Asia-Pacific Journal of Business (아태비즈니스연구)
Vol. 11, No. 1, March 2020 (pp. 17-26)
https://doi.org/10.32599/apjb.11.1.202003.17.

APJB
ISSN 2233-5900 (Print)
ISSN 2384-3934 (Online)

Welfare Impacts of Behavior-Based Price Discrimination with Asymmetric Firms*

Hoe-Sang Chung[a]

[a]Department of Economics, Kangwon National University, South Korea

Received 28 February 2020, Revised 18 March 2020, Accepted 25 March 2020

Abstract

Purpose - This paper studies the welfare impacts of behavior-based price discrimination (BBPD) when firms are asymmetric in quality improvement costs.
Design/methodology/approach - To this end, we consider a differentiated duopoly model with an inherited market share, where firms first make quality decisions and then compete in prices according to the pricing scheme, namely, uniform pricing or BBPD.
Findings - We show that BBPD increases social welfare relative to uniform pricing if the firms' cost gap is large enough. This is because BBPD induces more consumers to buy a high-quality product than under uniform pricing, and because a low-cost firm's profit loss from BBPD decreases as the cost difference increases.
Research implications or Originality - Our analysis offers policy implications for markets where BBPD raises antitrust concerns, and quality competition prevails.

Keywords: Difference in Quality Costs, Behavior-Based Price Discrimination, Quality Choice, Social Welfare

JEL Classifications: D43, L13

Ⅰ. Introduction

Due to the development of more sophisticated techniques for acquiring, storing, and analyzing information on the customers' past shopping behavior, firms can offer different prices to their own customers and to rivals' previous customers. This form of price discrimination, termed behavior-based price discrimination (BBPD), is now widely used in many industries such as web retailing, supermarkets, air travel, telecommunication, restaurants, electricity, gas, banking, and insurance.

* This study was supported by 2018 Research Grant from Kangwon National University.
[a] First Author, E-mail: hschung@kangwon.ac.kr

BBPD has been investigated within various frameworks.[1] For example, Chen and Pearcy (2010) investigate BBPD with the dependence of consumers' intertemporal preferences, while Esteves (2014) studies BBPD when firms use retention strategies as an attempt to avoid consumer switching. In addition, Esteves and Reggiani (2014) explore BBPD with elastic demand, Chung (2016) studies BBPD with experience goods, and Carroni (2018) studies the pricing scheme with cross-group externalities. Moreover, Colombo (2016) and Colombo (2018) examine BBPD when firms have incomplete information about consumers' purchase histories and when firms retain additional information about the price sensitivity of their own consumers, respectively.

A common prediction in the literature on BBPD is that in markets exhibiting best-response asymmetry, price discrimination based on consumers' purchase history yields lower social welfare than uniform pricing when aggregate output is constant and some consumers are poached (e.g. Chen, 1997; Fudenberg and Tirole, 2000).[2]

This paper aims to examine the welfare impacts of BBPD when firms are asymmetric in quality costs. To this end, we consider a differentiated duopoly with an inherited market share, and a two-stage game where firms first simultaneously choose their product qualities, and then compete in prices according to the pricing scheme, namely, uniform pricing or BBPD. In our model, BBPD is analyzed in static settings where information used to segment consumers is exogenously given (e.g. Gehrig et al., 2012). We also assume that quality investment is a longer-run decision than price choice, since it usually involves technological decisions.

The paper closest to ours is Gehrig et al. (2012) who use an asymmetric duopoly model with inherited market dominance to analyze the effects of BBPD on social welfare. However, they do not consider quality choices of the firms. Ikeda and Toshimitsu (2010) examine the welfare effects of third-degree price discrimination with quality choice, but they focus on a monopolist in a vertically differentiated product market. We fill this gap by investigating competitive price discrimination with quality choice.

