*J. Appl. Math. & Informatics Vol.* **38***(2020), No. 3 - 4, pp. 201 - 209*
https://doi.org/10.14317/jami.2020.201

# ON EXTREMAL ROUGH $I$-CONVERGENCE LIMIT POINT OF TRIPLE SEQUENCE SPACES DEFINED BY A METRIC FUNCTION

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Abstract. We introduce and study some basic properties of rough $I$-convergent of triple sequence spaces and also study the set of all rough $I$-limits of a triple sequence spaces.

AMS Mathematics Subject Classification : 40F05, 40J05, 40G05.
*Key words and phrases* : Triple sequences, rough convergence, cluster points and rough limit points.

## 1. Introduction

The idea of rough convergence was first introduced by Phu [10, 11, 12] in finite dimensional normed spaces. He showed that the set $\mathrm{LIM}_x^r$ is bounded, closed and convex; and he introduced the notion of rough Cauchy sequence. He also investigated the relations between rough convergence and other convergence types and the dependence of $\mathrm{LIM}_x^r$ on the roughness of degree $r$.

Aytar [1] studied of rough statistical convergence and defined the set of rough statistical limit points of a sequence and obtained two statistical convergence criteria associated with this set and prove that this set is closed and convex. Also, Aytar [2] studied that the $r$-limit set of the sequence is equal to intersection of these sets and that $r$-core of the sequence is equal to the union of these sets. Dündar and Cakan [9] investigated of rough ideal convergence and defined the set of rough ideal limit points of a sequence The notion of $I-$ convergence of a triple sequence spaces which is based on the structure of the ideal $I$ of subsets of $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$, where $\mathbb{N}$ is the set of all natural numbers, is a natural generalization of the notion of convergence and statistical convergence.

In this paper we investigate some basic properties of rough $I$-convergence of a triple sequence spaces in three dimensional matrix spaces which are not earlier.

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Received April 9, 2019. Revised March 22, 2020. Accepted March 23, 2020. *Corresponding author.

We study the set of all rough $I$-limits of a triple sequence spaces and also the relation between analytic ness and rough $I$-convergence of a triple sequence spaces.

Let $K$ be a subset of the set of positive integers $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ and let us denote the set $K_{ij\ell} = \{(m, n, k) \in K : m \leq i, n \leq j, k \leq \ell\}$. Then the natural density of $K$ is given by

$$\delta(K) = \lim_{i,j,\ell \to \infty} \frac{|K_{ij\ell}|}{ij\ell},$$

where $|K_{ij\ell}|$ denotes the number of elements in $K_{ij\ell}$.

Throughout the paper, $\mathbb{N}$ denotes the set of all positive integers , $\chi_A$-the characteristic function of $A \subset \mathbb{N}$, $\mathbb{R}$ the set of all real numbers. A subset $A$ of $\mathbb{N}$ is said to have asymptotic density $d(A)$ if

$$d(A) = \lim_{i,j,\ell \to \infty} \frac{1}{ij\ell} \sum_{m=1}^{i} \sum_{n=1}^{j} \sum_{k=1}^{\ell} \chi_A(K).$$

A triple sequence (real or complex) can be defined as a function $x : \mathbb{N}\times\mathbb{N}\times\mathbb{N} \to \mathbb{R}(\mathbb{C})$, where $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequence was introduced and investigated at the initial by *Sahiner et al.* [13, 14], *Esi et al.* [3, 4, 5], *Dutta et al.* [6], *Subramanian et al.* [15], *Debnath et al.* [7] and many others.

A triple sequence $x = (x_{mnk})$ is said to be triple analytic if

$$\sup_{m,n,k} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

The space of all triple analytic sequences are usually denoted by $\Lambda^3$.

## 2. Definitions and Preliminaries

Throughout the paper $\mathbb{R}^3$ denotes the real three dimensional case with the metric space. Consider a triple sequence spaces $x = (x_{mnk})$ such that $x_{mnk} \in \mathbb{R}^3$; $(m, n, k) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} = \mathbb{N}^3$. The following defintion are obtained:

**Definition 2.1.** A triple sequence spaces $x = (x_{mnk})$ of real numbers is said to be statistically convergent to $L \in \mathbb{R}^3$ if for any $\epsilon > 0$ we have $d(A(\epsilon)) = 0$, where

$$A(\epsilon) = \left\{(m, n, k) \in \mathbb{N}^3 : |x_{mnk} - L| \geq \epsilon\right\}.$$

**Definition 2.2.** A triple sequence spaces $x = (x_{mnk})$ is said to be statistically convergent to $L \in \mathbb{R}^3$, written as $st - \lim x = L$, provided that the set

$$\left\{(m, n, k) \in \mathbb{N}^3 : |x_{mnk} - L| \geq \epsilon\right\},$$

has natural density zero for every $\epsilon > 0$.

