Robustness of optimized FPIID controller against uncertainty and disturbance by fractional nonlinear model for research nuclear reactor

Nafiseh Zare a, Gholamreza Jahanfarnia b, Abdollah Khorshidi c, a, *, Jamshid Soltania a

Abstract

In this study, a fractional order proportional integral derivative (FOPID) controller is designed to create the reference power trajectory and to conquer the uncertainties and external disturbances. A fractional nonlinear model was utilized to describe the nuclear reactor dynamic behaviour considering thermal-hydraulic effects. The controller parameters were tuned using optimization method in Matlab/Simulink. The FOPID controller was simulated using Matlab/Simulink and the controller performance was evaluated for Hard variation of the reference power and compared with that of integer order a proportional integral derivative (IOPID) controller by two models of fractional neutron point kinetic (FNPK) and classical neutron point kinetic (CNPK). Also, the FOPID controller robustness was appraised against the external disturbance and uncertainties. Simulation results showed that the FOPID controller has the faster response of the control attempt signal and the smaller tracking error with respect to the IOPID in tracking the reference power trajectory. In addition, the results demonstrated the ability of FOPID controller in disturbance rejection and exhibited the good robustness of controller against uncertainty.

1. Introduction

One application of research reactors is the production of radioisotopes, which are necessary to operate the reactor at dissimilar power levels. Therefore, the output power of reactor must be accurately controlled by control system. The main goal of the power controller in the nuclear reactors is to pursue the reference power trajectory in order to decrease undesirable power fluctuation. Controlling the output power is a decisive way to guarantee a stable reactor operation. The first step in the design of the controller is the selection of a suitable model to delineate the reactor dynamics. Recently, the fractional calculus has been used on the nuclear reactor to describe the reactor dynamic behaviour. In 2011, the fractional neutron point kinetic (FNPK) model was introduced with a group of delayed neutrons established upon non-Ficken law suppositions to attain the best representation of a nuclear reactor dynamics [1]. By this model, relaxation time is taken into account proportionate with the swift alterations in the neutron flux, which holds a fractional order caused by the quick mutations of reactivity. The fractional order has a connection with non-Ficken impacts from the point of view of the neutron diffusion equation [2–5]. Lately, the analysis of the FNPK equation has been performed by Espinosa-Paredes et al. [6] and investigated on the uncertainties and sensitivity of the fractional diffusion coefficient in fractional order core using Monte Carlo modeling [7]. They have shown that a 1% change in fractional order leads to fluctuations in the neutron density of 0.17% and 0.012% for the short and long time intervals, correspondingly. On the other hand, Labarrios et al. [8] have analysed the reactor dynamic behaviour during the start-up process of the reactor using FNPK model. Also, the FNPK equations via the temperature feedback impacts have been examined for the fast insertion reactivity [9,10]. In addition to that, the numerical analysis of the FNPK model has been carried out in response to a sinusoidal and ramp reactivity input [11]. Consequently, short-term reactivity mutations were observed in the start-up, and the FNPK model predicted the neutron density in a larger way than the classical neutron point kinetic (CNPK) model.

Furthermore, the different integer order controller on the basis of the CNPK model has been designed for the nuclear reactor power...
control [12–19]. An adaptive fuzzy controller in a TRIGA-type research reactor has been proposed by Ramirez at al [12]. For control of the reactor power level, a novel nonlinear state-feedback controller has been reported on asymptotic closed-loop stability [13]. Moreover, the control of the core power distribution has been recently performed using the state-feedback controller established upon the linear quadratic regulator (LQR) method [14] on both linearized and nonlinear models. In addition to that, Eliasi et al. [15] have designed a model-based predictive control (MPC) for a PWR nuclear power plant throughout the load following that it was based on some restrictions on the input and output variables and may be reserved within satisfactory constraints. Another control technique has been projected on the basis of the combination of the linear quadratic Gaussian (LQG), the PID controller and the improved adaptive genetic algorithm (IAGA) by Li et al. [16]. They used a linear multi-scheme instead of the nonlinear scheme for simulation of core dynamic. Also, Coban R [17] has suggested a power trajectory controller based on the multi feedback layer neural network method and optimized the particle swarm besides the CNPK model to scrutinize the reactor dynamics. What's more, Munje et al., 2013 [18] has designed a sliding mode control (SMC) for spatial oscillation test of the advanced heavy water reactor (AHWR). Ansarifar et al. [19] have developed a sliding mode-observer that was created using a two-point nuclear reactor scheme to determine the xenon concentration and the delayed neutron precursor density of the PWR by reactor power measurement. Also, Ansarifar and Rafiei [20] have projected a higher-order sliding mode control to conquer the disadvantage of chattering phenomenon of SMC established upon the point kinetics equations and three delayed neutron groups by taking temperature feedbacks and xenon concentration into account in their controller design.

