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The Time-Varying Coefficient Fama - French Five Factor Model: A Case Study in the Return of Japan Portfolios

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Abstract

In this paper, we have developed a Fama - French five factor model (FF5 model) from Fama & French (2015) by using concept of time-varying coefficient. For a data set, we have used monthly data form Kenneth R. French home page, it include Japan portfolios (classified by using size and book-to-market) and 5 factors from July 1990 to April 2020. The first analysis, we used Augmented Dickey-Fuller test (ADF test) for the stationary test, from the result, all Japan portfolios and 5 factors are stationary. Next analysis, we estimated a coefficient of Fama - French five factor model by using a generalized additive model with a thin-plate spline to create the time-varying coefficient Fama - French five factor model (TV-FF5 model). The benefit of this study is TV-FF5 model which can capture a different effect at different times of 5 factors but the traditional FF5 model can't do it. From the result, we can show a time-varying coefficient in all factors and in all portfolios, for time-varying coefficients of Rm-Rf, SMB, and HML are significant for all Japan portfolios, time-varying coefficients of RMW are positively significant for SM, and SH portfolio and time-varying coefficients of CMA are significant for SM, SH, and BM portfolio.

Keywords: Nonparametric Regression, Japanese Stock Exchange, Time-Varying Coefficient, Thin Plate Spline, Fama – French Five Factor Model

JEL Classification Code: C01, C14, C22, C58, G12

1. Introduction

Capital Asset Pricing Model (CAPM) was discovered by Sharpe (1964) which showed correlation between the market risk premium and a stock return. CAPM have been improved and many of research have been carried out to verify it. Later, Fama & French (1993) developed the Fama - French three factor model (FF3 model) from CAPM by adding size, and book to market ratio. Therefore, FF3 model is a very significant part in analysis. Shaharuddin, Lau, & Ahmad (2018) also used the FF3 model and found the FF3 model is valid for Islamic unit trust funds before and after the collapse of Lehman Brothers. In addition, Asmarani & Wijaya (2020) found the market risk premium and value risk premium are significant positive relationship on retail banks stock return listed in Indonesia Stock Exchange. Later, Fama & French (2015) proposed including profitability and investment to create the Fama - French five factor models (FF5 model) as the following equation,

$$R_{it} - Rf_t = a_i + b_i\,(Rm - Rf\,)_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t; \quad (1)$$

where R_i is the return of portfolio i, Rf is the risk-free rate, Rm is return of the market portfolio, SMB is small minus big, HML is high minus low, RMW is robust minus weak, and CMA is conservative minus aggressive.

Although the CAPM, FF3 model, and FF5 model are popular and effective for explain and predicting the stock returns but those models are time-invariant parameters.

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Hence, the analysis does not come close to the fact that each factor has a different effect over time. For the publication about the time-varying coefficient, Hongsakulvasu & Liammukda (2020) had studied the time-varying coefficient autoregressive model by using a generalized additive model with thin-plate spline to predict 4 Asian stock price index, from a result of this study, the coefficients of the models were varied over time. Therefore, it is interesting to know if we can use the time-varying coefficient on the FF5 model.

2. Statistical Theory and Literature Review

2.1. Time-Varying Coefficient with the Generalized Additive Model

For the time-varying coefficient model, it assumed one time-varying coefficient (c_t) for the model to be estimated in equation (2) and using regression splines method to estimate the time-varying coefficient as the following equation (3).

$$r_t = c_t + \varepsilon_t \tag{2}$$

$$\hat{c}_t = \hat{\alpha}_1 R_1(t) + \hat{\alpha}_2 R_2(t) + \hat{\alpha}_3 R_3(t) + ... + \hat{\alpha}_K R_K(t); \tag{3}$$

where we have estimation $\hat{c}_t$ with term of K basis function $R_1(t)$, …, $R_k(t)$ and t represent the time, then $\hat{c}_t$ can be estimated by using a linear regression method. For shape of $\hat{c}$, It can be either linear or non-linear shape which is dependent on time. In practice, $\hat{c}_t$ has many approaches on the type of basis function such as a cubic regression splines and thin-plate regression splines. One can call this method as a data driven approach, since each basis function can be calculated from the data (Bringmann et al., 2017; Hongsakulvasu & Liammukda, 2020).

