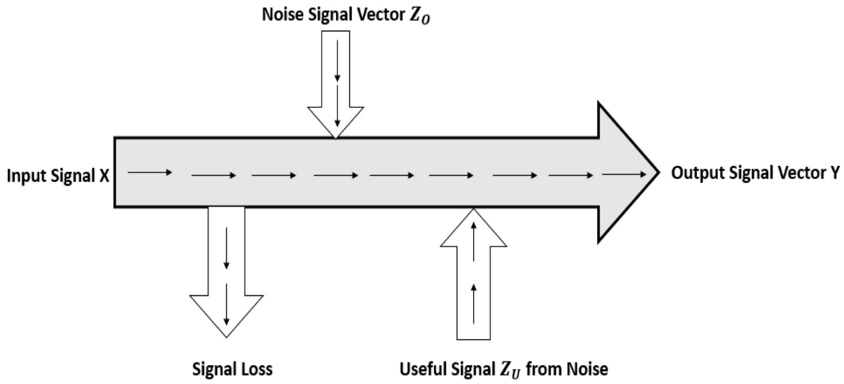

Fig. 1. Tree structure of the MIMO channel capacity maximization.

comparisons and the outage probability with cumulative distribution comparison are described in section 4. Finally, conclusions are stated in section 5.

2. MIMO Channel Capacity

In the existing MIMO system, the ergodic process is most convenient way to express the MIMO channel capacity which is calculated as statistical notation. The channel capacity of existing MIMO under the average transmitted power constraint is

$$\text{MIMO\_Cap} = \max_{\text{Tr}(\text{Rxx})=\text{N}_\text{T}} \log_2\left(\det\left(\mathbf{I}_{\text{N}_\text{R}} + \frac{\text{E}_\text{X}}{\text{N}_\text{T}\text{N}_\text{O}}\mathbf{H}\mathbf{H}^\text{H}\right)\right) \tag{1}$$

Where, **H** is the multiplication of number of transmitting (N_T) and receiving antenna **(N_R) i.e. $N_T \times N_R$** channel matrix. I_{N_R} denotes the identity matrix of size N_R, and $\mathbf{E_X}$ is the average signal to noise ratio (SNR) at each receiver branch. $\mathbf{H^H}$ is the transpose conjugate matrix. The elements of H are complex Gaussian with zero mean and unit variance.[2,7]

From the equation, we are going to add the method of channel capacity enhancement based on periodic circulant signal. To improve existing channel capacity of MIMO system, we are adding the non-negative eigenvalue of periodic circulant signal with its log determinant on the receiver.

3. Proposed Model

In the MIMO system, varying probability distribution function of transmits signal provides mutual information that can measures the information between two probabilities distributions functions (PDF) if the channel input and output are vector value instead of scalar one. The addictive white Gaussian noise (AWGN) channel removes the endowing phenomena of Ergodic quantities which contain each codeword. The definition of Entropy is also becoming same with mutual information when channel input and output are vector instead of scalar. Therefore eigenvalue from samples of complex exponentials is an excellent feature of circulant matrix. This is same fundamental idea used in eigenvalues of matrix and transfer function of linear systems. In this paper, we calculate the MIMO channel capacity based on matrix theory.

The complexity of the system increases with the increasing in super-polynomial value of codeword string n. The slice functions are used for measuring the circuit sizes.[10]

$$\textbf{For } \mathbf{n \geq 1} \qquad \textbf{complexity factor} = \mathbf{f_L}^{(\mathbf{n})}$$

If the value of n is sufficiently large then NP-complete estimates in complex circuits.

$$\mathbf{f_L}^{(\mathbf{n})} = \begin{cases} \mathbf{1} & \textbf{if codeword length} = \mathbf{n} \\ \mathbf{0} & \textbf{otherwise} \end{cases}$$

