Asia-Pacific Journal of Business (아태비즈니스연구) APJB
Vol. 11, No. 4, December 2020 (pp.37-48) ISSN 2233-5900 (Print)
https://doi.org/10.32599/apjb.11.4.202012.37. ISSN 2384-3934 (Online)

Risk Characteristic on Fat-tails of Return Distribution: An Evidence of the Korean Stock Market*

Cheoljun Eom[a]

[a]School of Business, Pusan National University, South Korea

Received 30 November 2020, Revised 16 December 2020, Accepted 21 December 2020

Abstract

Purpose - This study empirically investigates whether the risk property included in fat-tails of return distributions is systematic or unsystematic based on the devised statistical methods.
Design/methodology/approach - This study devised empirical designs based on two traditional methods: principal component analysis (PCA) and the testing method of portfolio diversification effect. The fatness of the tails in return distributions is quantitatively measured by statistical probability.
Findings - According to the results, the risk property in the fat-tails of return distributions has the economic meanings of eigenvalues having a value greater than 1 through PCA, and also systematic risk that cannot be removed through portfolio diversification. In other words, the fat-tails of return distributions have the properties of the common factors, which may explain the changes of stock returns. Meanwhile, the fatness of the tails in the portfolio return distributions shows the asymmetric relationship of common factors on the tails of return distributions. The negative tail in the portfolio return distribution has a much closer relation with the property of common factors, compared to the positive tail.
Research implications or Originality - This empirical evidence may complement the existing studies related to tail risk which is utilized in pricing models as a common factor.

Keywords: Fat-tails of Return Distribution, Principal Component Analysis, Portfolio Diversification, Random Matrix Theory, Singular Value Decomposition
JEL Classifications: C10, G10, G11

Ⅰ. Introduction

The extensive losses arising from the financial market crashes such as the 1997 Asian financial crisis, the 2008 U.S. credit risk and the 2011 European debt crisis have been raising basic questions about the capability of effective risk management based on the existing models in the field of finance. The expansion of quantitative models on risk management is dependent on how well the various properties observed in a financial time series can be reflected in the models. The returns that are located in the tails of return distribution stem from large-scale fluctuations. The risk property included in the tails of return distributions needs to be identified to improve traditional risk management. The representative variables that are used to introduce

* This study was supported by the Fund for Humanities & Social Studies at Pusan National University 2019.
[a] First Author, E-mail: shunter@pusan.ac.kr

the characteristic of fat-tails in the model are skewness and kurtosis. Although they might indicate the deviation of normality in return distributions, they cannot describe the risk property included in the fat-tails of return distributions. Especially, these measurements cannot explain the downside risk (e.g, Ang, Chen and Xing, 2006) on the large losses. On the other hand, the pricing models in the field of finance usually struggle to explain the changes of returns using systematic risks of common factors. Determining whether to include the risk property in the fat-tails of return distributions in the models as an additive explanatory variable will require preferentially defining the economic implications of the risk property in the fat-tails of return distributions, i.e., whether to have economic meanings of common factors that can explain the changes of returns, and whether to have the property of systematic risk not be removed by portfolio diversification. Taleb (2007) has proposed the Black Swan Theory associated with an event that is extremely unlikely but has the strong influence, i.e., an unexpected event in the past has an extreme deviation from the expected normal level, is positioned in the tails of the distribution, and causes an important change of the market regime. Bhansali (2008) mentioned that fat-tails in return distributions have the property of systematic risk based on the strong relation between market crashes and the liquidity problem in macroeconomic analysis. That is, the risk property in the tails of the return distributions is highly related to macroeconomic exposure. Kelly and Jiang (2014) show that tail risk has a significant impact on aggregate market returns and stock returns in both time series and cross section. However, no consensus has yet been reached about whether the risk property in the fat-tails of return distributions systematically affects the changes of returns as a common factor. This necessitates research which in advance defines the economic implications of the risk property included in the fat-tails of return distributions in order to expand the pricing model reflecting the characteristic of fat-tails.