2.2. Thin Plate Regression Splines

The general solution of estimating a smooth function, there are many ways and one of them is "Thin-plate splines". For a simple explanation, the estimation of y_i has one independent variable **x** with smooth function g (**x**) where **x** is a d-vector, from n ($\geq d$) observations (y_i, $\mathbf{x}_i$) such that

$$y_i = g(\mathbf{x}_i) + \varepsilon_i \tag{4}$$

where ε_i is a random residual of estimation. Function g can smoothen estimates with thin-plate spline by optimization base on function g minimizing.

$$\sum_{i=1}^{n} \|\mathbf{y} - \mathbf{g}\|^2 + \lambda J_{md}(g) \tag{5}$$

where **y** is the vector of y_t, $\mathbf{g} = (g(x_1), \ldots, g(x_n))'$, λ is a parameter that represents a trade-off between smoothness and fitting of g, J_{md} (g) is a penalty functional measuring the wiggliness of g. J_{md} (g)which is defined as:

$$J_{md} = \int \ldots \int_{\mathbb{R}^d} \sum_{v_t + \ldots + v_t = m} \frac{m!}{v_1! \ldots v_d!} \left(\frac{\partial^m g}{\partial x_1^{v_1} \ldots \partial x_d^{v_d}} \right)^2 dx_1 \ldots dx_d \tag{6}$$

The analysis has a one technical restriction is $2m > d$, it can be shown that the function minimizing expression (5) has the form

$$g(\mathbf{x}) = \sum_{i=1}^{n} \delta_i \eta_{md} \left(\|\mathbf{x} - \mathbf{x}_i\| \right) + \sum_{j=1}^{M} \alpha_j \phi_j(\mathrm{x}) \tag{7}$$

where δ and α are unknown parameter vectors under condition that $T_{ij} = \varphi_j(xi)$ and. $T'\delta = 0$.The $M = \binom{m+d-1}{d}$ functions φ_i are linearly independent polynomials spanning the space of polynomials $\mathbb{R}^d$ in d of degree less than m (wood, 2006).

$$\eta_{md}(r) = \begin{cases} \dfrac{(-1)^{m+1+d/2}}{2^{2m-1}\pi^{d/2}(m-1)!(m-d/2)!} r^{2m-d} \log(r) & d \ \ event, \\ \dfrac{\Gamma(d/2-m)}{2^{2m-1}\pi^{d/2}(m-1)!} r^{2m-d} & d \ \ odd, \end{cases} \tag{8}$$

Next, defining matrix E by $E_{ij} \equiv \eta_{md}(\|x - x_i\|)$, the thin-plate spline fitting problem are defined as the following equation:

$$\text{Minimize } \|\mathbf{y} - E\delta - T\alpha\|^2 + \lambda\delta' E\delta \ \text{ Subject to } T'\delta = 0 \tag{9}$$

With respect to δ and α. The function g estimated with system of equation (9) is something of an ideal smoother because it considered the smoothness of the system of equations by using λ to weight a penalty functional J_{md} (g).

The last, estimate an optimal smoothing parameter λ and the number of basis dimensions. This can be done in generalized additive model by Generalized Cross Validation score (GCV). It minimizes as the following equation:

$$GCV = \frac{T\sum_{t=1}^{T}\left(y_t - \hat{y}_t\right)^2}{\left[tr\left(I - A\right)\right]^2} \tag{10}$$

where A is the projection matrix. If a value of smoothing parameter λ is close to 1 then spline will be over-smoothed, On the opposite side, when value of smoothing parameter λ is close to zero than spline isn't penalized, so the method behaves like a classical ordinary least squares (OLS). With a number of basis dimensions of Estimated Degrees of Freedom (EDF), it is opposite. Higher EDF refers to that fit will be overfit (less smoothed), on the other side lower EDF refers to more smoothed behavior of fitted values (Wahba, 1980; Wood, 2006).