In this case, $L$ is called the statistical limit of the sequence $x$.

**Definition 2.3.** Let $x = (x_{mnk})_{(m,n,k) \in \mathbb{N}^3}$ be a triple sequence spaces in a metric space $(X, |.,.|)$ and $r$ be a nonnegative real number. A triple sequence spaces $x = (x_{mnk})$ is said to be $r$-convergent to $L \in X$, denoted by $x \to^r L$, if for any $\epsilon > 0$ there exists $N_\epsilon \in \mathbb{N}^3$ such that for all $m, n, k \geq N_\epsilon$ we have

$$|x_{mnk} - L| < r + \epsilon.$$

In this case $L$ is called an $r$-limit of $x$.

**Remark 2.1.** We consider $r$-limit set $x$ which is denoted by $\mathrm{LIM}_x^r$ and is defined by

$$\mathrm{LIM}_x^r = \{L \in X : x \to^r L\}.$$

**Definition 2.4.** A triple sequence spaces $x = (x_{mnk})$ is said to be $r-$ convergent if $\mathrm{LIM}_x^r \neq \phi$ and $r$ is called a rough convergence degree of $x$. If $r = 0$ then it is ordinary convergence of triple sequence spaces.

**Definition 2.5.** Let $x = (x_{mnk})$ be a triple sequence spaces in a metric space $(X, |.,.|)$ and $r$ be a nonnegative real number is said to be $r-$ statistically convergent to $L$, denoted by $x \to^{r-st_3} L$, if for any $\epsilon > 0$ we have $d(A(\epsilon)) = 0$, where

$$A(\epsilon) = \left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - L| \geq r + \epsilon\right\}.$$

In this case $L$ is called $r$-statistical limit of $x$. If $r = 0$ then it is ordinary statistical convergent of triple sequence spaces.

**Definition 2.6.** A class $I$ of subsets of a nonempty set $X$ is said to be an ideal in $X$ provided

(i) $\phi \in I$.
(ii) $A, B \in I$ implies $A \bigcup B \in I$.
(iii) $A \in I$, $B \subset A$ implies $B \in I$.
$I$ is called a nontrivial ideal if $X \notin I$.

**Definition 2.7.** A nonempty class $F$ of subsets of a nonempty set $X$ is said to be a filter in $X$. Provided

(i) $\phi \in F$.
(ii) $A, B \in F$ implies $A \bigcap B \in F$.
(iii) $A \in F$, $A \subset B$ implies $B \in F$.

**Definition 2.8.** $I$ is a non trivial ideal in $X$, $X \neq \phi$, then the class

$$F(I) = \{M \subset X : M = X \backslash A \text{ for some } A \in I\}$$

is a filter on $X$, called the filter associated with $I$.

**Definition 2.9.** A non trivial ideal $I$ in $X$ is called admissible if $\{x\} \in I$ for each $x \in X$.

**Note 2.10.** *If $I$ is an admissible ideal, then usual convergence in $X$ implies $I$ convergence in $X$.*

**Remark 2.2.** If $I$ is an admissible ideal, then usual rough convergence implies rough $I$-convergence.

**Definition 2.11.** Let $x = (x_{mnk})$ be a triple sequence in a metric space $(X, |.,.|)$ and $r$ be a nonnegative real number is said to be rough ideal convergent or $rI$-convergent to $L$, denoted by $x \rightarrow^{rI} L$, if for any $\epsilon > 0$ we have

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - L| \geq r + \epsilon\right\} \in I.$$

In this case $L$ is called $rI$-limit of $x$ and a triple sequence spaces $x = (x_{mnk})$ is called rough $I$-convergent to $L$ with $r$ as roughness of degree. If $r = 0$ then it is ordinary $I$-convergent.