This study empirically investigates the risk characteristic of fat-tails in return distributions in order to determine the evidence that supports the economic implications of the risk property included in the fat-tails of return distributions based on statistical methods. The fatness of the tails in return distributions is quantitatively measured by statistical probability, in which the number of return data belonging to each end-area deviated from central section of 99% in the distribution is divided by the total number of return data in the distribution. This study devised empirical designs based on two traditional methods: principal component analysis (PCA) in multivariate statistics by King (1966) and Ross (1976) and the testing method of portfolio diversification effect by Evans and Archer (1968). As reported by Chen, Roll and Ross (1986), Brown (1989), Plerou et al. (2002), and Eom et al. (2009), the eigenvalue with a value greater than 1 among all eigenvalues extracted by PCA has the economic meaning of being able to explain the changes of return in pricing model, such as market, industry and macroeconomic factors. The study tests whether the magnitude of eigenvalues is positively correlated with the fatness of the tails in the distribution of the eigenvalue time series. This is evidence suggesting that the risk property included in the fat-tails of return distributions may have the economic meanings of eigenvalues with a value greater than 1 statistically. As a test of the portfolio diversification effect in Elton and Gruber (1977) and Staman (1987), the type of systematic and unsystematic risk can be visually determined by changing the pattern of portfolio risks as the number of stocks in a portfolio increases. The tails of portfolio return distributions have return data from large-scale price fluctuations deviated from the average value and, therefore, the change of the fatness of the tails in portfolio return distributions is very closely related with the portfolio risks. A continuous decreasing pattern of portfolio risks with an increasing number of stocks in a portfolio indicates a property of unsystematic risk, whereas a pattern of converging into a certain level of portfolio risks indicates a property of systematic risk. Identifying the type of changing patterns of the fatness of the tails in portfolio return distributions might be evidence determining whether the risk property in the fat-tails of return distributions is systematic or unsystematic risk.

The results are summarized briefly as follows. The relationship between the magnitude of eigenvalue and the fatness of the tails in the distribution from the eigenvalue time series is positive through PCA. This means that the eigenvalues with higher value tend to have much fatter tails in the distribution of the eigenvalue time series. When using the testing method for portfolio diversification effect, this study discovered that the fatness of the tails in portfolio return distributions tends to converge to a certain level. This evidence may support that the risk property included in the fat-tails of return distributions has the systematic risk that cannot be removed by portfolio diversification. An interesting finding is that the negative tail is much fatter than the positive tail in the portfolio return distribution according to increasing number of stocks in a portfolio. This result is due to the different strength of relation with common factors in each tail of the portfolio return distribution; that is, the negative tail has a much closer relation with the properties of the common factors rather than the positive tail in the portfolio return distribution. Also, this evidence suggests the potential defect of traditional portfolio investment that suffers from the limitations of missing the opportunity for large profits in the positive tail and of taking large losses in the negative tail. In this study, the uncovered economic implications of the risk property included in the fat-tails of return distributions will complement existing studies (e.g., Kelly and Jiang, 2014) of tail risk which is utilized in pricing models as a common factor. Therefore, this study expects that main findings will provide new insights.

The rest of this paper is organized as follows: Chapter 2 describes the data, the periods and methods for testing the research purpose. Chapter 3 presents the results on the economic implications of the risk property included in the tails of return distributions, and the discussion on the main findings is also presented. Chapter 4 summarizes and concludes.

II. Empirical Design

1. Data and Periods

This study utilized the returns of the market and individual stocks traded in the Korean stock market from July 2006 to June 2015 (2,231 trading days). Three stock groups are classified by market capitalization (=price×number of outstanding shares) of firms. Since the study of Banz (1981), firm size is one of the key factors to explain the changes of stock returns, and so this study divided the stocks into three groups: the all stocks group, the large stocks group (stocks belonging to the top 40% among market capitalization of all firms), and the small stocks group (stocks belonging to the bottom 40% among market capitalization of all firms). The classification of stock groups is based on the average value of monthly firm market capitalization within a period. The basic statistics for each of three-type groups is presented in 〈Table 1〉.