The output of AWGN channel with MIMO coded sequence of transmitting signal is not effected by the fading, amplitude loss, and phase distortion problem.[9] Although the considered signal will not suffer, we choose a high probability non-confusable bit sequence from those set of the sequence. In MIMO system, by using Zero-Mean Circular Symmetric Complex Gauss-

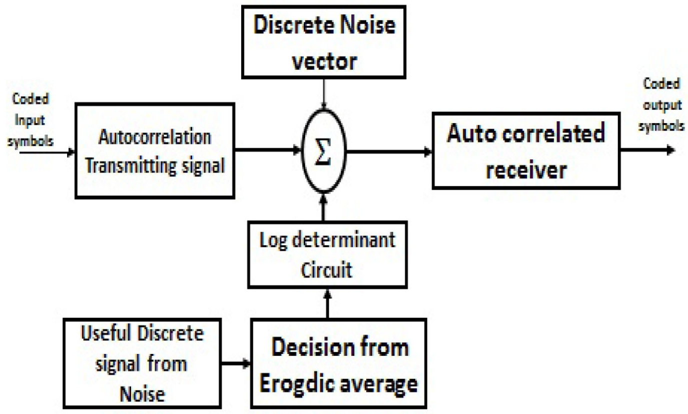

Fig. 2. Maximization of MIMO channel capacity using noise vector.

ian (ZMCSCG) signal, we can estimate maximum channel capacity. So we assume that all transmitting, receiving and noise vector are ZMCSCG.

In this system, we achieve maximum capacity when receiver receives an auto correlated signal from a transmitter which is a combination of autocorrelation function of transmitting signal, receiving signal and log determinant signal. In this circumstance Ergodic average provides the same PDF signals with comparing acceptable signals. Due to the random nature of signal, we generate the log determinant. Here, our proposed system model is shown in Figure 2.

We assume that discrete time signal vectors are positive Hermitian, validating the function of a wide-sense stationary random process with circulant approximation. The expression of circular convolution of discrete linear system has a key feature which is same as multiplication between circulant matrixes. Here, the circular convolution of the periodic circulant input signal is X, random matrix H with noise vector Z then discrete linear system can be express as

$$\mathbf{Y} = \sqrt{\frac{\mathbf{E_X}}{\mathbf{N_T}}}\mathbf{Hx} + \mathbf{Z_O}.$$

Now, we are adding circulant periodic signal ($\mathbf{Z}_\mathrm{U}$), which was taken from discrete noise vector ($\mathbf{Z_O}$). Therefore the useful noise vector is linear and dependence on the eigenvectors of Hermitian matrix of orthonormal, so the noise vector can be represented as squared Frobenius norm of MIMO channel. In this case, the transmitting power of each antenna is equal to 1. The useful signal Z_U is calculated from noise vector as a statistically dependence with transmitting signal vector, which is calculated as Monte Carlo equation from set of expected value of the function discrete noise signal. Depending upon X and $\mathbf{Z_O}$, our expected signal becomes ZMCSCG, circulant and periodic. The Monte Carlo equation from which useful signal calculated is

$$\mathbf{E}\big(\mathbf{Z_U}(\mathrm{Z_O})\big) = \textstyle\sum_{\mathbf{m} \epsilon \mathbf{Z}} \mathbf{Z_U}\, \mathbf{f}_{\mathbf{Z_0}}(\mathbf{m}) \tag{2}$$

where, **m** is our expected value from noise through function $\mathbf{f_{Z_O}}$ and element of circulant periodic matrix. By Comparing useful signal with the ergodic average, the decision circuit provides similar coded useful elements matrix signal $\mathbf{Z_U(k)} = \sum_{l=0}^{\infty}\mathbf{Z_O(k+l.N)}$, which gives similar value coded elements of **X**. The elements of generated signal make periodic signal which require additional hardware for the same codeword circuit, although it is not complex due to its same polynomial order. Then the useful matrix is,

$$\mathrm{Z_U} = \begin{bmatrix} \mathrm{Z_{U_0}} & \mathrm{Z_{U_{-1}}} & \cdots & \mathrm{Z_{U_{(-m+1)}}} \\ \mathrm{Z_{U_1}} & \cdots & \cdots & \mathrm{Z_{U_{(-m+2)}}} \\ \cdots & \cdots & \cdots & \cdots \\ \mathrm{Z_{U_{m-1}}} & \cdots. & \mathrm{Z_{U_1}} & \mathrm{Z_{U_0}} \end{bmatrix}$$

