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TRANSIENT THERMOELASTIC STRESS ANALYSIS OF A THIN CIRCULAR PLATE DUE TO UNIFORM INTERNAL HEAT GENERATION

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ABSTRACT. The present work aims to analyzed the transient thermoelastic stress analysis of a thin circular plate with uniform internal heat generation. Initially, the plate is characterized by a parabolic temperature distribution along the z-direction given by $T = T_0(r, z)$ and perfectly insulated at the ends $z = 0$ and $z = h$. For times $t > 0$, the surface $r = a$ is subjected to convection heat transfer with convection coefficient h_c and fluid temperature T_∞. The integral transform method used to obtain the analytical solution for temperature, displacement, and thermal stresses. The associated thermoelastic field is analyzed by making use of the temperature and thermoelastic displacement potential function. Numerical results are carried out with the help of computational software PTC Mathcad Prime-3.1 and shown in figures.

1. INTRODUCTION

Nowacki [1] has studied the steady-state thermoelastic problem of a thick circular plate subject to axisymmetric temperature distribution on the upper surface with the lower surface is kept at zero temperatures and the fixed circular edge is thermally insulated. The direct thermoelastic problem of normal deflection due to axisymmetric heat supply on a circular plate in the case of fixed and simply supported edges have been considered in [2]. The approximate analytical and the exact solutions of the one-dimensional transient thermoelastic problems of heat flux and temperature determination on the surface of an isotropic infinite slab presented in [3]. Theoretical analysis of a three-dimensional transient thermoelastic problem of a non-homogeneous hollow circular cylinder due to a moving heat source in the axial direction from the inner and outer surfaces were presented in [4]. The transient thermoelastoplastic bending problems making use of the strain increment theorem and thermoelastic deformation of the circular plate due to a partially distributed heat supply was studied in [5]. The transient heat conduction and analysis of thermal stresses in a thin circular plate subjected to some different types of boundary conditions were presented in [6]. Some contributions of this theory are given

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in [7, 8, 9, 10, 11, 12]. The nonhomogeneous heat conduction problem of a thin hollow circular disk and its thermal deflection under heat generation was solved in [13]. The thermoelastic analysis and its deformation of a thin hollow circular disk subject to a partially distributed and axisymmetric heat supply on the upper surface studied in [14]. Recently, many thermoelastic problems have been discussed [15, 16, 17, 18, 19, 20].

In this work, a two-dimensional transient thermoelastic problem of a thin circular plate due to uniform internal heat generation was investigated. Initially, the plate is characterized by a parabolic temperature distribution along the z-direction given by $T = T_0(r, z)$ and perfectly insulated at the ends $z = 0$ and $z = h$. For times $t > 0$, the surface $r = a$ is subjected to convection heat transfer with convection coefficient h_c and fluid temperature T_∞. The integral transform method was used to obtain the analytical solution for temperature, displacement, and thermal stresses. The associated thermoelastic field is analyzed by making use of the temperature and thermoelastic displacement potential function. Numerical results are carried out with the help of computational software PTC Mathcad Prime-3.1 and shown in figures. No work has been carried out so far dealing with a transient thermoelastic stress analysis of a thin circular plate with uniform internal heat generation to the best of my knowledge.

2. The Problem Formulation and Governing Equations

Consider a thin circular plate is depicted as shown in Figure 1. The plate is of radius a and thickness h and may be considered perfectly insulated at the ends $z = 0$ and $z = h$. Initially, the plate is characterized by a parabolic temperature distribution along the z-direction given by $T = T_0(r, z)$. For times $t > 0$, the surface $r = a$ is subjected to convection heat transfer with convection coefficient h_c and fluid temperature T_∞, while the plate is also subjected to uniform internal energy generation g_0 (W.m^{-3}). Under these conditions, the thermoelasticity in a thin circular plate due to uniform internal heat generation is required to be determined.

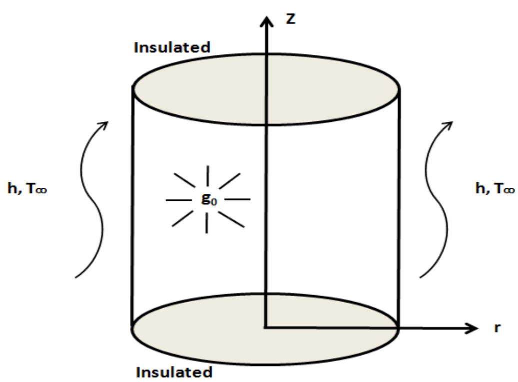

Figure 1: Geometry of the problem.