Table 1. Basic Statistics

	all stocks	large stocks (top 40%)	small stocks (bottom 40%)
number of stocks	580	232	232
market capitalization (million won)			
average	2,343,594	5,700,963	42,124
maximum	304,848,742	304,848,742	107,818
minimum	788	392,275	788
descriptive statistics			
ave.(average)	0.000735	0.000675	0.000786
ave.(standard deviation)	0.030087	0.027308	0.032767
ave.(skewness)	0.5592	0.4054	0.6949
ave.(kurtosis)	8.8625	8.3898	9.1762

2. Methods

This section describes the main contents of the methods for identifying the economic implications of the risk property included in the fat-tails of return distributions. The empirical distribution observed from the return data has a more peaked central section and much fatter tails, compared to the normal distribution, and therefore is a leptokurtic distribution. The tails of return distributions have large losses and large profits that deviate from the average value. This study standardizes the stock return data for all periods by subtracting the average value and dividing by the standard deviation. By using the standardized return data, the tails are defined by the area of each end-side deviated from central section of 99% in the frequency distribution. The statistical probability is utilized as a measurement to assess the fatness of the tails in return distributions. The statistical probability is the relative frequency (f_N/f_T) ratio, calculated as the number of data (f_N) included in the tails of the distribution divided by the total number of data (f_T). The bin size of the frequency distribution is determined according to Scott (1992). This study assesses the existence of fat-tails in return distributions by the statistical probability having a value greater than 0.5%. To identify the economic implications of the risk property included in the fat-tails of return distributions, the devised methods are based on traditional methods of PCA and the portfolio diversification effect. PCA is used to explore statistically the common factors based on eigenvalues having a value greater than 1, and the portfolio diversification method presents a theoretical basis for the pricing models that is strongly reliant on systematic risk.

The first devised method is based on the PCA method. King (1966) and Ross (1976) reported that PCA is a useful tool to extract potential common factors from the return data of all stocks. Herein, the common factor is a factor that commonly affects the changes of returns. The number of eigenvalues (λ_k, $k=1,2,...,N$) extracted by PCA using the returns of all stocks is the same as the number of N stocks. As reported by Chen, Roll and Ross (1986), Brown (1989), Plerou et al. (2002), and Eom et al. (2009), the eigenvalues with a value greater than 1 based on the Kaiser (1960) criterion have economic meanings such as market, industry, and macroeconomic factors. This study empirically examines the relationship between the magnitude of eigenvalues and the fatness of the tails in the distribution of the eigenvalues time series. If the magnitude of eigenvalues is positively correlated with the fatness of the tails in the distribution of the eigenvalue time series, this is evidence suggesting that the risk property in the fat-tails of return distributions has economic meanings of eigenvalues with a value greater than 1, statistically. The testing process is as follows. All eigenvalues are extracted using PCA, and then create the time series ($R^{\lambda}_{k,t}$) of each eigenvalue, as defined.

$$R^{\lambda}_{k,t} = \sum_{j=1}^{N} v_k R_{k,t} \tag{1}$$

In the equation, the time series of each eigenvalue is created by multiplying eigenvectors (v_k) assigned to the k-th stock to stock return ($R_{j=k,t}$) in time $t=1,2,...,T$. This study standardizes the eigenvalue time series by subtracting the average value and dividing by the standard deviation. Using standardized data, the statistical probability (f_N/f_T) is calculated as the measurement on the fatness of the tails in the distribution. This study then tests the relationship between the magnitude of eigenvalue and the statistical probability on the tails of the eigenvalue distribution using correlation analysis.

The second devised method is based on the testing method of portfolio diversification effect. The effect of portfolio diversification based on Evans and Archer (1968) is defined by the reduction of portfolio risk as the number of stocks in a portfolio increases. The risk of a well-diversified portfolio is determined by the systematic risk. The tails of a portfolio return distribution have large-scale price fluctuations that deviate from the average value. As a result, the fatness of the tails in portfolio return distributions is very closely related with portfolio risk. This study investigates the changing pattern of the fatness in the tails of portfolio return distributions through the testing method for the portfolio diversification effect. If the fatness of the tails in portfolio return distributions clearly converges to a certain level with an increasing number of stocks in the portfolio, then this is evidence suggesting that the risk property in the fat-tails of portfolio return distributions is a systematic risk that cannot be removed by portfolio diversification. On the other hand, if the fatness in the tails of portfolio return distribution disappears, this is evidence suggesting that the risk property in the fat-tails of portfolio return distribution is an unsystematic risk that can be reduced by portfolio diversification. The testing process is as follows. The number of stocks in a portfolio is set using the range from at least 2 to a maximum of 50 based on the previous studies of Evans and Archer (1968), Elton and Gruber (1977) and Staman (1987). In each stock in a portfolio, this study calculates the portfolio returns from each of 100 cases of a portfolio constructed by randomly selected stocks based on the sampling with non-replacement. All portfolio returns are standardized by subtracting the average value and dividing by the standard deviation. Using standardized portfolio returns, the statistical probability (f_N/f_T) is calculated as the measurement on the fatness of the tails in the portfolio return distribution, and the average value of the 100 statistical probabilities is calculated in each stock in a portfolio. This study then investigates the changing patterns of the average value from statistical probability according to the increasing number of stocks in the portfolio.