Our analysis can provide policy implications for markets where BBPD raises antitrust concerns, and quality competition prevails. For example, in Korea, telecommunication companies actively investing in R&D are not permitted to price discriminate between their own and rivals' customers by the Mobile Device Distribution Improvement Act.[3]

---

1) There are two main approaches to the analysis of price discrimination based on purchase history. In the switching cost approach, consumers' past purchases reveal information about their switching costs (e.g. Chen, 1997). In the brand preference approach, consumers' past purchases reveal information about their brand preferences (e.g. Fudenberg and Tirole, 2000).

2) A market exhibits best-response asymmetry when one firm's strong market is the other's weak market (Corts, 1998). This best-response asymmetry often arises in Hotelling's model of product differentiation, where consumers are heterogeneous on a single dimension. A market exhibits best-response symmetry when firms price discriminate according to 'choosiness' (see Armstrong, 2006).

3) See Gehrig et al. (2011) for European anti-

We show that BBPD raises consumer surplus but reduces industry profits relative to uniform pricing. Then, if the difference in costs of quality improvement between the firms is sufficiently large, the gain to consumers associated with BBPD exceeds the associated profit loss to the firms.

The rest of the paper unfolds as follows. In Section II, we set up the model. In Section III, we solve the game and compare social welfare under the two pricing schemes. Section IV concludes the paper.

II. The Model

Consider two firms 1 and 2 producing differentiated products at zero marginal cost. The brands produced by firms 1 and 2 are located at point 0 and point 1, respectively, of an interval $[0,1]$ representing the product characteristic space. They play a two-stage game. In the first stage, each firm $i \in \{1,2\}$ simultaneously chooses a quality level $q_i \geq 0$. We assume that firm i incurs a fixed cost of the form $F_i(q_i) = \frac{c_i}{2} q_i^2$ to achieve the level of quality q_i.[4] Without loss of generality, we normalize c_1 to be 1 and assume that $c_2 = c \in (0,1)$. That is, firm 2 is more efficient than firm 1 in improving the product quality. In the second stage, the firms simultaneously choose their prices.

trust cases concerned with BBPD.

4) Unlike Ikeda and Toshimitsu (2010) with fixed costs of quality, Nguyen (2014) considers variable costs of quality and obtains results opposite to Ikeda and Toshimitsu (2010). In this regard, it would be interesting to see how introducing variable costs of quality affects results that we will present below.

There are a unit mass of consumers who are uniformly distributed on the interval $[0,1]$. A consumer's location $x \in [0,1]$ measures how well each firm's product matches her tastes. A consumer indexed by $x \in [0,1]$ incurs a disutility of x when buying from firm 1, and of $1-x$ when buying from firm 2 (unit transportation cost is normalized to 1). Let v be the basic value of each firm's product, which is assumed to be sufficiently large to ensure that all consumers purchase from one of the two firms. Each consumer demands at most one unit of the product. Hence, if a consumer indexed by x buys firm i's product of quality q_i at price p_i, then she enjoys the net benefit

$$v + q_i - (|x - i + 1|) - p_i.$$

Let m $(1-m)$ denote firm 1's (firm 2's) inherited market share so that all consumers with $x \in [0,m]$ $(x \in (m,1])$ have purchased from firm 1 (firm 2) before. Finally, the following assumption will be maintained throughout the paper.

Assumption 1. (i) $\frac{1}{3} < m < \frac{2}{3}$ and (ii) $c > \underline{c} \equiv \frac{6m-8}{30m-25}$.

Assumption 1 implies that the inherited market share of each firm and the cost difference between the firms are not too large, and guarantees that in equilibrium where BBPD is used, both firms poach some of their competitor's former consumers by charging them lower prices.

Ⅲ. Analysis

To derive the subgame perfect equilibrium, the game is solved using backward induction from the second stage. Let the superscript u (d) identify the uniform pricing (BBPD) case.