**Note 2.12.** *Generally, a triple sequence $y = (y_{mnk})$ is not $I$-convergent in usual sense and $|x_{mnk} - y_{mnk}| \leq r$ for all $(m,n,k) \in \mathbb{N}^3$ or*

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - y_{mnk}| \geq r\right\} \in I$$

*for some $r > 0$. Then the triple sequence $x = (x_{mnk})$ is $rI$-convergent.*

**Note 2.13.** *It is clear that $rI$-limit of $x$ is not necessarily unique.*

**Definition 2.14.** Consider $rI$-limit set of $x$, which is denoted by

$$I - \mathrm{LIM}_x^r = \left\{L \in X : x \rightarrow^{rI} L\right\},$$

then the triple sequence $x = (x_{mnk})$ is said to be $rI$-convergent if $I - \mathrm{LIM}_x^r \neq \phi$ and $r$ is called a rough $I$-convergence degree of $x$.

**Definition 2.15.** A triple sequence $x = (x_{mnk}) \in X$ is said to be $I$-analytic if there exists a positive real number $M$ such that

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk}|^{1/m+n+k} \geq M\right\} \in I.$$

**Definition 2.16.** A point $L \in X$ is said to be an $I$-accumulation point of a triple sequence $x = (x_{mnk})$ in a metric space $(X, d)$ if and only if for each $\epsilon > 0$ the set

$$\left\{(m,n,k) \in \mathbb{N}^3 : d\left(x_{mnk}, L\right) = |x_{mnk} - L| < \epsilon\right\} \notin I.$$

We denote the set of all $I$-accumulation points of $x$ by $I\left(\Gamma_x\right)$.

**Definition 2.17.** A point $L \in X$ is said to be an $I$-accumulation point of a triple sequence $X = (X_{mnk})$ in a metric space $(X, d)$ if and only if the set

$$\left\{(m,n,k) \in \mathbb{N}^3 : d\left(x_{mnk}, L\right) = |x_{mnk} - L| < \infty\right\} \notin I.$$

We denote the set of all $I$-accumulation points of $x$ by $I\left(\Lambda_x\right)$.

**Example 2.18.** Let $I = \left\{A \subset \mathbb{N}^3 : d\left(A\right) = 0\right\}$. We define a triple sequence $x = (x_{mnk})$ in the following way

$$x_{mnk} = \begin{cases} 1, & \text{if } m = n = k \\ k, & \text{otherwise} \end{cases}.$$

$I$-limit point does not exist, i.e., $I\left(\Lambda_x\right) = \phi$.

**Definition 2.19.** A nontrivial ideal $I$ on $\mathbb{N}^3$ is called strongly admissible if $\{i\} \times \mathbb{N}^3$belongs to $I$ for each $i \in \mathbb{N}$.

**Definition 2.20.** A triple sequence $x = (x_{mnk})$ is said to be rough $I$-convergent if $I - \mathrm{LIM}^r\, x \neq \phi$. It is clear that if $I - \mathrm{LIM}^r\, x \neq \phi$ for a triple sequence spaces $x = (x_{mnk})$ of real numbers, then we have

$$I - \mathrm{LIM}^r\, x = [I - \limsup x - r, I - \liminf x + r].$$

For a triple sequence spaces $x = (x_{mnk})$ of real numbers, the notions of ideal limit superior and ideal limit inferior are defined as follows:

$$I - \limsup\ x = \begin{cases} \sup\ B_x, & \text{if } B_x \neq \phi \\ -\infty, & \text{if } B_x = \phi \end{cases},$$

and

$$I - \liminf x = \begin{cases} \inf\ A_x, & \text{if } A_x \neq \phi \\ +\infty, & \text{if } A_x = \phi \end{cases},$$

where

$$A_x = \left\{a \in \mathbb{R} : \left\{(m,n,k) \in \mathbb{N}^3 : x_{mnk} < a\right\} \notin I\right\},$$

and

$$B_x = \left\{b \in \mathbb{R} : \left\{(m,n,k) \in \mathbb{N}^3 : x_{mnk} > b\right\} \notin I\right\}.$$

**Definition 2.21.** A triple sequence $x = (x_{mnk})$ is said to be rough $I$-convergent if $I - \mathrm{LIM}^r\, x \neq \phi$. It is clear that if $I - \mathrm{LIM}^r\, x \neq \phi$ for a triple sequence spaces $x = (x_{mnk})$ of real numbers, then we have

$$I - \mathrm{LIM}^r\, x = [I - \limsup x - r, I - \liminf x + r].$$

## 3. Main Results

**Theorem 3.1.** *If $I - \mathrm{LIM}^r\, x \neq \phi$ for a triple sequence $x = (x_{mnk})$ of real numbers, and $I - \mathrm{LIM}^r\, x = [I - \limsup x - r, I - \liminf x + r]$ then* $\mathrm{diam}\,(\mathrm{LIM}^r\, x) \leq \mathrm{diam}\,(I - \mathrm{LIM}^r\, x)$.