2.3. Augmented Dickey-Fuller test

An augmented Dickey-Fuller test apply for the testing of a complicated sets of time series models, the disturbances of the Augmented Dickey-Fuller test require to be white noise, however whether the disturbances of the model have some forms of serial correlation, the stationary will be executed by ADF Unit Root Test. The descriptive equation as follows:

$$y_t = c_t + \gamma\, y_{t-1}\sum_{i=1}^{p}\phi_i\, \Delta y_{t-i} + e_t \tag{12}$$

where c_t is a deterministic function of time index and it can be divided into three type as the flowing, there is no intercept term, then $c_t = 0$, there is intercept term, then c_t = constant, and there is intercept and trend, then $c_t = \mu + \beta_t$.

Therefore, the null and alternative hypothesis can be written as, $H_0 : \gamma = 1$ for non-stationary (there is unit root in time series data) and $H_1 : \gamma < 1$ for stationary. According to the first differencing the above equation by deduction y_{t-1} on both side, then we get the following equation;

$$\Delta y_t = c_t + \gamma_c\, y_{t-1}\sum_{i=1}^{p-1}\phi_i\, \Delta y_{t-i} + e_t; \tag{13}$$

where $\gamma_c = \gamma - 1$. After first differencing equation (13), the null and alternative hypothesis are $H_0 : \gamma_c = 0$ for non-stationary and $H_1 : \gamma_c < 0$ for stationary. The ADF test statistic is defined as

$$ADF = \frac{\hat{\gamma} - 1}{std\left(\hat{\gamma}\right)} \tag{14}$$

where $\hat{\gamma}$ is the coefficient estimation and $std\left(\hat{\gamma}\right)$ is its corresponding estimation of standard error for each type of linear model. The p-value is calculated by interpolating the test statistics from the corresponding critical values tables.

2.4. Literature Review

After Fama & French proposed FF5 model by adding RMW and CMA factors on FF3 model, many researchers examined this model and used it to explain a return of portfolios. We can conclude publication about FF5 model as the following

Sundqvist (2017) studied average returns in the Nordic markets during the period from December 1997 to June 2016 by using CAPM, FF3 model, and FF5 model. They can explain the average returns of portfolios sorted on size and book-to-market ratio, and portfolios sorted on size and investment, but it is not happening for portfolios sorted on size and profitability. While the FF5 model provides a more mean-variance efficient portfolio from its explanatory variables which is close to a study by Huang (2019), He compared the CAPM, FF3 model, Carhart4, and FF5 model in explaining individual stock returns in China from January 1994 to December 2016. RMW and CMA factors were significant and the FF5 model was the highest performing compared to other traditional asset pricing models in explaining individual stock returns in China. In addition, Mosoeu & Kodongo (2019) used a Generalized Method of Moments (GMM) for estimating parameters in the FF5model and used average stock returns for emerging and selected developed equity markets from January 2010 to December 2015. From the result, RMW was found to be most effective for explaining average equity returns in emerging markets.

But the results of many researchers' studies are not the same as those of the above studies. Nguyen (2016) studied the average return of stock in Vietnam stock exchange for the period of January 2011 to December 2015 by using CAPM, FF3 model, and FF5 model. The result of this study shows that from CAPM to FF5 model, the R-square increases gradually and FF5 model has the highest R-square, but it was only 34 percent and RMW and CMA proved insignificant in explaining the stock returns. Which is close to the study by Kubota and Takehara (2018), They investigated the FF5 model for explaining return of stocks in Tokyo Stock Exchange (TSE) from January 1978 to December 2014, they used a Generalized Method of Moments (GMM) for estimating parameter in FF5 model. From the result, they found that RMW and CMA are insignificant in explaining return from stocks. Thus, they concluded that the original version of the FF5 model was not the best benchmark pricing model for the Japanese. In addition, Wijaya, Irawan, & Mahadwartha (2018) explained the stocks listed in the LQ-45 Index since January 2013 to December 2015 by using FF5 model. Rm-Rf, HML, and CMA had a positively significant effect on return and SMB had negatively significant effect on return, but RMW was insignificant.