The unsteady-state temperature of the plate $T(r,z,t)$ satisfies the following model:

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\frac{g_0}{k_t}=\frac{1}{\alpha}\frac{\partial T}{\partial t}, \quad \text{in} \quad 0\le r\le a,\ 0\le z\le h,\ t>0 \tag{2.1}$$

with the boundary conditions,

$$T(r\to 0)\Rightarrow \text{finite} \tag{2.2}$$

$$-k_t\frac{\partial T}{\partial r}=h_c[\,T-T_\infty] \qquad \text{at } r=a,\ t>0, \tag{2.3}$$

$$\frac{\partial T}{\partial z}=0 \qquad \text{at } z=0,\ t>0, \tag{2.4}$$

$$\frac{\partial T}{\partial z}=0 \qquad \text{at } z=h,\ t>0, \tag{2.5}$$

and the initial condition,

$$T=T_0(r,z) \quad \text{in} \quad 0\le r\le a,\ 0\le z\le h,\ t=0. \tag{2.6}$$

where k_t is the thermal conductivity, the thermal diffusivity is defined as $\alpha=k_t/\rho c$ with ρ and c denoting the density and specific heat of the material of the circular plate respectively.

According Roy Choudhary [21], we assume that a circular plate of small thickness h is in a plane state of stress. In fact "the smaller the thickness of the circular plate compared to its diameter, the nearer to a plane state of stress is the actual state". The displacements equations of thermoelasticity have the form

$$U_{i,kk}+\left(\frac{1+\nu}{1-\nu}\right)e_{,i}=2\left(\frac{1+\nu}{1-\nu}\right)a_tT_{,i}$$

$$e=U_{k,k};\quad k,i=1,2$$

where U_i is the displacements component, e is the dilatation, T is the temperature and ν and a_t are respectively, the Poisson's ratio and linear coefficients of thermal expansion of the circular plate material.

Introducing

$$U_i=\psi_{,i}\quad i=1,2,$$

we have

$$\nabla_1^2\psi=(1+\nu)a_tT$$

$$\nabla_1^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}$$

$$\sigma_{ij}=2\mu(\psi_{,ij}-\delta_{ij}\psi_{,kk}) \qquad i,j,k=1,2,$$

where μ is Lamé's constant and δ_{ij} is the well-known Kronecker delta symbol.

In the axially symmetric case

$$\psi=\psi(r,z,t), \qquad T=(r,z,t)$$

and the differential equation governing the displacements potential function $\psi(r,z,t)$ is expressed as

$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}=(1+\nu)a_t\Delta T \tag{2.7}$$

where $\Delta T(=T-T_0)$ is the temperature deviation from the initial temperature T_0.

The stress functions σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr}=-\frac{2\mu}{r}\frac{\partial\psi}{\partial r} \tag{2.8}$$

$$\sigma_{\theta\theta}=-2\mu\frac{\partial^2\psi}{\partial r^2} \tag{2.9}$$

Initially

$$T=\psi=T_0(r,z) \qquad \text{at } t\text{=0.} \tag{2.10}$$

Also, in the plane state of stress within the circular plate

$$\sigma_{rz}=\sigma_{zz}=\sigma_{\theta z}=0 \tag{2.11}$$

Equations (2.1)-(2.11) constitute the mathematical formulation of the problem under consideration.

3. Solution of the Heat Conduction Problem

First we solve the nonhomogeneous boundary conditions by shifting the temperature scale, letting $\Upsilon(r,z,t)=T(r,z,t)-T_\infty$, in Eqs. (2.1)-(2.6), the new formulation as follows

$$\frac{\partial^2\Upsilon}{\partial r^2}+\frac{1}{r}\frac{\partial\Upsilon}{\partial r}+\frac{\partial^2\Upsilon}{\partial z^2}+\frac{g_0}{k_t}=\frac{1}{\alpha}\frac{\partial\Upsilon}{\partial t}, \quad \text{in} \quad 0\le r\le a,\ 0\le z\le h,\ t>0 \tag{3.1}$$