1. Uniform Pricing

Before proceeding to the BBPD analysis, we consider the benchmark case where there is no BBPD in the second stage, either because it is not permitted or because the firms do not have information about consumers' past purchases. When the firms engage in uniform pricing, firm $i \in \{1,2\}$ charges the same price, p_i, to all consumers. Let $\hat{x}$ denote a consumer who is indifferent between buying from either firm. This consumer is determined from

$$v + q_1 - \hat{x} - p_1 = v + q_2 - (1 - \hat{x}) - p_2.$$

That is,

$$\hat{x} = \frac{p_2 - p_1 + \Delta q + 1}{2},$$

where $\Delta q \equiv q_1 - q_2$.

Thus, at the pricing stage, firm 1 chooses p_1 to maximize

$$\pi_1 = p_1 \hat{x}$$

and firm 2 chooses p_2 to maximize

$$\pi_2 = p_2 (1 - \hat{x}),$$

for a given quality pair (q_1, q_2). The equilibrium prices are

$$p_1^u = \frac{3 + \Delta q}{3} \text{ and } p_2^u = \frac{3 - \Delta q}{3}.$$

The market allocation is described by

$$\hat{x}^u = \frac{3 + \Delta q}{6}.$$

At the quality choice stage, firm 1 chooses q_1 to maximize

$$\pi_1^u = p_1^u \hat{x}^u - F_1(q_1)$$

and firm 2 chooses q_2 to maximize

$$\pi_2^u = p_2^u (1 - \hat{x}^u) - F_2(q_2).$$

We then obtain the equilibrium quality levels as

$$q_1^{u*} = \frac{9c - 2}{24c - 3} \text{ and } q_2^{u*} = \frac{7}{24c - 3}.$$

Substituting them into the equilibrium prices and market allocation yields

$$p_1^{u*} = \frac{9c - 2}{8c - 1}, \quad p_2^{u*} = \frac{7c}{8c - 1}, \quad \text{and} \quad \hat{x}^{u*} = \frac{1}{2} + \frac{c - 1}{16c - 2}.$$

We can see that the low-cost firm chooses a higher quality level and charges a higher price than the high-cost firm ($q_1^{u*} < q_2^{u*}$ and $p_1^{u*} < p_2^{u*}$), and that the market share of the low-cost firm is larger than that of the high-cost firm ($\hat{x}^{u*} < \frac{1}{2}$).

2. Behavior-Based Price Discrimination

Now assume that BBPD is feasible. We first investigate the firms' price competition, taking their quality choices as given. When the two firms engage in BBPD, firm $i \in \{1,2\}$ charges a price p_i to consumers who have purchased its product before, and r_i to consumers who have purchased from the rival firm. Let $\hat{x}_1$ denote a consumer who has purchased from firm 1 before and is now indifferent between being loyal to firm 1 and switching to firm 2. Similarly, let $\hat{x}_2$ denote a consumer who has purchased from firm 2 before and is now indifferent between being loyal to firm 2 and switching to firm 1. These consumers are, respectively, determined from

$$v + q_1 - \hat{x}_1 - p_1 = v + q_2 - (1 - \hat{x}_1) - r_2,$$

$$v + q_1 - \hat{x}_2 - r_1 = v + q_2 - (1 - \hat{x}_2) - p_2.$$

That is,

$$\hat{x}_1 = \frac{r_2 - p_1 + \Delta q + 1}{2} \quad \text{and} \quad \hat{x}_2 = \frac{p_2 - r_1 + \Delta q + 1}{2}.$$

Thus, consumers with $x \in [0, \hat{x}_1)$ ($x \in (\hat{x}_2, 1]$) continue to buy from firm 1 (firm 2), and those with $x \in [\hat{x}_1, m)$ ($x \in [m, \hat{x}_2]$) switch to firm 2 (firm 1).