On the other hand, some researchers developed FF5 model to increase the performance of this model. Chiah, Chai, Zhong, & Li (2016) investigated the performance of FF5 model in pricing Australian equities since January 1982 to December 2013. They found that the FF5 model can explain more asset pricing anomalies than a range of competing asset pricing models, which supports the superiority of the FF5 model. Moreover, they can capture the volatility of equities by used the GARCH model. Furthermore, de la O González & Jareño (2019) studied U.S. sector returns from November 1989 to February 2014 by using the FF5 model base on Quantile linear regression. From the result, the extreme value at quantile 0.1 of the return distribution has the best results. In addition, Jan & Ayub (2019) forecasted the stock returns in the emerging markets by using the FF5 model with Artificial Neural Networks (ANN). Their study reinforces the financial concept that says "high risk - high return" and the FF5 model base on ANN will significantly improve the return on investments.

From the literature review, every publication used the FF5 model base on a multiple linear regression model with time-invariant parameters, so those models cannot capture and show a different effect at different times for the 5 factors. By this point, it is interesting to study that we can capture and show a different effect at different times for 5 factors or not?

3. Research Methods and Materials

3.1. Scope of the Study and Data Used

This study was conducted by using monthly data set from Kenneth R. French home page *mba.tuck.dartmouth.edu/pages/faculty/ken.french/*. The data starts from July 1990 to April 2020, the details are as follows:

1. Japan portfolios are weight return of stocks in Tokyo Stock Exchange (TSE) which is sorted on size and book-to-market ratio. The portfolios are the intersections of 2 portfolios formed on size (market equity, ME) and 3 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoint for year t is the median TSE market equity at the end of June of year t. The BE/ME breakpoints are the 30th and 70th TSE percentiles (see Table 1).

Table 1: Japan portfolios

Size	Book-to-Market		
	Bottom 30%	*40%*	*Top 30%*
Top 50%	big value (*BL*)	big neutral (*BM*)	big growth (*BH*)
Bottom 50%	small value (*SL*)	small neutral (*SM*)	small growth m(*SH*)

2. Return of risk-free rate (Rf) is Return on a 1-Month Treasury Bill (short-term U.S. government debt obligation).
3. Return of the market portfolio (Rm) is value-weight return of firms in the Tokyo Stock Exchange.
4. Fama - French 5 Factor are internal factor including Market Risk Premium, Small Minus Big, High Minus Low, Robust Minus Weak, and Conservative Minus Aggressive with details as follows:

Market Risk Premium (Rm-Rf) describes the relationship between returns from an equity market portfolio and treasury bond yields. It reflects required returns, historical returns, and expected returns. The historical market risk premium will be the same for all investors since the value is based on what has happened. It can be calculated by using the return of the market portfolio minus the return of the market portfolio.

Small Minus Big (SMB) reflects a size risk premium, It can be calculated by using the average return on the nine small stock portfolios minus the average return on the nine big stock portfolios.

High Minus Low (HML) reflects a value risk premium, It can be calculated by using the average return on the two value portfolios minus the average return on the two growth portfolios.

Robust Minus Weak (RMW) reflects an profitability risk premium, It can be calculated by using the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios (operating profit is profits from a core business function of company).

Conservative Minus Aggressive (CMA) reflects an investment risk premium, It can be calculated by using the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios.

3.2. Empirical Methods

For the first analysis of the time-series data, we must check stationary conditions on an endogenous and exogenous variable by using the Augmented Dickey-Fuller test (ADF test) with 3 equations including no intercept, Intercept, and intercept & trend because if data is nonstationary, it maybe spurious regression.

For the next analysis, we use time-varying coefficient Fama - French five factor model (TV-FF5 model) to explain the return of Japan portfolios as the following equation:

$$R_{it} - Rf_t = a_{it} + b_{it}(Rm - Rf)_t + s_{it}SMB_t + h_{it}HML_t + r_{it}RMW_t + c_{it}CMA_t + \varepsilon_t; \quad (18)$$

where the time-varying coefficient $\hat{a}_{it}$, $\hat{b}_{it}$, $\hat{s}_{it}$, $\hat{h}_{it}$, $\hat{r}_{it}$, and $\hat{c}_{it}$ can be estimated by using generalized additive model with thin-plate spline as the following equation:

$$\hat{g}_t = \hat{\gamma}_1 TP_1(t) + \hat{\gamma}_2 TP_2(t) + \hat{\gamma}_3 TP_3(t) + \ldots + \hat{\gamma}_K TP_K(t); \quad (19)$$