Then the receiving signal becomes

$$\mathbf{Y} = \sqrt{\frac{\mathbf{E_X}}{\mathbf{N_T}}}\mathbf{Hx} + \mathbf{Z_U} + \mathbf{Z_O} \tag{3}$$

Let us assume signal transmitting on AWGN channel of MIMO with noise has discrete i.i.d complex Gaussian sequence. We assume, here, the periodic circulant signal X and $\mathbf{Z_O}$ are in coded form and can be express as $\mathbf{X(k)} = \sum_{1=0}^{\infty}\mathbf{X(k+l.N)}$, with non-negative complex elements and noise vector $\mathbf{Z_O(k)} = \sum_{1=0}^{\infty}\mathbf{Z_O(k+l.N)}$, where the value of K and N should be properly selected. Discrete linear system utilizes random matrix theory as forms of convolution. The elements of noise vector $\mathbf{Z_O}$ can be written in matrix form

$$Z_O = \begin{bmatrix} Z_0 & Z_{(-1)} & Z_{(-2)} & \dots & \dots & Z_{(-N+1)} \\ Z_1 & Z_0 & Z_{(-1)} & \dots & \dots & Z_{(-N+2)} \\ \dots & \dots & Z_0 & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ Z_{N-2} & \dots & \dots & \dots & \dots & Z_1 \\ Z_{N-1} & Z_{N-2} & Z_{N-3} & \dots & \dots & Z_0 \end{bmatrix}$$

Here each element is coded and have equal power spectrum with plausibly asymptotically equal distribution from left or right corners as

$\mathbf{Z_{N-1} \approx Z_{(-1)}}$ **and** $\mathbf{Z_{(-N+1)} \approx Z_1}$ and so on.

Where the maximum mutual information obtains by varying transmitting PDF signals in receiver which estimate channel capacity of the system. It can be expressed as $\mathbf{I(X;Y) = H(Y) - H(Y|X)}$. We assume **Z** and **X** are statistically dependence discrete vector signal then, $\mathbf{H(X/Y) = H(Z) - H(X|Z)}$ and mutual information taken as

$$\mathbf{I(X;Y) = H(Y) + H(Y|X) - H(Z)} \tag{4}$$

By taking mutual information of respected signal in terms of entropy we can get following relationship

$$\mathrm{I}\,(X;Y) = \log_2(\det(\pi e R_{YY})) + \log_2\left(\det\left(\pi e R_{\frac{X}{Z}}\right)\right) - \log_2(\det(\pi e R_Z))$$

which results in the mutual information such as

$$\mathbf{I\,(X;Y) = \log_2\left(\det(R_{YY})\,(R_{\frac{x}{z}})\,(R_Z{}^{-1})\right)} \tag{5}$$

Currently Available Channel State Information **(CSI)** on transmitter cannot improve channel capacity of MIMO when SNR is high. In this regard, we assume **CSI** is not available and **H** is also unknown to transmitter then the energy is equally spreading in all direction and then autocorrelation function of the transmitting signal vector X becomes,

$$\mathbf{R_{XX} = I_{N_T}} \tag{6}$$

We expect that we can receive maximum amount of mutual information only when the receiving signal has also autocorrelation then,

$$R_{YY} = E((\sqrt{\tfrac{E_X}{N_T}}Hx + Z_U + Z_O)(\sqrt{\tfrac{E_X}{N_T}}Hx + Z_U + Z_O)^H)$$

$$= \frac{E_X}{N_T}(E(Hxx^H H^H + Z_U Z_U{}^H + Z_O Z_O{}^H))$$

To make it simpler, we assume independently transmitting energy equals to 1. which leads us,

$$R_{YY} = \frac{E_X}{N_T}((R_E + R_U + R_O)$$

After that we can denote useful signal as $R_{X/Z} = Z_U Z_U{}^H = R_U$ & $R_Z = Z_O Z_O{}^H = R_O$