*Proof.* We know that $I - \mathrm{LIM}^r\, x = \phi$ for an unbounded triple sequence spaces $x = (x_{mnk})$. But such a sequence might be rough $I$-convergent. For instance, let $I$ be the $I_d$ of $\mathbb{N}$ and define

$$x_{mnk} = \left\{ \begin{array}{cc} \cos\,(mnk)\,\pi, & \text{if } (m,n,k) \neq \left(i^2, j^2, \ell^2\right) : (i,j,\ell \in \mathbb{N}) \\ (mnk)\,, & \text{otherwise} \end{array} \right\},$$

in $\mathbb{R}^3$. Because the set $\{1, 64, 739, \ldots\}$ belong to $I$, we have

$$I - \mathrm{LIM}^r\, x = \left\{ \begin{array}{cc} \phi, & \text{if } r < 1 \\ [1-r, r-1]\,, & \text{otherwise} \end{array} \right\},$$

and $\mathrm{LIM}^r\, x = \phi$, for all $r \geq 0$. The fact that $I - \mathrm{LIM}^r\, x \neq \phi$ does not imply $\mathrm{LIM}^r\, x \neq \phi$. Because $I$ is a admissible ideal

$$\mathrm{LIM}^r\, x \neq \phi \Longrightarrow I - \mathrm{LIM}^r\, x \neq \phi,$$

i.e., if $x = (x_{mnk}) \in \mathrm{LIM}^r x$, then by Remark 2.2, $(x_{mnk}) \in I - \mathrm{LIM}^r x$, for each triple sequences. Also, if we define all the rough convergence sequences by $\mathrm{LIM}^r$ and rough $I$-convergence sequences by $I - \mathrm{LIM}^r$, then we get $\mathrm{LIM}^r \subseteq I - \mathrm{LIM}^r$.

$$\{r \geq 0 : \mathrm{LIM}^r x \neq \phi\} \subseteq \{r \geq 0 : I - \mathrm{LIM}^r x \neq \phi\}.$$

Hence the sets yields immediately

$$\inf \{r \geq 0 : \mathrm{LIM}^r x \neq \phi\} \geq \{r \geq 0 : I - \mathrm{LIM}^r x \neq \phi\},$$

for each triple sequences. Moreover, it also yield directly

$$\mathrm{diam}\,(\mathrm{LIM}^r x) \leq \mathrm{diam}\,(I - \mathrm{LIM}^r x).$$

□

**Note 3.2.** *The rough $I$-limit of a triple sequence is unique for the roughness degree $r > 0$.*

**Theorem 3.3.** *If $I \subset 3^{\mathbb{N}}$ be an strongly admissible ideal and $x = (x_{mnk})$ be a triple sequence, then we have $I(\Lambda_x) \subseteq I(\Gamma_x)$.*

*Proof.* Let $c \in I(\Lambda_x)$. If $c \notin \mathrm{LIM}^r x$ then there exists a set

$$M = \left\{(m,n,k) \in \mathbb{N}^3 : (u_m, v_n, w_k)\right\} \notin I,$$

such that

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{u_m v_n w_k} - c|^{1/m+n+k} \geq r + \epsilon\right\} \notin I. \tag{1}$$

Let $\epsilon > 0$. Then by equation (1) there exists $(r_0, s_0, t_0) \in \mathbb{N}^3$ such that $u_m \geq r_0, v_n \geq s_0, w_k \geq t_0$, we have

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - c|^{1/m+n+k} \geq r + \epsilon\right\} \supset M \setminus \left\{(m,n,k) \in \mathbb{N}^3 : \right.$$
$$\left.(u_m, v_n, w_k),\ \text{either } u_m \leq (r_0 - 1)\ \textit{or}\ v_m \leq (s_0 - 1)\ \text{or } w_m \leq (t_0 - 1)\right\}.$$

since $I$ is strongly admissible, so

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - c| \geq r + \epsilon\right\} \notin I.$$

This implies $c \in I(\Gamma_x)$. Hence $I(\Lambda_x) \subseteq I(\Gamma_x)$. □

**Theorem 3.4.** *If $I \subset 3^{\mathbb{N}}$ be an strongly admissible ideal, $I - \mathrm{LIM}^r x \neq \phi$ for a triple sequence $x = (x_{mnk})$ of real numbers, then*

*(i) $I - \limsup x = \alpha$ if and only if for any $\epsilon > 0$,*

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - \alpha| \geq r + \epsilon\right\} \notin I,$$