where $\hat{\mathbf{g}}$ is the time-varying coefficient $\hat{a}_{it}$, $\hat{b}_{it}$, $\hat{s}_{it}$, $\hat{h}_{it}$, $\hat{r}_{it}$, and $\hat{c}_{it}$ and we have estimation $\hat{\mathbf{g}}$ with term of K basis function $TP_1(t), \ldots, TP_k(t)$ and t represents the time. In this study, we used Generalized Additive Model (GAM) for the estimation of time-varying coefficient $\hat{a}_{it}$, $\hat{b}_{it}$, $\hat{s}_{it}$, $\hat{h}_{it}$, $\hat{r}_{it}$, and $\hat{c}_{it}$ and used thin-plate splines for basis function $TP_1(t), \ldots, TP_k(t)$ because there are no need to choose the knot locations and it performs very well when we have many independent variables (Bringmann et al., 2017; Hongsakulvasu & Liammukda, 2020). The last, we optimal value of estimated time-varying coefficient $\hat{a}_{it}$, $\hat{b}_{it}$, $\hat{s}_{it}$, $\hat{h}_{it}$, $\hat{r}_{it}$, and $\hat{c}_{it}$ can be found when we perform the minimization of this penalized least squares loss function as the following equation

$$\sum_{i=1}^{n} \|\varepsilon_t\|^2 + \lambda J_{md}\left(R_{it} - Rf_t - \varepsilon_t\right) \quad (20)$$

where $\varepsilon_t = R_{it} - Rf_t - a_{it} - b_{it}(Rm - Rf)_t - s_{it}SMB_t - h_{it}HML_t - r_{it}RMW_t + c_{it}CMA_t$. The last, we are finding a smoothing parameter λ as by using Generalized Cross Validation score (GCV) for optimize equation (20) as the following equation

$$GCV = \frac{T\sum_{t=1}^{T}\left(\widehat{\varepsilon_t}\right)^2}{\left[tr\left(I - A\right)\right]^2} \quad (21)$$

4. Results and Discussion

4.1. Descriptive Analysis and Stationary Test

For the first analysis, we study by using descriptive analysis for showing a basic information including mean, standard deviation, skewness, and kurtosis of data as the following (Table 2). To check the stationary condition of the data, we perform the Augmented Dickey-Fuller test (ADF test) with 3 equations including 1.no intercept, 2.intercept, and 3.intercept & trend. According to the results of ADF statistics in Table 2, we reject the null hypothesis of non-stationarity, so, we can conclude that all data is stationary.

4.2. Time-Varying Coefficient Fama - French Five Factor Model

The results of the time-varying coefficient Fama - French five factor model by using generalized additive model with thin-plate spline are reported in Table 3, Table 4, and Figure 1.

According to Table 3. The values in the row of a_t, b_t, s_t, h_t, r_t, and c_t are Effective Degrees of Freedom (EDF), It refers to the number of basis functions that are used to estimate the time-varying coefficient. If the value of EDF is high, it means that the model uses many basis functions to estimate the time-varying coefficient, so shape of the time-varying coefficient is non-linear and wiggly. On the other hand, if the value of EDF is close to 1, the shape of the time-varying coefficient is linear (Shadish, Zuur, & Sullivan, 2014). In addition, asterisks in Table 4 refer to the rejection of null-hypothesis which means that the time-varying coefficient is significantly different from zero (Wood, 2013).

Table 2: Descriptive statistics and ADF test results.

Variable	Mean	St. Dev.	Skewness	Kurtosis	ADF statistics		
					None	Intercept	Trend & Intercept
Rm-Rf	0.057	5.657	0.306	4.317	-7.703***	-7.701***	-7.710***
SMB	0.108	3.185	0.103	4.826	-8.152***	-8.171***	-8.247***
HML	0.255	2.890	-0.202	5.035	-6.943***	-7.043***	-7.144***
RMW	0.133	2.117	0.000	4.969	-8.010***	-8.076***	-8.068***
CMA	0.046	2.364	-0.738	7.179	-6.605***	-6.596***	-6.594***
SL-Rf	0.005	7.598	0.365	4.009	-7.125***	-7.117***	-7.164***
SM-Rf	0.133	6.570	0.399	5.074	-7.816***	-7.824***	-7.876***
SH-Rf	0.279	6.529	0.344	4.421	-7.959***	-8.006***	-8.031***
BL-Rf	-0.017	6.061	0.246	4.766	-7.627***	-7.617***	-7.658***
BM-Rf	0.085	5.511	0.402	4.615	-8.044***	-8.047***	-8.050***
BH-Rf	0.218	6.042	0.424	4.064	-7.849***	-7.888***	-7.864***