with the boundary conditions,

$$\begin{aligned}
&\Upsilon(r\to 0)\Rightarrow \text{finite}\\
-k_t&\frac{\partial\Upsilon}{\partial r}=h_c\,\Upsilon \qquad \text{at } r=a, t>0\\
&\frac{\partial\Upsilon}{\partial z}=0 \qquad \text{at } z=0,\ t>0\\
&\frac{\partial\Upsilon}{\partial z}=0 \qquad \text{at } z=h,\ t>0
\end{aligned}$$

and the initial condition,

$$\Upsilon=T_0(r,z)-T_\infty \quad \text{in} \quad 0\le r\le a,\ 0\le z\le h,\ t=0.$$

We reduce Eq. (3.1) to the homogeneous PDE and a nonhomogeneous ODE by defining a new dependent variable $\Upsilon(r,z,t)$ is defined as

$$\Upsilon(r,z,t)=\Psi(r,z,t)+\Phi(r)$$

where $\Psi(r,z,t)$ takes the homogeneous form of the PDE and $\Phi(r)$ corresponds to the one-dimensional nonhomogeneous ODE, containing the energy generation term.

The formulation of the nonhomogeneous $\Phi(r)$ problem becomes

$$\frac{d^2\Phi}{dr^2} + \frac{1}{r}\frac{d\Phi}{dr} + \frac{g_0}{k_t} = 0, \quad \text{in} \quad 0 \leq r \leq a \tag{3.2}$$

with the boundary conditions,

$$\Phi(r \to 0) \Rightarrow \text{finite}$$

$$-k_t\frac{d\Phi}{dr} = h_c\,\Phi \qquad \text{at } r = a,$$

The solution of Eq. (3.2) becomes

$$\Phi(r) = \frac{a g_0}{2h_c} + \frac{g_0}{4k_t}(a^2 - r^2)$$

The formulation of $\Psi(r, z, t)$ transient problem becomes

$$\frac{\partial^2\Psi}{\partial r^2} + \frac{1}{r}\frac{\partial\Psi}{\partial r} + \frac{\partial^2\Psi}{\partial z^2} = \frac{1}{\alpha}\frac{\partial\Psi(r,t)}{\partial t}, \quad \text{in} \quad 0 \leq r \leq a,\ 0 \leq z \leq h,\ t > 0 \tag{3.3}$$

with the boundary conditions,

$$\Psi(r \to 0) \Rightarrow \text{finite} \tag{3.4}$$

$$-k_t\frac{\partial\Psi}{\partial r} = h_c\,\Psi \qquad \text{at } r = a, t > 0, \tag{3.5}$$

$$\frac{\partial\Psi}{\partial z} = 0 \qquad \text{at } z = 0,\ t > 0, \tag{3.6}$$

$$\frac{\partial\Psi}{\partial z} = 0 \qquad \text{at } z = h,\ t > 0, \tag{3.7}$$

and the initial condition,

$$\Psi = T_0(r,z) - T_\infty - \Phi(r) = H(r,z) \quad \text{in} \quad 0 \leq r \leq a,\ 0 \leq z \leq h,\ t = 0. \tag{3.8}$$

To obtain the expression of the function $\Psi(r, z, t)$, following [23], we develop the finite Fourier transform and finite Hankel transform and their respective inverses and operate them on Eqs. (3.3)-(3.8):

$$\Psi(r,z,t) = \left(\frac{2}{h}\right)\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{J_0(\beta_m r)\cos(\eta_p z)}{N(\beta_m)}.e^{-\alpha(\beta_m^2+\eta_p^2)t}.\bar{\bar{H}}(\beta_m, \eta_p)$$

where

$$\bar{\bar{H}}(\beta_m, \eta_p) = \int_{z'=0}^{h}\int_{r'=0}^{a} r'\ J_0(\beta_m r')\ \cos(\eta_p z')\ H(r', z')\ dr'\ dz'$$

and

$$\frac{1}{N(\beta_m)} = \frac{2}{J_0^2(\beta_m a)}\frac{\beta_m^2}{a^2\left(\dfrac{h_c^2}{k_t^2} + \beta_m^2\right) - \nu^2}$$

and $\beta_1, \beta_2, \beta_3, \ldots$ are the positive root of transcendental equation