At stage 2, firm 1 chooses p_1 and r_1 to maximize

$$\pi_1 = p_1 \hat{x}_1 + r_1 (\hat{x}_2 - m),$$

and firm 2 chooses p_2 and r_2 to maximize

$$\pi_2 = p_2 (1 - \hat{x}_2) + r_2 (m - \hat{x}_1),$$

for a given quality pair (q_1, q_2). The equilibrium prices are

$$p_1^d = \frac{1 + 2m + \Delta q}{3}, \quad r_1^d = \frac{3 - 4m + \Delta q}{3},$$

$$p_2^d = \frac{3 - 2m - \Delta q}{3}, \quad r_2^d = \frac{-1 + 4m - \Delta q}{3}.$$

The market allocation is described by

$$\hat{x}_1^d = \frac{1 + 2m + \Delta q}{6} \quad \text{and} \quad \hat{x}_2^d = \frac{3 + 2m + \Delta q}{6}.$$

Now we turn to the firms' quality choice. At stage 1, firm 1 chooses q_1 to maximize

$$\pi_1^d = p_1^d \hat{x}_1^d + r_1^d (\hat{x}_2^d - m) - F_1(q_1)$$

and firm 2 chooses q_2 to maximize

$$\pi_2^d = p_2^d (1 - \hat{x}_2^d) + r_2^d (m - \hat{x}_1^d) - F_2(q_2).$$

We then obtain the equilibrium quality levels as

$$q_1^{d*} = \frac{(12 - 6m)c - 4}{21c - 6} \quad \text{and} \quad q_2^{d*} = \frac{2 + 6m}{21c - 6}.$$

Substituting them into the equilibrium

prices and market allocation yields

$$p_1^{d*} = \frac{(11+12m)c-4-6m}{21c-6},$$
$$p_2^{d*} = \frac{(17-12m)c-4+6m}{21c-6},$$

$$r_1^{d*} = \frac{(25-30m)c-8+6m}{21c-6},$$
$$r_2^{d*} = \frac{(-11+30m)c+4-6m}{21c-6},$$

$$\hat{x}_1^{d*} = \frac{1}{6}+\frac{m}{3}+\frac{(2-m)c-1-m}{21c-6},$$
$$\hat{x}_2^{d*} = \frac{1}{2}+\frac{m}{3}+\frac{(2-m)c-1-m}{21c-6}.$$

Note that firm 2 (the low-cost firm) chooses a higher quality level than firm 1 (the high-cost firm) if their cost gap is large enough. Formally, for $\frac{1}{3}< m \le \frac{1}{2}$, $q_1^{d*} < q_2^{d*}$ if $c < \frac{1+m}{2-m}$, and for $\frac{1}{2}< m < \frac{2}{3}$, it always holds. It can be also checked that each firm charges a lower price to the rival's consumers than to its own consumers ($p_i^{d*} > r_i^{d*}$), and that some of each firm's former consumers are poached by the rival ($0 < \hat{x}_1^{d*} < m < \hat{x}_2^{d*} < 1$). The number of switching consumers is $\hat{x}_2^{d*} - \hat{x}_1^{d*} = \frac{1}{3}$. The market share of the low-cost firm (denoted by MS_2) is larger than that of the high-cost firm (denoted by MS_1) if their cost gap is large enough. Formally, for $\frac{1}{3}< m \le \frac{1}{2}$, $MS_1 = \hat{x}_1^{d*} + (\hat{x}_2^{d*} - m)$ $\langle MS_2 = (1-\hat{x}_2^{d*}) + (m-\hat{x}_1^{d*})$ if $c < \frac{2}{5-6m}$, and for $\frac{1}{2}< m < \frac{2}{3}$, it always holds.

3. Welfare

This subsection provides the welfare consequences of BBPD when the firms are asymmetric in quality costs.