Notes: Asterisks indicate the rejection of null hypothesis statistical at the 10% (*), 5% (**) or 1% (***) level.

Table 3: Time-varying coefficient Fama - French five factor model results.

Coefficient	Portfolio					
	SL	*SM*	*SH*	*BL*	*BM*	*BH*
a_t	1.0008	1.3249	1.0001	1.5602	1.0008	1.0002
b_t	2.0003***	3.6161***	2.0000***	2.0000***	4.7687***	2.0003***
s_t	4.6532***	3.1778***	6.4261***	2.0009***	2.2563***	5.9723***
h_t	4.2221***	4.0462*	4.3723***	3.7617***	4.1508***	2.1085***
r_t	2.0003	2.0006**	7.8928***	2.0006	2.0002	2.0080
c_t	3.1420	4.9372***	2.0002***	2.0006	5.2953**	2.4651

Note: The values in the row of a_t, b_t, s_t, h_t, r_t, and c_t are EDF which refer to the number of basis functions that are used to estimate the time-varying coefficient in the model. Asterisks indicate the rejection of null hypothesis statistical at the 10% (*), 5% (**) or 1% (***) level

Table 4: Timeline of start and end significant time-varying coefficients for the TV-FF5 model.

Coefficient	Positive effect		Negative effect	
	Portfolios	**Date**	**Portfolios**	**Date**
a_t	-	-	-	-
b_t	*SL, SM, SH, BL ,BM ,BH*	July 1990 to April 2020	-	-
s_t	*SL, SM, SH,* *BH*	July 1990 to April 2020 May 1998 to December 2008	*BL* *BM* *BH*	July 1990 to April 2020 September 1991 to April 2020 July 1992 to November 1995
h_t	*SH, BH* *BM*	July 1990 to April 2020 March 1994 to April 2010	*SL, BL* *SM*	July 1990 to April 2020 July1990 to November 1994
r_t	*SM* *SH*	July 1990 to September 1996 July 1990 to September 1991	*SH*	January 1994 to April 1996, February 2008 to January 2010
c_t	*SM* *SH* *BM*	September 1992 to October 2003 August 1999 to October 2018 May 1997 to March 1999	-	-