In recent decades, the fractional calculus has been attracted in scientific and technical areas such as physics, control engineering and signal processing [21,22]. The fractional calculus was used in controller design in order to attain the most robust performance of the systems [21]. Podulubny 1994 [22] presented a fractional PID controller for a fractional order system. After this work, the different fractional order controllers such as fractional order proportional integral derivative (FOPID) and fractional order proportional integral (FOPI) were planned for the power control and tested on PHWR under the step-back circumstances [23–25]. In an irregular or a "load following" circumstances, the fast drop of the reactor power during a small finite time interval is referred to as a step-back. The control rod props and variation in the initial power level cause to irregular and "load following" circumstances, respectively. A fuzzy PID controller proposes that the controller parameters are tuned using GA optimization method, and the nonlinear state-space model is linearized around an operating point [23]. In addition, the PHWR reactor dynamics is fractionally modelled by the various linearized designs around the dissimilar operating points which are used for the closed loop controller scheme under the step-back condition [24]. These recent outcomes show that the fractional order controller has improved performance than the integer order controllers. In another research carried out by Saha et al., 2010 [26], a phase shaping by means of Autoregressive Exogenous (ARX) algorithm has been designed using fractional order (FO) phase shapers that a control rod drop scenario is modelled as a fast power drop in a 500 MWe Canadian Deuterium Uranium (CANDU) reactor. They have demonstrated that the fractional order phase shaper along with a PID controller provides a better response compared to the present reactor regulating system. Also, Lamba et al., 2017 [27] have presented an interval fractional order proportional integral derivative (INFOPID), which is intended for the power control of a PHWR under step-back circumstance. In addition to that, Bongulwar and Patre 2017 [28] have proposed a controller of Fractional Order Proportional Integral Derivative (FOPI) for global power control of a Pressurized Heavy Water Reactor (PHWR) under the step-back situation. By their work, the stability region of NIOPTD-I plants is obtained by using stability boundary locus technique in \( \left( K_p, K_i, K_d \right) \) parameter space. Their simulation results showed that the proposed \( P^d \ D^k \) controller has a robust performance against the power alterations with 30% and 50% global power drop compared to the initial 100%. Furthermore, Salehi et al., 2019 [29] have proposed a control system with gain-scheduled fractional order PID (FOPID) for the steam generator level control system in the whole operating range, and the stability analysis of the controller has been performed under the operating circumstances using the Nyquist method. Their obtained results show that the controller is robust in a wide range of operating circumstances.

In this research, a fractional nonlinear model for Tehran Research Reactor (TRR) is studied to explain the nuclear reactor dynamics besides a FOPID controller. Then, the obtained parameters are tuned using optimization methods in Simulink of Matlab software. This proposed controller is set up on the basis of the tangibly gaugeable feedbacks and would be able to generate the time-varying reference signal by conquering the disturbances and to ensure the system stability.