Then, the resulting mutual information can be written as,

$$\mathbf{I\,(X;Y) = \log_2\left(\det\left(\frac{E_X}{N_T}((R_E + R_U + R_O))(R_U)(R_O{}^{-1})\right)\right)} \tag{7}$$

In the useful circulant periodic signal ($\mathbf{Z_U}$), the diagonal elements provide its eigenvalues which has equal probability.

$$\mathbf{R_{X/Z} = E(Z_U Z_U{}^H) = E\textstyle\sum_{l=1}^{M}\gamma_i = \sum_{l=1}^{M}\gamma_i} \tag{8}$$

Assuming that the PDF of useful signals follows chi-square distribution. This gives better performance when M tends to infinity. Then, the probability of useful signal calculated as

$$\mathbf{(\gamma_M) = \frac{1}{p}\textstyle\sum_{m\epsilon Z} q(m)P = \frac{1}{P}\,E(q(Z_U))} \tag{9}$$

where $\mathbf{Z_U}$ is the random variables that takes expected value of Z, and q is sum of the function over countable values. The matrix vector $\mathbf{Z_U}$ is circulant and periodic so we can solve the matrix similar to covariance matrix of both signal, divided by N_{Min}. The value of γ_m is taken as log basis due to random nature of useful signal.

$$\mathbf{\gamma_m = \log_2(\det(real(Z_U))} \tag{10}$$

Useful co-variance receiving matrix gives the value of $\mathbf{Z_U}$ as $Z_U = \frac{E_X}{N_M} I_M$, in case of transmitting orthonormal signals, $\mathbf{Z_U}$ becomes $Z_U = \frac{1}{N_M} I_M$ then the resulting equation (8) is

$$\gamma_M = \frac{1}{N_M}\log_2(\det(\mathrm{real}(I_M))) \quad (11)$$

where R_O is complex circulant periodic sequence, it gives eigenvalues when it multiplies by its inverse. By assigning all found values in an equation provides a new improved mutual information of the MIMO system.[3]

$$I(X;Y) = \log_2\left(\det\left(\frac{E_X}{N_T}\left((R_E R_U R_O^{-1} + R_U^2 R_O^{-1} + I_{N_R} R_U)\right)\right)\right) \quad (12)$$

The resulting useful circulant signal has non-negative values which works as increasing factor. The value of R_U is scalar and can be replace by γ_M. Then, the final mutual information can be,

$$I(X;Y) = \log_2\left(\det\left(I_{N_R}\gamma_M + \frac{E_X}{N_T N_O}\gamma_M^2 + \frac{E_X}{N_T N_O}\gamma_M HH^H\right)\right) \quad (13)$$

Hence, the calculated channel capacity of the MIMO from the mutual information is

$$\text{channel capacity}(C) = \underset{Tr(Rxx)=NT}{\mathrm{Max}} \log_2\left(\det\left(I_{N_R}\gamma_M + \frac{E_X}{N_T N_O}\gamma_M^2 + \frac{E_X}{N_T N_O}\gamma_M HH^H\right)\right) \quad (14)$$

This channel capacity of the system depends upon non zero, non-negative real value of γ_M. These values give overall MIMO system to some additional computation. Even by providing proper selection of **n** creates some complexity due to requirement of more super-polynomial circuit design in receiver side which gives bulk size estimation then the existing system.[5,10]