$$-\beta_m J_1(\beta_m a) + \frac{h_c}{k_t} J_0(\beta_m a) = 0$$

and $\eta_1, \eta_2, \eta_3, \ldots$ are the positive roots of the transcendental equation

$$\sin(\eta_p h) = 0, \quad p = 1, 2, 3, \ldots.$$

Finally, the temperature distribution is the sum of the homogeneous transient solution $\Psi(r, z, t)$ and the nonhomogeneous, steady-state solution $\Phi(r)$, with the additional re-shifting of temperature by T_∞,

$$T(r, z, t) = T_\infty + \left(\frac{2}{h}\right) \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \frac{J_0(\beta_m r)\cos(\eta_p z)}{N(\beta_m)} . e^{-\alpha(\beta_m^2 + \eta_p^2)t} . \tilde{\bar{H}}(\beta_m, \eta_p) + \frac{a g_0}{2h_c} + \frac{g_0}{4k_t}(a^2 - r^2) \tag{3.9}$$

3.1. **Special Case.** Setting, $T_0(r, z) = T_\infty$ in Eq. (3.9), one obtains the expression of the temperature distribution function as

$$T(r, z, t) = T_0 + \left(\frac{2}{h}\right) \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \frac{J_0(\beta_m r)\cos(\eta_p z)}{N(\beta_m)} . e^{-\alpha(\beta_m^2 + \eta_p^2)t} . \tilde{\bar{H}}(\beta_m, \eta_p) + \frac{a g_0}{2h_c} + \frac{g_0}{4k_t}(a^2 - r^2) \tag{3.10}$$

The temperature is represented in the following form:

$$\Delta T = \left(\frac{2}{h}\right) \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \frac{J_0(\beta_m r)\cos(\eta_p z)}{N(\beta_m)} . e^{-\alpha(\beta_m^2 + \eta_p^2)t} . \tilde{\bar{H}}(\beta_m, \eta_p) + \frac{a g_0}{2h_c} + \frac{g_0}{4k_t}(a^2 - r^2) \tag{3.11}$$

4. DISPLACEMENT POTENTIAL FUNCTION AND THERMAL STRESSES

Using Eq. (3.11) in Eq. (2.7), one obtains

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} = (1 + \nu) a_t \left[\left(\frac{2}{h}\right) \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \frac{J_0(\beta_m r)\cos(\eta_p z)}{N(\beta_m)} . e^{-\alpha(\beta_m^2 + \eta_p^2)t} . \tilde{\bar{H}}(\beta_m, \eta_p) + \frac{a g_0}{2h_c} + \frac{g_0}{4k_t}(a^2 - r^2) \right] \tag{4.1}$$

Solving Eq. (4.1), by using the result,

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right) J_0(\beta_m r) = -\beta_m^2 J_0(\beta_m r)$$

one obtains,

$$\frac{\psi}{X} = \left[\left(\frac{2}{b}\right)\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{J_0(\beta_m r)\cos(\eta_p z)}{\beta_m^2 N(\beta_m)}.e^{-\alpha(\beta_m^2+\eta_p^2)t}.\bar{\tilde{H}}(\beta_m,\eta_p) + \frac{a g_0 r^2}{8h_c} + \frac{g_0}{64k_t}(4a^2r^2 - r^4)\right] \tag{4.2}$$

Using Eq. (4.2) in Eqs. (2.8) and (2.9), one obtains the expressions of thermal stresses as

$$\frac{\sigma_{rr}}{Y} = \left[\left(\frac{2}{h}\right)\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{J_1(\beta_m r)\cos(\eta_p z)}{r\,\beta_m\,N(\beta_m)}.e^{-\alpha(\beta_m^2+\eta_p^2)t}.\bar{\tilde{H}}(\beta_m,\eta_p) + \frac{a g_0}{4h_c} + \frac{g_0}{16k_t}(2a^2 - r^2)\right] \tag{4.3}$$

$$\frac{\sigma_{\theta\theta}}{Y} = \left[\left(\frac{2}{h}\right)\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\left(J_0(\beta_m r) - \frac{J_1(\beta_m r)}{\beta_m r}\right)\frac{\cos(\eta_p z)}{N(\beta_m)}.e^{-\alpha(\beta_m^2+\eta_p^2)t}.\bar{\tilde{H}}(\beta_m,\eta_p) + \frac{a g_0}{4h_c} + \frac{g_0}{16k_t}(2a^2 - 3r^2)\right] \tag{4.4}$$

where X and Y are the constants and set for convenience, as

$$X = -(1+\nu)a_t, \qquad Y = -2(1+\nu)a_t\mu.$$

5. Numerical Results and Discussion

5.1. **Dimensions.** The constants associated with the numerical calculation are taken as:
Radius of a circular plate a = 1 m,
Thickness of a circular plate h = 0.1 m.