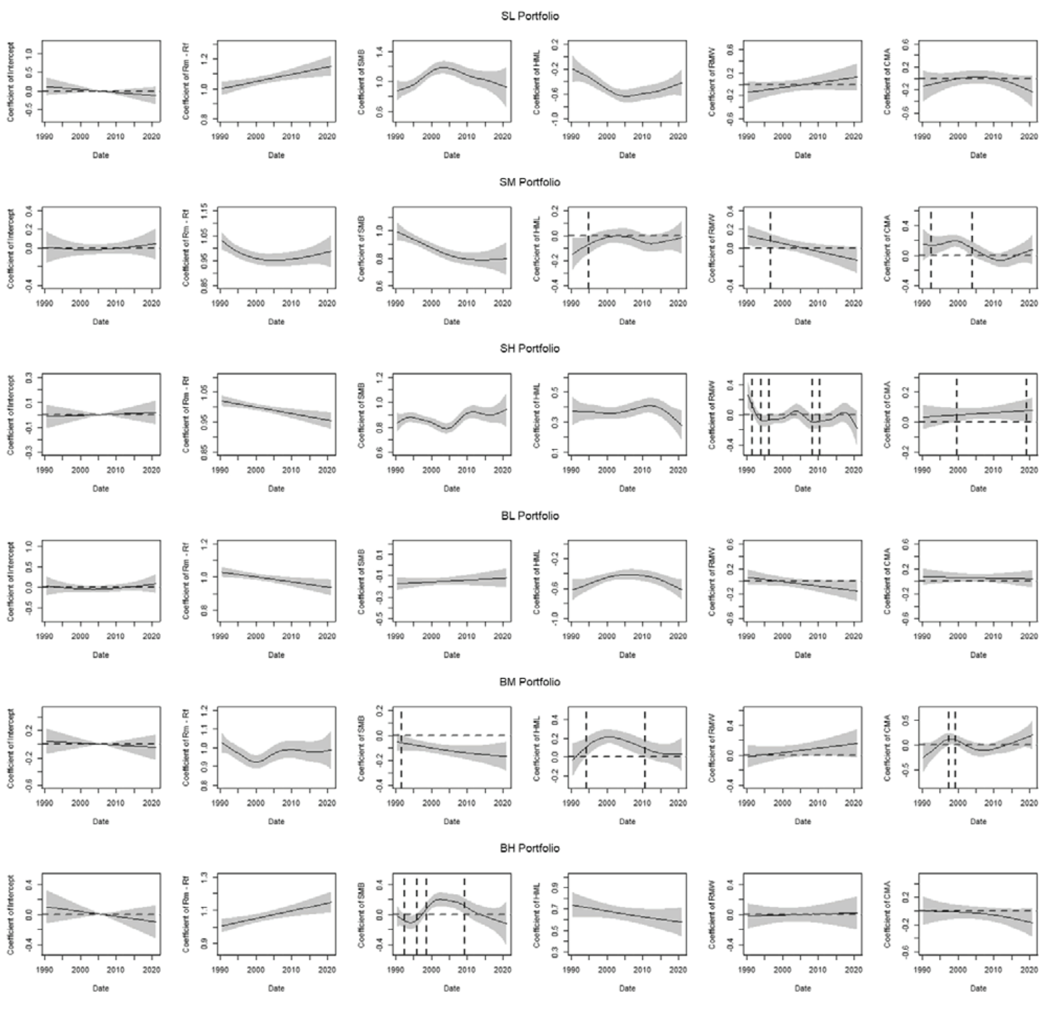
Figure 1 shows the estimated time-varying coefficients $\hat{a}_{it}$, $\hat{b}_{it}$, $\hat{s}_{it}$, $\hat{h}_{it}$, $\hat{r}_{it}$, and $\hat{c}_{it}$ of all portfolios since July 1990 to April 2020, horizontal dotted lines are crossed with the gray areas (95% confidence interval) which means that the time-varying coefficient is insignificantly different from zero, vertical dotted lines are crossed with the gray areas (95% confidence interval) which means that the time-varying coefficient is start or end of significant time-varying coefficients, and timeline of start and end significant time-varying coefficients are reported in Table 4.

According to Table 3, Table 4, and Figure 1. Time-varying coefficients of intercept ($\hat{a}_{it}$) are insignificant for all Japan portfolios since July 1990 to April 2020 and the value of EDF close to 1, it means the shape of $\hat{a}_t$ are linearly.

Time-varying coefficients of Rm − Rf ($\hat{b}_t$) are positive significant for all Japan portfolios since July 1990 to April 2020, the value of EDF for SL, SH, BL, and BH portfolio are close to 2 and the value of EDF for SM and BM portfolio are 3.6 and 4.7 respectively, so the shape of $\hat{b}_t$ for SL, SH, BL, and BH portfolio is close to linearly but the shape of $\hat{b}_t$ for SM and BM portfolios are nonlinearly.

Time-varying coefficients of SML ($\hat{s}_t$) are positive significant for SL, SM, and SH portfolio and negative significant for BL portfolio since July 1990 to April 2020, the value of EDF for SL, SM, and SH portfolio are 4.6, 3.1, and 6.4 respectively, so the shape of $\hat{s}_t$ for SL, SM, and SH portfolio are nonlinearly, and the value of EDF for BL portfolio is close to 2, so the shape of $\hat{s}_t$ for BL portfolio is close to linearly. $\hat{s}_t$ of BM and BH portfolio are partial significant, $\hat{s}_t$ of BM portfolio is significant since September 1991 to April 2020 and the shape of $\hat{s}_t$ for BM portfolio is close to linearly becuse the value of EDF is 2.2, $\hat{s}_t$ of BH portfolio is positively significant since May 1998 to December 2008 and negative significant since July 1992 to November 1995 and the shape of $\hat{s}_t$ for BH portfolio is nonlinear becuse the value of EDF is 5.9.