2. Tehran Research Reactor (TRR) attributes

The TRR is a pool type reactor and has been constructed for the maximum output of 5 MW. Normally, the light water is employed as moderator and coolant and also for shielding. The solid fuel of \( U_3O_8 \) is regularly utilized in the reactor, and aluminum is considered as the protective layer. The reactor core consists of Standard Fuel Elements (SFE) plus Control Fuel Elements (CFE). By the First Operating Core (FOC), the reactor is operated with the 14 SFE—including 19 fuels plates— and 5 CFE—including 14 fuel plates—which is shown in Fig. 1. The general characteristics of TRR have been listed in Table 1 [30]. The control of the reactor is executed using the absorbent rods. There are two different types of absorbent rods, Fine Regulating Rod (FRR) and Shim Safety Rod (SSR) that are employed in a fork design.

3. Model description

In this research, the nonlinear behaviour of TRR core is designed by FNPK nonlinear model. The effect of temperature feedback and xenon concentration is also taken into account. The system variables have been explained in Nomenclature. The nonlinear dynamic model fractionally normalized regard to the equilibrium condition is given by Eqs. (1) and (2) [31].

\[
\begin{align*}
\frac{d^k}{dt^k} D^k C_F &+ r^k \left( \frac{1}{T} + \beta \right) D^k P_R + D P_R = \beta (t) - \beta \frac{d^k}{dt^k} P_R + \lambda C_F \\
&+ \frac{D^k}{dt^k} C_F 
\end{align*}
\]

where, \( d^k/dt^k = D^k \) and \( k \) is anomalous diffusion coefficient (fractional order) by \( 0 < k < 1 \).

\[
DC_F = \frac{\beta}{A} P_R - \lambda C_F
\]

The thermal-hydraulic equations of the reactor core are characterized by:
General attributes of TRR.

<table>
<thead>
<tr>
<th>parameters</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal Power (Initial)</td>
<td>10^9 Watt</td>
</tr>
<tr>
<td>Thermal Power</td>
<td>5 MW</td>
</tr>
<tr>
<td>Fuel</td>
<td>MTR type of low enriched 235U plus Al clad</td>
</tr>
<tr>
<td>Number of Plate for each Fuel</td>
<td>19 for SFE; 14 for CFE</td>
</tr>
<tr>
<td>Core dimensions (FDC)</td>
<td>40.5 x 38.54 x 89.7 cm³</td>
</tr>
<tr>
<td>Flow of Coolant</td>
<td>500 m³/h</td>
</tr>
<tr>
<td>Moderator</td>
<td>Light water</td>
</tr>
<tr>
<td><strong>Inlet temperature of Coolant at:</strong></td>
<td></td>
</tr>
<tr>
<td>Cold and clean (at 10 °C)</td>
<td>20°C</td>
</tr>
<tr>
<td>Full power (at 5 MW)</td>
<td>37.8°C</td>
</tr>
<tr>
<td><strong>Outlet temperature of Coolant at:</strong></td>
<td></td>
</tr>
<tr>
<td>Cold and clean (at 10 °C)</td>
<td>20°C</td>
</tr>
<tr>
<td>Full power (at 5 MW)</td>
<td>46°C</td>
</tr>
<tr>
<td><strong>Control rods by:</strong></td>
<td></td>
</tr>
<tr>
<td>Regulating rods</td>
<td>1 stainless steel</td>
</tr>
<tr>
<td>Shim safety rods</td>
<td>4 Ag-In-Gd</td>
</tr>
</tbody>
</table>

\[ DT_f = \frac{f_t P_0}{\mu_f} n_r - \frac{\mu}{2\mu_f} T_f(t) + \frac{\mu}{2\mu_f} T_{\text{min}} \]  

(3)

\[ DT_i = \left( 1 - \frac{1 - f_t P_0}{\mu_c} n_r + \frac{\mu}{\mu_c} T_f(t) \right) \frac{2M}{2\mu_c} + \frac{2M}{2\mu_c} T_{\text{min}} \]  

(4)

The variation of iodine (DI) and xenon (DX) densities are obtained from Eqs. (5) and (6).

\[ DI = \frac{\gamma f P_0}{\gamma f R_{NVf} S_f} n_r - \lambda_d I(t) \]  