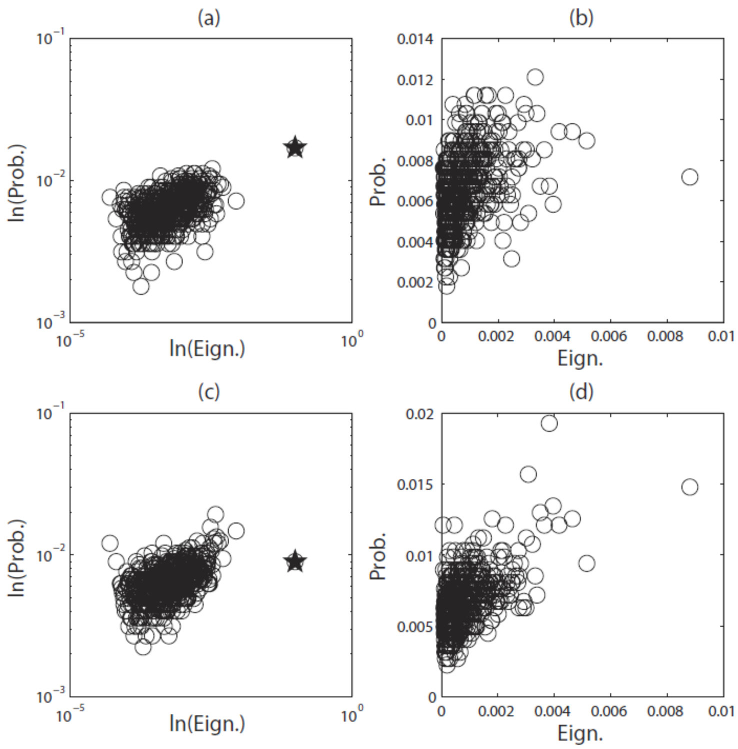
Ⅲ. Results

1. The implications of fat-tails in risk management

In this section, this study presents the results for the economic implications of the risk property included in the fat-tails of return distributions. In the field of finance, determining whether or not to include a new explanatory variable in the pricing model is needed to assess whether the new variable has an economic meaning that affects the changes of returns, and then whether the risk property of the new variable is the systematic risk or unsystematic risk. This study utilizes two traditional methods: PCA and the testing method of portfolio diversification. The results observed from each method are presented in ⟨Fig. 1⟩ and ⟨Fig. 2⟩.

First, the results uncovering the risk property in the fat-tails of return distributions through PCA are presented. Determining whether or not eigenvalues extracted from PCA have an economic meaning is strongly reliant on the magnitude of the eigenvalues. Thus, this study investigates the relationship between the magnitude of eigenvalues and the fatness of the tails in the distribution from the eigenvalue time series. The positive relationship means that eigenvalues with a higher value tend to have fatter tails in the eigenvalue distribution, which is evidence suggesting that the risk property in the fat-tails of return distributions has economic meaning of eigenvalues with a value greater than 1, statistically.