Time-varying coefficients of HML ($\hat{h}_t$) are positive significant for SH and BH portfolio and negative significant for SL and BL portfolio since July 1990 to April 2020, the value of EDF for SL, SH, BL, and BH portfolios are 4.2, 4.3, 3.7, and 2.1 respectively, so the shape of $\hat{h}_t$ for SL, SH, and BL portfolio are nonlinearly but the shape of $\hat{h}_t$ for BH portfolio is close to linearly, $\hat{h}_t$ for SH portfolio is positively significant since July 1990 to September 1991 and the shape of $\hat{h}_t$ for SH portfolio is nonlinearly becuse the value of EDF is 4.3, $\hat{h}_t$ for SM portfolio is negatively significant since February 2008 to January 2010 and the shape of $\hat{h}_t$ for SM portfolio is close to linearly becuse the value of EDF is 2.1.



Note: The columns are referred to time-varying coefficients in order as follows $\hat{a}_{it}, \hat{b}_{it}, \hat{s}_{it}, \hat{h}_{it}, \hat{r}_{it}$, and $\hat{c}_{it}$ and rows are referred to Japan portfolio in order as follows SL, SM, SH, BL, BM, and BH portfolio. The solid black line is the estimated time varying coefficient and the gray area are 95% confident interval.

Figure 1: The estimated Time-varying coefficients $\hat{a}_{it}, \hat{b}_{it}, \hat{s}_{it}, \hat{h}_{it}, \hat{r}_{it}$, and $\hat{c}_{it}$ of all portfolios.

Time-varying coefficient of RMW ($\hat{r}_t$) is positive significant for SM since July 1990 to September 1996 and the shape of $\hat{r}_t$ is close to linearly becuse the value of EDF is 2. For SH portfolio, $\hat{r}_t$ have 2 effect on this portfolio, $\hat{r}_t$ is positive significant since July 1990 to September but January 1994 to April 1996 and February 2008 to January 2010 are negatively significant, and the shape of $\hat{r}_t$ is heavy nonlinearly becuse the value of EDF is 7.8.

Time-varying coefficient of CMA ($\hat{c}_t$) are positive significant for SM, SH, and BM portfolios in difference time, $\hat{c}_t$ for SM portfolio is significant since September 1992 to October 2003, $\hat{c}_t$ for SH portfolio is significant since August 1999 to October 2018, and $\hat{c}_t$ for BM portfolio is significant since May 1997 to March 1999, and the value of EDF are 4.9, 2, and 5.2 respectively, so the shape of $\hat{c}_t$ for SM and BM portfolios are nonlinearly but SH portfolio is close to linearly.

5. Conclusions

From the result of the study, we can estimate time-varying coefficients of Fama - French five factor model for explaining the return of Japan portfolio from July 1990 to April 2020. For the time-varying coefficients of intercept are insignificant for all Japan portfolios. The time-varying coefficients of Rm-Rf are positively significant for all Japan portfolios. The time-varying coefficients of SMB are positively significant for SL, SM, SH, and BH portfolio but negative significant for BL, BM, and BH portfolio. The time-varying coefficients of HML are positively significant for SH, BM, and BH portfolio but negatively significant for SL, SM, and BL portfolio. The time-varying coefficients of RMW are positively significant for SM, and SH portfolio but negatively significant for the SH portfolio. The time-varying coefficients of CMA are positively significant for SM, SH, and BM portfolio.

In conclusion, the benefit of time-varying coefficients Fama - French five factor model over than traditional model is that the time-varying coefficients model can capture and show a different effect at different times of 5 factors on Japan portfolios and some factors such as SMB and RMW have a positive and negative effect on portfolios in a different time but the traditional model can't capture and show a different effect at different times. For developing the model in the next time, we will capture and show a time-varying volatility by plugin Generalized Autoregressive Conditional Heteroscedasticity model (GARCH model) on time-varying coefficients Fama - French five factor model.

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