Under uniform pricing, consumer surplus is calculated as

$$\begin{aligned} &CS^{u*} \\ &= \int_0^{\hat{x}^{u*}} \left[v+q_1^{u*}-x-p_1^{u*}\right]dx + \\ &\quad \int_{\hat{x}^{u*}}^1 \left[v+q_2^{u*}-(1-x)-p_2^{u*}\right]dx \\ &= v-\frac{813c^2-296c+22}{12(8c-1)^2}. \quad (1) \end{aligned}$$

Consumer surplus under BBPD is calculated as

$$\begin{aligned} CS^{d*} = &\int_0^{\hat{x}_1^{d*}} \left[v+q_1^{d*}-x-p_1^{d*}\right]dx \\ &+\int_{\hat{x}_1^{d*}}^{m} \left[v+q_2^{d*}-(1-x)-r_2^{d*}\right]dx \\ &+\int_m^{\hat{x}_2^{d*}} \left[v+q_1^{d*}-x-r_1^{d*}\right]dx \\ &+\int_{\hat{x}_2^{d*}}^{1} \left[v+q_2^{d*}-(1-x)-p_2^{d*}\right]dx \\ &= v-\frac{\begin{array}{c}(391-531m+621m^2)c^2\\ -(232-270m+378m^2)c\\ +(34-36m+54m^2)\end{array}}{9(7c-2)^2}. \quad (2) \end{aligned}$$

Subtracting (1) from (2) yields

$$CS^{d*}-CS^{u*} = \frac{M_4c^4-4M_3c^3+6M_2c^2-8M_1c+8M_0}{36(8c-1)^2(7c-2)^2} > 0$$

where

$M_4 = 19415 + 135936m - 158976m^2$, $M_3 = 6847 + 25776m - 34128m^2$, $M_2 = 2123 + 4770m - 6750m^2$, $M_1 = 287 + 423m - 621m^2$, and $M_0 = 16 + 18m - 27m^2$. This result shows that consumers benefit from BBPD, regardless of the quality cost difference between the firms.

The equilibrium profit of each firm under uniform pricing is

$$\pi_1^{u^*} = p_1^{u^*}\hat{x}^{u^*} - F_1(q_1^{u^*}) = \frac{4(9c-2)^2}{9(8c-1)^2}$$

$$\pi_2^{u^*} = p_2^{u^*}(1-\hat{x}^{u^*}) - F_2(q_2^{u^*}) = \frac{49c(9c-1)}{18(8c-1)^2} \quad (3)$$

and under BBPD it is

$$\pi_1^{d*} = p_1^{d*}\hat{x}_1^{d*} + r_1^{d*}(\hat{x}_2^{d*} - m) - F_1(q_1^{d*}) = \frac{7(43-78m+72m^2)c^2 - 28(7-9m+9m^2)c + 4(8-6m+9m^2)}{9(7c-2)^2}$$

$$\pi_2^{d*} = p_2^{d*}(1-\hat{x}_2^{d*}) + r_2^{d*}(m-\hat{x}_1^{d*}) - F_2(q_2^{d*}) = \frac{(205-534m+522m^2)c^2 - 6(19-54m+45m^2)c + 4(4-12m+9m^2)}{9(7c-2)^2}. \quad (4)$$

From (3) and (4), we have

$$(\pi_1^{d*} + \pi_2^{d*}) - (\pi_1^{u^*} + \pi_2^{u^*}) = \frac{N_4c^4 - N_3c^3 + 12N_2c^2 + N_1c - 16N_0}{18(8c-1)^2(7c-2)^2} < 0$$

where $N_4 = 11407 - 138240m + 131328m^2$, $N_3 = 8867 - 108288m + 99648m^2$, $N_2 = 143 - 2484m + 2331m^2$, $N_1 = 88 + 3456m - 3348m^2$, and $N_0 = 2 + 9m - 9m^2$. This result shows that industry profits are always lower under BBPD than under uniform pricing. Thus, we draw the following.

Proposition 1. *Consider a quality-then-price game played by the two firms with different costs of quality improvement, and suppose that in equilibrium, each firm poaches some of its competitor's former consumers by charging them a lower price. Then, behavior-based price discrimination raises consumer surplus but reduces industry profits relative to uniform pricing.*

From Proposition 1, we can see that BBPD has distributional effects between consumers and the firms.