Fig. 1. Economic Implications Using Eigenvalues through the PCA Method

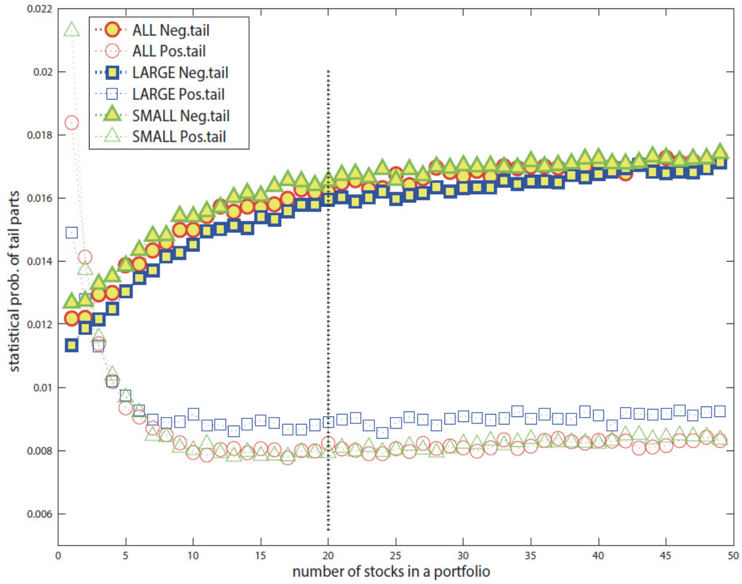


The results are shown in ⟨Fig. 1⟩. The figure shows N=580 stocks belonging to all stocks group over the period from July 2006 to June 2015. Thus, the number of eigenvalues extracted from PCA is K=580, and the time series data with the property of each of the eigenvalues has the same length of the sub-period from July 2006 to June 2015. The statistical probability is used to measure the fatness of the tails in the distribution of the eigenvalue time series. The X-axis indicates the magnitude of eigenvalues and the Y-axis denotes the statistical probability in the tails of the eigenvalue distributions. ⟨Fig. 1a⟩ and ⟨Fig. 1b⟩ presents results for the negative tail, and ⟨Fig. 1c⟩ and ⟨Fig. 1d⟩ for the positive tail in the eigenvalue distribution. In addition, this study controls the effect from the largest eigenvalue that can substantially affect the result. This is because the difference of the value between the largest eigenvalue and the second largest eigenvalue is very high, as known in Brown (1989) and Eom et al. (2009). ⟨Fig. 1a⟩ and ⟨Fig. 1c⟩ show the results of the relationship including the largest eigenvalue (☆) using the double-log plot, while ⟨Fig. 1b⟩ and ⟨Fig. 1d⟩ show the relationship excluding the largest eigenvalue using the scatter plot.

According to the results, the magnitude of eigenvalues has a clearly positive relationship with the fatness of the tails in the distribution of the eigenvalue time series. In ⟨Fig. 1a⟩ and ⟨Fig. 1b⟩, the strength of the relationship for the negative tail is statistically significant 31.12% and 57.14% in the correlations, respectively, and the strength of relationship for the positive tail in ⟨Fig. 1c⟩ and ⟨Fig. 1d⟩ is statistically significant 14.41% and 63.41% correlations, respectively. The relationship excluding the largest eigenvalue has a higher value compared to the relationship including the largest eigenvalue; that is, the positive relation is obvious in figures. The results on large stocks group ((a)&(b), 36.45% & 50.05%; (c)&(d), 16.26% & 44.75%) and small stocks group (33.36% & 50.59%; 12.22% & 50.61%) that are not reported in this paper also present the significant positive relationship. These results are evidence suggesting that the risk property included in the fat-tails of return distributions has economic meaning of eigenvalues with a value greater than 1, statistically.

Next, this study presents results of testing whether the risk property in the fat-tails of return distributions is the systematic risk or unsystematic risk through the testing method of portfolio diversification. By portfolio diversification, a well-diversified portfolio can eliminate the unsystematic risk. The returns that are located in the tails of return distributions come from large-scale price fluctuations. As the number of stocks in a portfolio increases, observing the changes in the fatness of the tails in portfolio return distributions is very closely related with observing the changes of portfolio risk. Therefore, this study empirically investigates the changes in the fatness of the tails in portfolio return distributions according to the increasing number of stocks in a portfolio. A pattern converging into a certain level indicates that the fat-tails of the portfolio return distribution have the property of systematic risk. The results are shown in ⟨Fig. 2⟩. The figure presents results using all return data of three stock groups over the period from July 2006 to June 2015. The measurement on the fatness of the tails in the portfolio return distribution is the statistical probability. The X-axis indicates the number of stocks in a portfolio within the range from 2 to 50, and the Y-axis presents the average values of all statistical probabilities calculated from 100 iterated simulations in each number of stocks, 2~50, in a portfolio through the random selecting method. The results are using the stock return data of all stocks group (○), large stocks group (□), and small stocks group (△). and are separately presented for the negative tail (●, ■, ▲) and positive tail (○, □, △).