4. Simulation Results

To verify the theoretical model proposed in section 3 and section 4, MATLAB implementation is employed to showing capacity rate and obtain estimates of the source and mixing discrete signal through a matrix of N_T and N_R followed the algorithm of channel capacity. However, adding of a useful logarithmic determinant on the receiver estimates higher capacity rate. Simulation results show that linearly increasing in channel capacity is similar way of increasing in value of N_T and N_R which also help to reduce the necessity of the requiring high channel rank for high spectral efficiency. In MIMO system channel capacity is increases with increasing in N_T and N_R values, however, the addition of the determinant signal on receiver provides linear shifting of capacity rate to some positive extent as shown in Figure 3. The Simulation results shows the shifting value is linearly increasing with number of increased transmitting and receiving antennas. N_T has a greater impact on achieving new capacity rates in the receivers. Statistical notation arises the floating point round error, to recover this error, we take convenient arbitrary number known as binning technique. The comparison between the existing and proposed MIMO channel capacity rate using MATLAB is shown in Figure 3.[10]

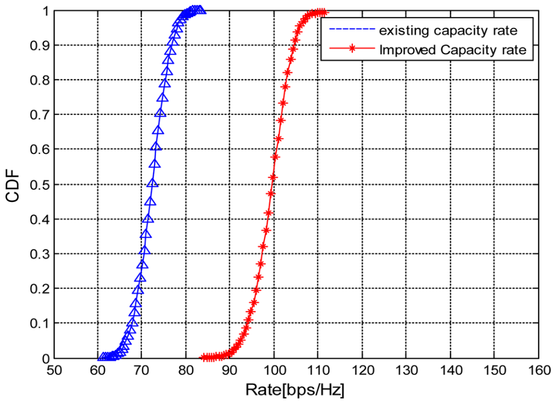

Fig. 3. Comparison of MIMO channel Capacity rate between existing and proposed system.

Bit-Error-Rate (**BER**) and Signal-to-Noise Ratio (**SNR**) of the receiving signal have inverse relation. SNR is ratio of useful signal and noise signal as form of voltage or power.[7] The SNR formula relating with BER in terms of diversity is $\mathbf{BER} \propto \frac{1}{\mathbf{SNR}^d}$ where **d** represents order of diversity of the MIMO system which provides better reliability of the system. But on the other hand spatial multiplexing of MIMO uses different antennas with differential stream which estimates high data rate. In this regard, the proposed model has linearly increasing SNR with high value, then existing MIMO

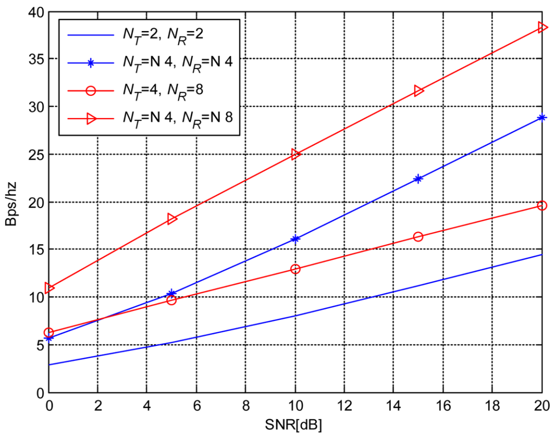

Fig. 4. Comparison of SNR between existing and proposed Model.

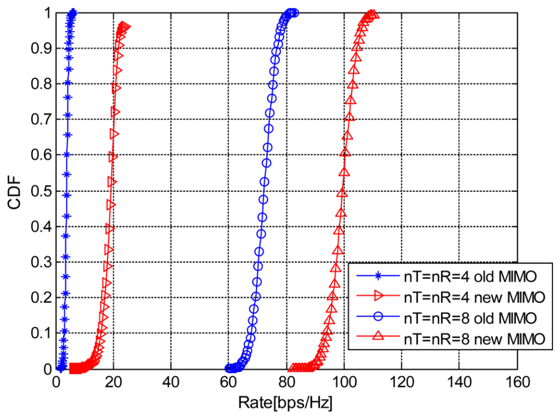

Fig. 5. CDF comparison between Existing and proposed MIMO system

system model estimates low BER as shown in Figure 4.[10]