5.2. **Material Properties.** The copper material was chosen for purposes of numerical evaluations. The parameters of the problem are thus given in SI units by [24]:

Table 1: Material Constants

ρ = 8954 kg/m^3	$\alpha = 112.34(10)^{-6}$ m^2/s	k_t = 386 W/(m K)
E = 128 Gpa	c_p = 383 J/(kg K)	a_t = 16.5(10)$^{-6}$ K^{-1}
ν = 0.35	μ = 26.67	g_0= 1(10^4) W/m^3

5.3. **Roots of the Transcendental Equations.** The first five positive root of the transcendental equation $\beta_m J_1(\beta_m a) - \frac{h}{k} J_0(\beta_m a) = 0$ as defined in [23] are $\beta_1 = 3.8317$, $\beta_2 = 7.0156$, $\beta_3 = 10.1735$, $\beta_4 = 13.3237$, $\beta_5 = 16.470$ and $\eta_1 = 6.28$, $\eta_2 = 12.56$, $\eta_3 = 18.84$, $\eta_4 = 25.12$, $\eta_5 = 31.40$ are positive roots of the transcendental equation $\sin(\eta_p h) = 0$. The numerical calculations and graphs are plotted with the help of computational mathematical software PTC Mathcad Prime-3.1.

In this paper, a thin circular plate is considered and determined the expressions for temperature change, displacements, and stress functions due to the internal heat generation at a constant rate. As a special case $T_0(r, z) = T_\infty$, the mathematical model is constructed for copper material and performed numerical calculations. The thermoelasticbehaviour is examined such as temperature change, displacements, and stresses with different time parameters under internal heat generation at a constant rate. The obtained expressions for the temperature, displacement and thermal stresses provide important intuition into the role of the thermomechanical material properties in elastic behaviors of the thin circular plate under the internal energy generation at a constant rate. The temperature distribution in the plate is only dependent on its thermal properties, on the other hand, the plate displacement and stresses are dependent on both thermal and mechanical properties. We have used the first 50 terms (p=1–50) for the inner series summation, as given by Eq. (3.10), and have used the first 10 terms (m=1–10) of the outer series summation to achieve greater accuracy.

Figure 2 shows the variation of the temperature distribution along the radial direction for different values of time t = 50, 100, ..., 250 s. It is assumed that, the initial temperature of the plate is $T_0(r, z) = 0$ with the rate of internal energy generation is 1×10^4. We observed that, near the centreline ($r \sim 0$), the temperature is increasing primarily due to the internal heat generation at a constant rate and when the radii increase the temperature starts decreasing towards the outer circular edge (r=1) for different times.

Figure 3 shows the variation of the displacement potential function along the radial direction for different values of time t = 50, 100, ..., 250 s. It is clear that the displacement is maximum at the center ($r = 0$) due to the internal heat generation at a constant rate and when the radii increase the displacement starts decreasing towards the outer circular edge ($r = 1$) for different times. Also from the figures of temperature and displacement, we observed that the direction of heat flow and the direction of body displacement are the same and are proportionate to each other.

Figure 4 shows the variation of radial stress along the radial direction for different values of time t = 50, 100, ..., 250 s. It is observed that the radial stress increases with increasing the radii and its maximum towards the outer circular edge ($r = 1$). It develops the tensile stresses in a radial direction.

Figure 5 shows the variation of axial stress along the radial direction with different time parameters. It is observed that the axial stress is maximum at the center ($r = 0$) due to the internal heat generation at a constant rate and when the radii increase the axial stress starts decreasing towards the outer circular edge ($r = 1$) for different times.