Fig. 2. Economic Implications Using the Testing Method of Portfolio Diversification Effect

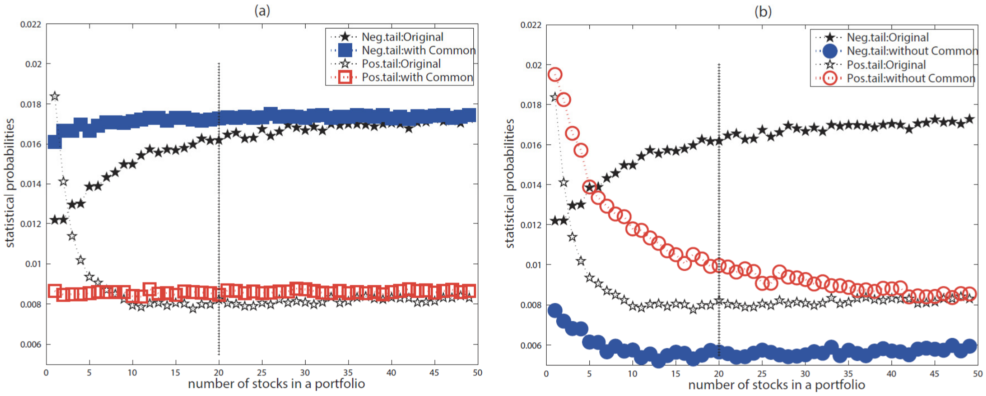


From ⟨Fig. 2⟩, regardless of stock groups, the statistical probability on the fatness of the tails in the portfolio return distribution tends to converge into a certain level as the number of stocks in the portfolio increases. This means that the risk property included in the tails of the portfolio return distribution is not completely removed through portfolio diversification, that is, the property of systematic risk. Moreover, the interesting finding is that the changing patterns in the fatness of the tails in the portfolio return distribution show opposite behavior between the negative and positive tails. As the number of stocks in a portfolio increases, the statistical probability in the positive tail has a decreasing pattern converging to a certain level, while the statistical probability in the negative tail has an increasing pattern converging to a certain level. The decreasing pattern in the positive tail means that the traditional portfolio diversification misses the opportunity for large profits from frequent large-scale price changes in the financial market, whereas the increasing pattern in the negative tail means that the tradition portfolio diversification cannot avoid the possibility of large losses from large-scale price fluctuations in market crashes. Consequently, this finding suggests that the traditional portfolio diversification suffers from the limitations of not effectively controlling for the possibility of large losses and the opportunity of large profits.