As we know that, there is no possibility of completely removing decoding error however it can be minimizing to some level by using advance decoding scheme with certain threshold value. If random error bit rate is greater than threshold value, the corresponding probability is called as outage probability. Here, the mutual information and outage probability can be related as

$$\mathbf{P_{out} = P(I < R).}$$

Flat fading estimates capacity is random instantaneous value and taking constant for coded block of information. Using the probability law, the outage probability of capacity in terms of total probability is

$$\mathbf{P = P(I < R) = \textstyle\sum_C P[\,I < R|C]P[C]}$$

And its complex exponential outage probability is calculating statically as Monte Carlo estimation. When **BER** and signal-to-Noise-interference ratio (**SINR**) are equals, the threshold value depends upon the detection and diversity order of MIMO. The outage probability has inverse relation with SNR.[8] Therefore, we can be optimized the transmitting signal by reducing uncertainty in channel distribution. Simulation results in Figure 5 shows the higher value of **M** provides good cumulative distribution (CDF) of outage probability. From our simulation results we found that the statistical bin values for calculating CDF is directly affect the capacity rate of MIMO system.[10]

5. Conclusion

We proposed MIMO system capacity enhancement by using convolution of periodic circulant matrix-vector signal. The expression we derived from the statistical dependence of discrete transmitting and noise signal with AWGN channel is beneficial to carry information of MIMO at a higher rate than the existing system with the requirement of a higher number of transmitting and receiving antennas. The linear shifting of capacity mainly depends on the higher value of N_T in a random matrix. The higher SNR with higher channel capacity has been obtained due to the interception of digital communication signals. Our model predicts proper selection of transmitting signal codeword that can reduce the complexity of the system. However, proposed MIMO system is quite complex than existing onc. Our results show that the proposed system can increase channel capacity in MIMO system.[10]

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References

[1] Y. S. Cho, J. Kim, W. Y. Yang, C. G. Kang, MIMO-OFDMA Wireless Communication with MATLAB, John Wiley & Sons, 2010.

[2] B. Holter, "ON The CAPACITY OF THE MIMO CHANNEL – A Tutorial Introduction -", Technical Report, Norwegian University, 2010.

[3] DG. Messerschmitt, "Autocorrelation matrix eigenvalues and the power spectrum", Technical Report, UC Berkeley, 2006.

[4] EC. Anderson, "Monte Carlo Methods and Importance Sampling" Lecture Notes for Stat 578C Statistical Genetics, 1999.

[5] AD. Poularikas, Discrete Random Signal Processing and Filtering Primer with MATLAB, CRC Press, 2009.

[6] D. Tse, P. Viswanath, Fundamentals of Wireless Communication, 8 – MIMO II: capacity and multiplexing architectures, Cambridge University Press, pp. 332-382, 2005.

[7] A. S. Rao, S. Srinivas, "A New BER and SNR Calculations for MIMO System", International Journal of Inventive Engineering and Science (IJIES), ISSN: 2319-9598, Vol.2, Iss.8, 2014.

[8] Y. Zhao, R. Adve, and T. J. Lim, "Outage Probability at Arbitrary SNR in Cooperative Diversity Networks", IEEE Communications Letters, Vol.9, Iss.8, 2005.

[9] J. Singh, O. Dabeer, U. Madhow, "Capacity of the Discrete-Time AWGN Channel Under Output Quantization", arXiv:0801.1185, 2008.

[10] P. Poudel, A. Shrestha, P. Adhikaree, "Channel Capacity Maximization in MIMO System via Periodic Circulant Discrete Noise Vector", Journal of Sci. and Eng., Vol.1, Iss.1, pp. 1-6, 2020.