(5)

\[ DX = \frac{P_0}{\gamma f R_{NVf} S_f} I(t) - \left( \lambda_x + \frac{\sigma_n^0}{\gamma f R_{NVf} S_f} P_0 \right) X(t) \]  

(6)

The total reactivity (ρ) is obtained by the alteration of control rod reactivity and the feedback reactivity by virtue of fuel, coolant temperature variations, xenon concentration dissimilarity and coolant density alteration as demonstrated in Eq. (7).

\[ \rho(t) = \delta \rho_{\text{ex}} + \alpha_f \left[ T_f(t) - T_{\text{ref}} \right] + \alpha_c \left[ T_c(t) - T_{\text{ref}} \right] - \sigma_n^0 (X(t) - X_0) \]  

(7)

The control rod reactivity alteration (ρ_{ex}) is obtained by Eq. (8),

\[ D\rho_{\text{ex}} = G_t Z_t \]  

(8)

where the reactor power P(t) may be described as:

\[ P(t) = P_0 n_r = P_0 P_r = \gamma_f \Sigma_f \rho_{NVf} \]

Eqs. (1)–(8) were simulated in Matlab/Simulink software. In this simulation of TRR, the toolbox of Nid-block/Ninteger was employed for fractional derivative and integral by fractional nonlinear modeling. This toolbox has been developed by Valerio and Costa 2005 [32] that the Crone approximation in order 10 is fractionally used for simulation.

### 4. Design of fractional PID controller

The classic PID controller is frequently used in process control industries because of simplicity of the design and fairly good performance. In general, the good performance indexes are the low percentage of overshoot and the short settling time. The IOPID controllers are special cases of fractional controllers.

In 1994, Podulbny [22] has introduced a fractional PID controller for a fractional order system. The transfer function of FPID is given by:

\[ C(s) = \frac{U(s)}{e(s)} = K_p + K_s \alpha + K_d \beta \]  

(9)

\[ U(t) = K_p e(t) + K_s D^{-\alpha} e(t) + K_d D^\beta e(t) \]  

(10)

where ratio of control signal U(s) to error signal e(s) determines the controller output C(s). Also, this ratio can be given by combining three different K parameters, proportional constant gain Kp, integral constant gain Ki, derivative constant gain Kd in the direction of differentiator order \( \beta \) beside order of integration \( \alpha \). As shown in Fig. 2, the PID controller jumps between fixed points in the plane while the FOPID moves continuously in this plane [29]. It is worth noting that fractional PID controller has some features compared to conventional PID controller [31], such as: the most robust performance, less sensitivity to variations of parameters, better creation of higher order systems and a system with a long time delay, creation of more robust stability, the ability to steer the system with nonlinearities, achieving a better response to a minimum phase system, capability to control the nonlinearity system without requiring linearization of the system.

There are five parameters in the design of FPID controller to be tuned. So far, diverse tuning techniques such as Ziegler-Nichols scheme, self-tuning, auto-tuning, and based on the optimization method have been used to derive the controller parameters [31,33].

In this section, a PID controller is designed established upon the fractional calculus. The control system is comprised of a plant and a FOPID controller in a feedback loop. Fig. 3 depicts the inclusive structure of a closed loop system. The FOPID controller is simulated using Nipid block/Ninteger toolbox in Matlab/Simulink software.

There are several approximation methods, namely: Crone, Carlson, and Matsuda for a class of MIMO fractional order systems [32].
A. In the first step, the block “Check Step Response Properties” added in Fig. 3, is shown in Fig. 4.

B. In stage 2, the Parameters of the Check Step Response Block including rise time, settling time, overshoot, initial value, and final value are specified.

C. In stage 3, Five parameters of controller $K_p, K_i, K_d, \alpha$ and $\beta$ are selected for optimization.

D. Finally, the optimization method is determined. In this simulation, the Gradient descent method was selected.

Furthermore, Fig. 5a and b shows the disturbance input and output schemes in the reactor system.