2. Robustness and Discussion

This study empirically verified that the fat-tails of return distributions have the economic meanings of eigenvalues having a higher value than 1 statistically, and the property of systematic risk that cannot be removed through portfolio diversification. This evidence reveals that the property included in the fat-tails of return distributions has a close relationship with the properties of the common factor. This section presents the results of the additional test conducted to determine the reliability of findings. The testing hypothesis is as follows: if the property of the fat-tails in return distributions is closely related to the properties of the common factors, the results are significantly dependent on whether or not to include the properties of the common factor in the return data. Based on the empirical design established for the research goal, this study devises a method for testing the hypothesis. That is, the number of eigenvalues having the properties of the common factors is identified from the return data, and then, through the number of common factors, both the return data having only properties of the common factors in the original return data and the return data removing only properties of the common factors in the original return data are separately generated. Then, the same testing process employed in 〈Fig. 2〉 is performed using the two types of return data generated. The number of eigenvalues having the properties of the common factors is identified by the random matrix theory (RMT, Mehta (1995)), and the two types of return data with and without the properties of the common factors from the original return data are generated by the singular vector decomposition (SVD, Leon (2002)). All three methods (PCA used for the second research goal and RMT and SVD for the additional test) share the use of eigenvalues to control for the various properties included in the return data. Also, each method has a unique comparative advantage. RMT can mathematically define the eigenvalues that are deviated from the range of eigenvalues having random properties in the distribution of eigenvalues estimated from the return data. In other words, the eigenvalues that have a higher value than an eigenvalue having the maximum value among the random eigenvalues are well known to have economic meanings as common factors. SVD can generate the new return data that have only the properties of the eigenvalues included within the pre-specific range from the original return data. That is, this method can generate the return data having only the properties of the eigenvalues identified as common factors through RMT, and also the return data removing only the properties of the eigenvalues having the properties of the common factors. The specific details of each method are not presented here due to space considerations. The main testing procedures for the testing hypothesis are briefly as follows. This study checks the number of common factors through RMT, and then generate the two-type return data with and without the property of common factor through SVD based on the RMT results. Finally, using each of the two-type return data generated, this study performs the same testing process with 〈Fig. 2〉 of the portfolio diversification effect. The results are presented in 〈Fig. 3〉. The figure utilizes the return data of all stocks group in the period from July 2006 to June 2015. In the figures, 〈Fig. 3a〉 is the result from the return data having only the properties of the common factors, and 〈Fig. 3b〉 is the results from the return data removing only the properties of the common factors. The X-axis indicates the number of stocks constructing a portfolio, and the Y-axis denotes the average values of the statistical probabilities as the measurement to quantify the fatness of tails in the return distribution.

Fig. 3. Additional Test for the Effect of Common Factors on Portfolio Diversification



According to the results, this study empirically verifies the evidence to support the testing hypothesis. In other words, the properties included in the tails of return distributions are closely related to the properties of the common factors. In particular, the strength of relation with properties of common factors is much higher in the negative tail than in the positive tail of portfolio return distributions. The result of 〈Fig. 3a〉 is very similar to the finding that is confirmed in 〈Fig. 2〉. As the number of stocks in a portfolio increases, the statistical probability on the tails of portfolio return distributions shows the pattern of a constant level without a specific change. This is clearly the pattern of the systematic risk that is not removed through portfolio diversification, due to using the return data having only the properties of the common factors. In addition, as shown in 〈Fig. 2〉, the statistical probability from the negative tail in the distribution of portfolio return has the much higher value, compared to the positive tail. On the other hand, the result of 〈Fig. 3b〉 is clearly different from 〈Fig. 2〉. As the number of stocks in a portfolio increases, the statistical probability from the tails of the portfolio return distribution has a decreasing pattern and approaches a certain level. The pattern of the statistical probability on the positive tail in the portfolio return distribution does not differ from 〈Fig. 2〉. However, the pattern of statistical probability on the negative tail in the portfolio return distribution contrasts sharply with the results of 〈Fig. 3a〉 as well as 〈Fig. 2〉. In other words, the statistical probability in the negative tail in the portfolio return distribution has a much smaller value than that in the positive tail. This result is strongly dependent on using the return data that do not have only the properties of the common factors. Consequently, 〈Fig. 3〉 is evidence supporting the hypothesis that the properties included in the tails of return distributions are closely related to the properties of the common factors, i.e., evidence to obtain the robustness for the results from 〈Fig. 2〉. Moreover, the properties of the common factors have a closer relationship with the negative tail in return distributions, compared to the positive tail. That is the asymmetric relation of the common factor on the tails of return distributions.

Ⅳ. Conclusions

This study has investigated the characteristic of fat-tails in return distributions in order to identify the economic implications of the risk property included in the fat-tails of return distributions in the Korean stock market over the period from July 2006 to June 2015. The results are summarized as follows. The risk property included in the fat-tails of return distributions has the economic meanings of eigenvalues with a value greater than 1 statistically, and the property of systematic risk that cannot be removed by portfolio diversification. In other words, the risk property included in the fat-tails of return distributions has economic implications of common factors that may commonly explain the changes of returns in the pricing model. Interestingly, the properties of the common factors have a closer relation with the negative tail, compared to the positive tail, in the portfolio return distribution, i.e., the asymmetric relation of common factor on the tails in return distribution. The findings are robust regardless of the stock groups. Accordingly, the uncovered economic implications of risk property in the fat-tails of return distribution may complement the existing studies using tail risk as a common factor in pricing models.