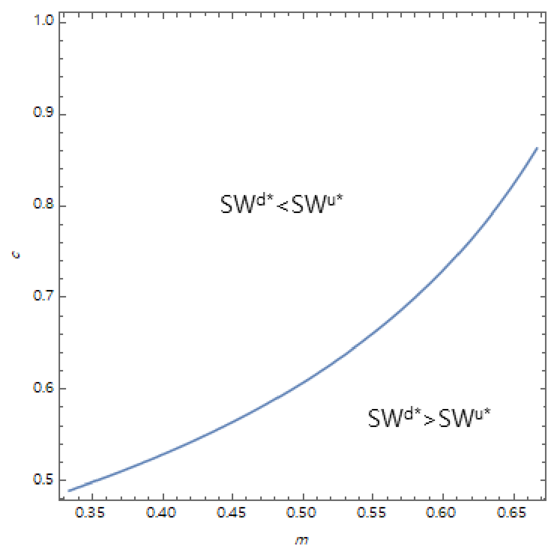
Next, to explore the welfare implications of using BBPD, we define social welfare as the sum of consumer surplus and industry profits. With (1), (2), (3), and (4), we can calculate social welfare under each pricing scheme as

$$SW^{u^*} = \frac{9(-29+256v)c^2 + (214-576v)c - 2 + 36v}{36(8c-1)^2}$$

$$SW^{d*} = \frac{(115+441v-549m+405m^2)c^2 - 6(13+42v-51m+24m^2)c + 2(7+18v-18m+9m^2)}{9(7c-2)^2}.$$

From the comparison of SW^{u^*} and SW^{d*}, we establish that (i) for

Fig. 1. Region of *c* wherein BBPD is welfare-improving



$\frac{1}{3} < m \leq \frac{1}{720}(488 - 7\sqrt{551}) \approx 0.4495$, $SW^{d*} > SW^{u*}$ if $c < c^{*}$; and (ii) for $\frac{1}{720}(488 - 7\sqrt{551}) < m < \frac{2}{3}$, $SW^{d*} > SW^{u*}$ if $c < c^{**}$, where c^{*} and c^{**} are the third and fourth roots of

$$G(z) = (42229 - 140544m + 103680m^2)z^4 - (45122 - 113472m + 62784m^2)z^3 + (16170 - 30996m + 15444m^2)z^2 - (2120 - 3528m + 1728m^2)z + (64 - 144m + 72m^2) = 0,$$

respectively. Hence, our analysis can be summarized as follows.

Proposition 2. *Consider a quality-then-price game played by the two firms with different costs of quality improvement, and suppose that in equilibrium, each firm poaches some of its competitor's former consumers by charging them a lower price. Then, social welfare is higher under behavior-based price discrimination than under uniform pricing if the firms' cost difference is sufficiently large.*

In ⟨Fig. 1⟩, we present the region of firm 2's cost parameter (c) in which BBPD increases social welfare. For example, when $m = \frac{1}{2}$, $SW^{d*} > SW^{u*}$ if $(\underline{c} = 0.5 <) c < 0.6074$.

The result follows from the fact that the benefit to consumers from BBPD increases and the industry profit loss from BBPD decreases as the firms' cost difference increases (beyond some level). Formally, for $\frac{1}{3} < m \leq \frac{1}{2}$, $\frac{\partial(CS^{d*} - CS^{u*})}{\partial c} < 0$ if