5. Simulation results and discussion

Here, the performance of FOPID controller is evaluated on the TRR. The dynamic behaviour of TRR is described by FOPID nonlinear model. The FOPID controller was simulated using Nipid/Ninteger toolbox in MATLAB Simulink. The Nid block/Ninteger toolbox is regularly used for simulation the fractional derivative and integral in FNPK model. There are various approximations in Ninteger toolbox. The Crone approximation by order 10 was used for simulation of FOPID controller and fractional order derivative and integral in FNPK nonlinear model. Three cases were simulated to evaluate the ability of FOPID controller in tracking control problem of nuclear reactor power for the desired trajectory. The performance indexes in this method were the control input, overshoot, settling time and rise time. In first simulation, the performance of FOPID controller was tested for the hard reference power trajectory. A comparative study between FOPID and IOPID controller were carried out for two FNPK and CNPK models. In the second and third simulation, the controller robustness was simulated and evaluated against the external disturbance and uncertainty. The simulation results have been depicted in Figs. 6–9 for diverse cases. In each case, the reference power trajectory, the error signal tracking, and also the speed of the control rod were separately evaluated and discussed. The TRR was used for numerical simulation, and general characteristics of the TRR were given in Table 1. The dynamic parameters amounts of TRR are given in Table 2.

Case 1. Hard alteration of the reference power

By Case 1, it was believed that the reactor operates at 100% nominal power and after that the reactor was kept at 10% low power level for about 60s. Then, the power rise from 10% to 100% was performed. The simulation results and the reference power tracking are illustrated in Fig. 6a for both controllers. Now, it is seen that the reference power trajectory by the FOPID controller alongside FNPK model properly followed without any significant overshoot or oscillation. The reactor output relative power was zoomed in time interval 50–120s in Fig. 6b. Moreover, the tracking error signal of FOPID was much smaller than that of IOPID controller, as shown in Fig. 6c. Similarly, the FOPID controller with FNPK model revealed a much less error than other cases. Fig. 6d shows the control rod speed of FOPID and IOPID controllers, and the outcomes revealed that the FOPID controller provides a faster response than that of IOPID controller.

Case 2. Disturbance Analysis

In this case, the robustness of the FOPID controller is appraised by an external disturbance in input and output of the reactor system that have been shown in Fig. 5. It was believed that the reactor is operating at 100% of its nominal power and then step reactivity of
Fig. 6. Simulation outcomes of FOPID and IOPID controllers in the output power trajectory tracking for Case 1: (a) relative power of the reactor output, (b) the relative power zoomed in time interval 50–120s, (c) power tracking error, (d) speed of the control rod.

Fig. 7. Simulation outcomes of FOPID controller when dealing with input disordered signal for Case 2: (a) the disordered signal by 0.1s step reactivity, (b) relative power of reactor output, (c) power tracking error, (d) speed of the control rod.
0.1 $ in an external disturbance was inserted at time 50s. In simulation of the external disturbance in output of reactor system, a disturbance signal as the relative power with amplitude 0.1 (500 kW) was inserted. The relative power increased up to 1.1, the rod was inserted to the reactor core in order to decline the relative power to 1 again. The trajectory of the reference power, the error of signal tracking, and the speed of the control rod for FOPID controller have been demonstrated in Figs. 7 and 8 for both input and output disturbances. The simulation outcomes demonstrate that the controller is robust to an external disorder and the relative power does not go over 1.1.

### Case 3. Uncertainty Analysis

Remarkably, the robustness of the FOPID controller against the unstructured uncertainties (non-modelled dynamics) was simulated and investigated. The equations for alteration of promethium and samarium (Pm and Sm) concentrations were assumed to be an unstructured uncertainty. The pertinent equations are given by:

\[
\frac{dS_m}{dt} = \lambda_{pm}P_m - \sigma_{Sm}^a \phi S_m
\]

\[
\frac{dP_m}{dt} = \gamma_{pm} \Sigma f \phi - \lambda_{pm} P_m
\]

Eqs. (11) and (12) are subjoined to the reactor dynamic system which are described with Eqs. (1)–(8) to simulate the non-modelled dynamics. The FOPID controller performance is evaluated in face to this uncertainty.