In addition, out findings might be evidence to explain the potential defect of traditional portfolio investment. Under the risk-return relation, the portfolio constructed using many stocks will be expected to take a lower return as a consideration of being capable of avoiding a higher risk. Portfolio investment always achieves a lower profit, compared to the high profit realized by some of stocks in a portfolio. The fatness of the positive tail in the portfolio return distribution shows a decreasing pattern according to an increasing number of stocks in a portfolio. The positive tail in the portfolio return distribution might have a closer relation with the properties of individual stocks than with the properties of common factors, while the negative tail in the portfolio return distribution might have a substantially closer relation with the properties of common factors. For example, during periods of large losses in the stock market, in particular, such as a market crash, most of the stocks in a portfolio tend to show a synchronized pattern with the decreasing trend of the market. Since most of the stocks in a portfolio simultaneously experience large losses in the declining period of the market. In this case, the fatness in the negative tail in the portfolio return distribution shows a constant pattern having a higher value than the positive tail. As a result, our findings might be empirical evidence to reveal the potential defect of the traditional portfolio investment in that it misses the potential opportunities to avoid the large losses as well as to achieve the large profits. Accordingly, investors may suffer from large losses in a market crash, despite having a well-diversified portfolio. This study expect our findings to give a new insight on effort to improve the practical portfolio investment.

References

Ang, A., J. Chen, and Y. Xing (2006), "Downside Risk", *Review of Financial Studies*, 19(4), 1191-1239.

Banz, R. W. (1981), "The Relationship between Return and Market Value of Common Stocks", *Journal of Financial Economics*, 9, 3-18.

Bhansali, V. (2008), "Tail Risk Management", *Journal of Portfolio Management*, 34, 68-75.

Brown S. T. (1989), "The Number of Factors in Security Returns", *Journal of Finance*, 44(5), 1247-1262.

Chen, N., R. Roll, and S. A. Ross (1986), "Economic Factors and the Stock Market", *Journal of Business*, 59(3), 383-403.

Elton, E. J. and M. J. Gruber (1977), "Risk Reduction and Portfolio Size: An Analytical Solution", *Journal of Business*, 50, 145-437.

Eom, C., W. Jung, T. Kaizoji, and S. Kim (2009), "Effect of Changing Data Size on Eigenvalues in the Korean and Japanese Stock Markets", *Physica A,* 388(22), 4780-4786.

Evans, J. L. and S. H. Archer (1968), "Diversification and the Reduction of Dispersion: An Empirical Analysis", *Journal of Finance*, 23(5), 761-767.

Kaiser, H. F. (1960), "The Application of Electronic Computers to Factor Analysis", *Educational and Psychological Measurement*, 20, 141-151.

Kelly, B. and H. Jing (2014), "Trail Risk and Asset Prices", *Review of Financial Studies*, 27(10), 2841-2871.

King, B. (1966), "Market and Industry Factors in Stock Price Behavior", *Journal of Business*, 39(1), 139-190.

Leon, S. J. (2002), *Linear Algebra with Application*. (6th ed.), Prentice Hall.

Mehta M. (1995), *Random Matrices*, Academic Press, New York.

Plerou V., P. Gopikrishnan, B. Rosenow, L. A. N. Amaral, T. Guhr and H. E. Stanley (2002), "Random Matrix Approach to Cross Correlations in Financial Data", *Physical Review E*, 65, 066126.

Ross, S. (1976), "The Arbitrage Theory of Capital Asset Pricing", *Journal of Economic Theory*, 13(3), 341-360.

Scott, D. W. (1992), *Multivariate Density Estimation: Theory, Practice, and Visualization*, New York: John Wiley.

Statman, M. (1987), "How Many Stocks Make a Diversified Portfolio?" *Journal of Financial and Quantitative Analysis*, 22(3), 353-363.

Taleb, N. N. (2007), *The Black Swan: The Impact of the Highly Improbable*, Random House (NY).