$c < c^{\dagger}$ and $\frac{\partial\left[(\pi_1^{d*}+\pi_2^{d*})-(\pi_1^{u*}+\pi_2^{u*})\right]}{\partial c} < 0$ if $c < c^{\dagger\dagger}$, where $c^{\dagger}$ and $c^{\dagger\dagger}$ are the second roots of

$$H(z) = (-35591+26624m+18432m^2)z^4 + (35634-14080m-6912m^2)z^3 - (12300-2784m-864m^2)z^2 + (1696-244m-36m^2)z - (72-8m) = 0$$

and

$$I(z) = (3130-32768m+51200m^2)z^4 - (11445-28672m+23296m^2)z^3 + (7554-7680m+3936m^2)z^2 - (1784-832m+292m^2)z + (144-32m+8m^2) = 0,$$

respectively. For $\frac{1}{2} < m < \frac{2}{3}$, it always holds. The low-cost firm has an incentive to improve the product quality more under BBPD than under uniform pricing with its cost efficiency, and this enables the firm to raise prices more under BBPD. Then, BBPD increases the market share of the low-cost firm more than under uniform pricing with the cost difference. As a result, more consumers are induced to buy the high-quality product of the low-cost firm under BBPD, and the low-cost firm's profit loss from BBPD falls.

Ⅳ. Conclusion

Considering a quality-then-price game played by the two firms with different costs of quality improvement, we investigate the welfare impacts of BBPD. We show that BBPD promotes social welfare compared to uniform pricing if the firms' cost difference is sufficiently large. This is due to that BBPD induces more consumers to buy the high-quality product than under uniform pricing, and that the low-cost firm's profit loss from BBPD decreases as the cost difference increases.

Our study suggests that a competition authority needs to consider the firms' technology of quality improvement when evaluating the effects of BBPD on social welfare.

In future research, it would be interesting to analyze BBPD in dynamic settings where information used to segment consumers is obtained by their past purchases.

References

Armstrong, M. (2006), "Recent Developments in the Economics of Price Discrimination", in Blundell, R., W. Newey and T. Persson (eds.), Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress, Vol. 2, Cambridge, UK, Cambridge University Press, 97-141.

Carroni, E. (2018), "Behaviour-Based Price Discrimination with Cross-Group Externalities", *Journal of Economics*, 125, 137-157.

Chen, Y. (1997), "Paying Customers to Switch", *Journal of Economics and Management Strategy*, 6(4), 877-897.

Chen, Y. and J. Pearcy (2010), "Dynamic Pricing: When to Entice Brand Switching and When to Reward Consumer Loyalty", *RAND Journal of Economics*, 41(4), 674-685.

Chung, H.S. (2016), "Behavior-Based Price Discrimination with Experience Goods", *Manchester School*, 84(5), 675-695.

Colombo, S. (2016), "Imperfect Behavior-Based Price Discrimination", *Journal of Economics and Management Strategy*, 25(3), 563-583.

Colombo, S. (2018), "Behavior- and Characteristic-Based Price Discrimination", Journal of Economics and Management Strategy, 27(2), 237-250.

Corts, K. (1998), "Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment", RAND Journal of Economics, 29(2), 306-323.

Esteves, R.B. (2014), "Behavior-Based Price Discrimination with Retention Offers", Information Economics and Policy, 27, 39-51.

Esteves, R.B. and C. Reggiani (2014), "Elasticity of Demand and Behaviour-Based Price Discrimination", International Journal of Industrial Organization, 32, 46-56.

Fudenberg, D. and J. Tirole (2000), "Customer Poaching and Brand Switching", RAND Journal of Economics, 31(4), 634-657.

Gehrig, T., O. Shy and R. Stenbacka (2011), "History-Based Price Discrimination and Entry in Markets with Switching Costs: A Welfare Analysis", European Economic Review, 55(5), 732-739.

Gehrig, T., O. Shy and R. Stenbacka (2012), "A Welfare Evaluation of History-Based Price Discrimination", *Journal of Industry, Competition and Trade*, 12(4), 373-393.

Ikeda, T. and T. Toshimitsu (2010), "Third-Degree Price Discrimination, Quality Choice, and Welfare", *Economics Letters*, 106(1), 54-56.

Nguyen, X. (2014), "Monopolistic Third-Degree Price Discrimination under Vertical Product Differentiation", *Economics Letters*, 125(2), 153-155.