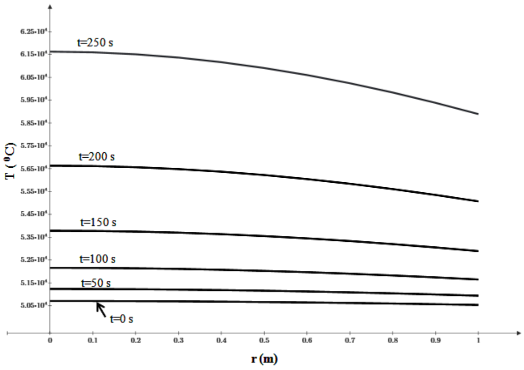

Figure 2: Variation of T versus r (r = 0, 0.1, 0.2, . . . , 1) for different values of time t = 50, 100, . . . , 250 s.

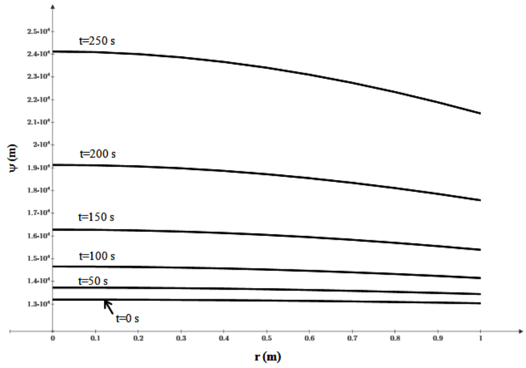

Figure 3: Variation of ψ versus r (r = 0, 0.1, 0.2, . . . , 1) for different values of time t = 50, 100, . . . , 250 s.

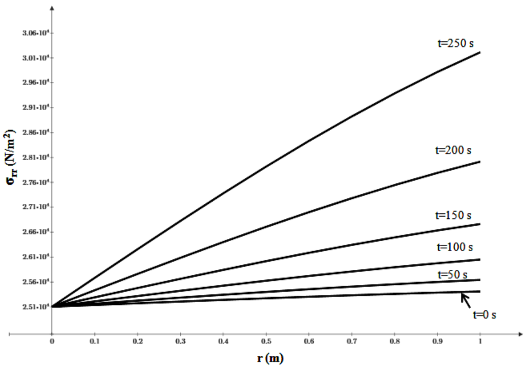

Figure 4: Variation of σ_{rr} versus r (r = 0, 0.1, 0.2, . . . , 1) for different values of time t = 50, 100, . . . , 250 s.

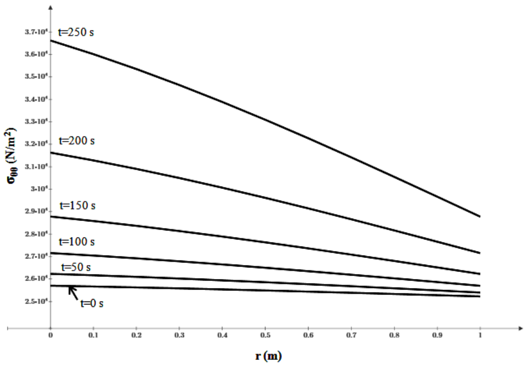

Figure 5: Variation of $\sigma_{\theta\theta}$ versus r (r = 0, 0.1, 0.2, . . . , 1) for different values of time t = 50, 100, . . . , 250 s.

6. Concluding Remarks

In this article, we analyzed a two-dimensional transient thermoelastic problem of a thin circular plate under uniform internal heat generation at a constant rate and investigates the temperature, displacement, and stresses. The upper and lower surfaces are thermally insulated, while the perimetric surface is characterized by parabolic temperature distribution along the z-direction given by $T = T_0(r, z)$. The mathematical model is constructed for copper material and an integral transform technique was used to obtain the analytical solution for temperature, displacement, and thermal stresses. The method used in this study provides a quite successful approach in dealing with transient thermoelastic stress analysis.

The results of the present work can be summarized as:

(1) The converging of the series summation is rapid for large time.
(2) The rate of temperature increase near the centerline $(r \sim 0)$, at very early times due to internal energy generation at a constant rate.
(3) From the figures of the temperature and displacement, we observe that the direction of heat flow and the direction of body displacement are the same and are proportionate to each other.
(4) The stress components and displacement occur near the heated region.

The results presented here will be useful in engineering problems, particularly in aerospace engineering for stations of a missile body not influenced by nose tapering. Also, any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions (3.10)-(4.4).

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