*and*

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - \alpha| \geq \epsilon\right\} \in I.$$

*(ii) $I - \liminf x = \beta$ if and only if for any $\epsilon > 0$,*

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - \beta| \leq r + \epsilon\right\} \notin I,$$

*and*

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - \beta| \leq \epsilon\right\} \in I.$$

*Proof.* The proof is straight forward. $\square$

**Theorem 3.5.** *If $I \subset 3^{\mathbb{N}}$ be an strongly admissible ideal, $I - \mathrm{LIM}^r x \neq \phi$ for a triple sequence $x = (x_{mnk})$ of real numbers, then $I - \liminf x \leq I - \limsup x$ holds.*

*Proof.* The proof is similar to the proof of Theorem 3 ([8]) and is omitted. $\square$

**Theorem 3.6.** *If $I \subset 3^{\mathbb{N}}$ be an strongly admissible ideal, $I - \mathrm{LIM}^r x \neq \phi$ for a triple sequence $x = (x_{mnk})$ of real numbers then $I - \mathrm{LIM}^r x \leq I - \liminf x + r \leq I - \limsup x + r \leq I - \mathrm{LIM}^r x$.*

*Proof.* We first prove that $I - \mathrm{LIM}^r x - \liminf x \leq I - \liminf x + r$. If $I - \mathrm{LIM}^r x - \liminf x = -\infty$, then it is obvious. Let $I - \mathrm{LIM}^r x - \liminf = r > -\infty$. Then

$$r_1 = \sup_{uvw} r_{1(uvw)}$$

where $r_{1(uvw)} = \inf \{m \geq u, n \geq v, k \geq w : x_{mnk}\}$. Then

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - L| < r_{1(uvw)}\right\} \subset \{\ (m,n,k) \in \mathbb{N}^3 : \\ (m,n,k)\,,\ \text{either } m \leq (u-1) \text{ or } n \leq (v-1) \text{ or } k \leq (w-1)\}\,.$$

The fact that $I - \mathrm{LIM}^r x \neq \phi$ does not imply $\mathrm{LIM}^r x \neq \phi$.

Since $I$ is strongly admissible ideal $\mathrm{LIM}^r x \neq \phi \Longrightarrow I - \mathrm{LIM}^r x \neq \phi$, we have

$$\left\{(m,n,k) \in \mathbb{N}^3 : (m,n,k)\,,\ \text{either } m \leq (u-1) \text{ or } n \leq (v-1) \text{ or } k \leq (w-1)\right\} \in I,$$

so, then there exists $\epsilon > 0$ such that

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - L| < r_{1(uvw)} + \epsilon\right\} \in I.$$

Now, let $r_2 = I - \liminf x = \inf\ A_1(\epsilon)$. Because $a \in I - \mathrm{LIM}^r x$, we have $A_1(\epsilon) \in I$ for every $\epsilon > 0$, where

$$A_1(\epsilon) = \left\{a \in \mathbb{R} : \left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - a| \geq a + \epsilon\right\} \notin I\right\}.$$

Now if $r_2 < r_{1(uvw)}$, then there exists $a^{'} \in A_1(\epsilon)$ such that $r_2 \leq a^{'} < r_{1(uvw)}$, we have,

$$\{r \geq 0 : \mathrm{LIM}^r x \neq \phi\} \subseteq \{r \geq 0 : I - \mathrm{LIM}^r x \neq \phi\}\,.$$

Hence the sets yields immediately

$$\inf \{r \geq 0 : \mathrm{LIM}^r x \neq \phi\} \geq \{r \geq 0 : I - \mathrm{LIM}^r x \neq \phi\}$$

for each triple sequences. However,

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - L| < a^{'} + \epsilon\right\} \\ \subset \left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - L| < r_{1(uvw)} + \epsilon\right\} \in I$$

which yields $a^{'} \notin A_1(\epsilon)$, which is a contradiction. Then $r_2 \geq r_{1(uvw)}$ for all $(u,v,w)$. Hence it also yield directly $r_1 \leq r_2$, i.e.,

$$I - \mathrm{LIM}^r x - \liminf x \leq I - \liminf x.$$

Similarly we can show $I - \limsup x \leq I - \mathrm{LIM}^r\, x - \limsup x$. $\square$

## 4. Conclusions and Future Work

We introduced triple sequence spaces of extremal rough $I$-convergence limit point. For the reference sections, consider the following introduction described the main results are motivating the research.

## Competing Interests

The authors declare that there is not any conflict of interests regarding the publication of this manuscript.

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