In simulation by 10% per-minute reduction ratio, the TRR reactor was firstly considered at 100% operation of the nominal power throughout 25 s, and subsequently arrived at 90%. A comparison between two cases, namely dynamic system with these new equations including Pm and Sm concentrations alterations (i.e. uncertainty) and system with Eqs. (1)–(8), was performed and the relevant outcomes are shown in Fig. 9. In Fig. 9a, the findings show a satisfactory performance of FOPID controller in pursuing the reference power against uncertainty. As seen the stable reactor relative power at 1 for the 0–25 s time interval and it declines to 0.9 Pr upon from 25 s to at about 80 s at which it reached the value 0.9 Pr. The signals of control attempt and error are displayed in Fig. 9b and c, respectively.

### Table 2

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Symbol</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.00813</td>
<td>$\lambda_f$</td>
<td>$3.55 \times 10^{-6}$ (s$^{-1}$)</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>45 ($\mu$)</td>
<td>$\sigma_{S_m}^a$</td>
<td>$4 \times 10^{-20}$ (m$^2$)</td>
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<tr>
<td>$f_f$</td>
<td>0.95</td>
<td>$g_i$</td>
<td>$2.65 \times 10^{-15}$ (m$^2$)</td>
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<td>$P_0$</td>
<td>5 (MW)</td>
<td>$\alpha_C$</td>
<td>$-6 \times 10^{-5} (kW/k^2)$</td>
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<tr>
<td>$\lambda$</td>
<td>0.17 (s$^{-1}$)</td>
<td>$\alpha_T$</td>
<td>$-2 \times 10^{-5} (kW/k^2)$</td>
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<tr>
<td>$\tau_l$</td>
<td>0.06386</td>
<td>$T_{min}$</td>
<td>37.8 (°C)</td>
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<tr>
<td>$\tau_X$</td>
<td>0.00228</td>
<td>$M$</td>
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<td>$\mu_c$</td>
<td>0.235 (MWs/°C)</td>
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<tr>
<td>$L$</td>
<td>45 ($m$)</td>
<td>$\lambda_X$</td>
<td>$0.0161 \times 10^{-3}$ (s$^{-1}$)</td>
</tr>
<tr>
<td>$S$</td>
<td>2.0916 $\times 10^{-3}$ (s$^{-1}$)</td>
<td>$\mu_f$</td>
<td>0.315 (MW/°C)</td>
</tr>
</tbody>
</table>

### 6. Conclusion

The reactor power control is the most important issues in the nuclear reactor assessment. In this paper, a FOPID controller was designed for power level control of TRR. A nonlinear FNPK model was used to describe the reactor dynamic behaviour and simulated using Nipid/Ninteger toolbox in MATLAB software. The parameters of FOPID controller were tuned using optimization block in Simulink/MATLAB, and Nipid/Ninteger block was utilized for simulation of fractional order integral and derivative impacts.

The FOPID controller performance was examined for a hard alteration of the reference power and the obtained outcomes were compared with those of IOPID controller. Our findings revealed that the FOPID controller with FNPK nonlinear model properly tracked
the reference power trajectory without any significant overshoot or oscillation. The robustness of FOPID controller engaging in external disturbance was investigated in input and output of the reactor system by the step reactivity. According to these results, it was shown that the FOPID controller is robust enough against the external disturbance and uncertainty. Future work will address the other methods such as Carlson and Matsuda to evaluate the domain frequency in comparison.

Declaration of competing interest

The authors have declared no conflicts of interest.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jnet.2020.03.002.

References


[23] S.S. Bhase, B.M. Patre, Robust FOPI controller design for power control of

Fig. 9. Simulation outcomes of FOPID controller in the face with unstructured uncertainty for Case 3: (a) relative power of reactor productivity, (b) power tracking error, (c) speed of the control rod.


