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An Introduction to the Edumatrix Set and Its Didactic Capabilities

Mikhail Semenov (Professor)[1], Yong S. Colen (Professor)[2*],
Jung Colen (Researcher)[3], Antony Pardala (Professor)[4]

[1]Tomsk Polytechnic University, sme@tpu.ru
[2]Indiana University of Pennsylvania, yscolen@iup.edu
[3]St. Margaret's School, jcolen@smsapps.org
[4]Rzeszow University of Technology, pardala@prz.edu.pl

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Learning through "recreational mathematics" has become a meaningful outlet to children of all ages. The Edumatrix set is a didactic tool for the development of logical and abstract reasoning among students. In this paper, we provide several illustrative exercises involving Edumatrix that teachers can utilize in their classrooms. We formulate students' expected learning outcomes by aligning each exercise to the CCSSM content standards as well as examining which Standards for Mathematical Practices (SMP) our proposed exercises promote.

Keywords: Edumatrix, recreational mathematics, algorithmic thinking, logical thinking, didactic tool, play and learn.
MESC Classification: A20, U60
MSC2010 Classification: 00A35, 97A20, 97U60

I. INTRODUCTION

Perhaps no one has popularized "recreational mathematics" more than Martin Gardner. As the columnist of "Mathematical Games" in *Scientific American* from 1957 and for three decades, his estimated 300 columns inspired "developments with real impact on science, technology and society" (Mulcahy, 2014). Some of the more popular articles comprise: "Flexagons, in which strips of paper are used to make hexagonal figures with unusual properties" (Gardner, 1956); "More about complex dominoes" (Gardner, 1957); "Extraordinary nonperiodic tiling that enriches the theory of tiles" (Gardner, 1977).

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* Corresponding Author: yscolen@iup.edu

Learning through "recreational mathematics" has become a meaningful outlet to children of all ages (Olson, 2007). To supplement the traditional school mathematics curriculum, educators need to consider the available and yet grade-appropriate resources, such as interactive tactile games, mobile applications, gaming technologies, programming environments, and the Internet. Decisions regarding learning pedagogies, classroom assessments, and the optimal balance between educational and "playing" components are bound to occur and reoccur for educators. Moreover, they must acquire the professional competencies to assess when it is beneficial to implement "recreational mathematics" that may or may not be aligned to the school curriculum.

Through the years, the popular card and board games like SET, Rush Hour, and Meta-Forms have engaged children to expand their logical and problem-solving skills through "playing." In creative programming environments (CPEs) that foster algorithmic thinking, development of programming knowledge, and actively cultivating as well as using mathematical reasoning, programs such as Alice, Kumir, Logo, Scratch, and Snap! have become popular. (An extensive list of the available CPEs can be found at http://juniorcodeacademy.eu/resources/.)

In this article, we introduce a popular Polish game, Edumatrix. While the Information and Communication Technologies (ICTs) have advanced worldwide and in turn are reflected in school curriculum, Edumatrix is more aligned with SET[1], Rush Hour[2], and Meta-Forms[3] in that the game does not require the use of computers nor programs. Implementing Edumatrix is most appropriate at the preschool and elementary school levels. Yet, as the below exercises ("The Reverse Polish Notation," "Binary Numbers," and "Area Calculation") show, Edumatrix can be incorporated into middle and high school mathematical lessons. In this paper, we will examine that the systematic use of Edumatrix permits students to articulate their original work and to develop their mathematical habits of mind. The purpose of the article is to demonstrate several Edumatrix exercises and their associated didactic capabilities, to formulate the expected learning outcomes, and to discuss the use of Edumatrix in developing students' mathematical content knowledge as well as the practice proficiencies advocated by the Common Core State Standards for Mathematics (CCSSM; NGA & CCSSO, 2010). Finally, a brief discussion on plausible future research steps follows.

[1] https://www.nytimes.com/puzzles/set
[2] https://www.crazygames.com/game/rush-hour-online
[3] http://www.foxmind.com/games/1257-meta-forms

II. LITERATURE REVIEW

The Edumatrix set is made of natural materials and can be easily found in markets throughout Poland and Russia (Pollak et al., 2016). Edumatrix is a didactic tool for the development of logical and abstract thinking among students (Wijata et al., 2016). As Ludorowska (2018) argues, Edumatrix can provide illustrative tasks on symmetry, statistical reasoning, logical puzzles, algorithmic thinking, and creative expressions. In their study, Pollak and her colleagues (2016) describe the educational experiments and methods for assessing the effectiveness in using the Edumatrix set in the development of computational skills. Specifically, the experimental group, consisting of seven- to nine-year-old children, utilized Edumatrix daily (15-30 minutes) for a six-week duration, while the control group solved the same mathematical problems employing the traditional methods. The researchers conclude that in addition to improvements in the quantitative characteristics, like the speed and accuracy in solving the problems, the qualitative factor, such as students' overall satisfactory response in using Edumatrix, enhanced as well. Similarly, Semenov and his colleagues (2016) confirm that students' academic performance increased noticeably due to the use of the projects' method.

For students, the transition process from empirical activities to logical and abstract reasoning is complex, yet necessary to broaden their intellectual development (Simon & Tzur, 2004). However, as most educators can attest, this process is not simple. Much work entails careful considerations of the subsequent factors: selection of cognitively challenging tasks and appropriate tools, students' prior knowledge relating to the tasks, and the degree to which the tasks supplement the curriculum and enrich the learning experience.

Welcoming this challenge, educators explore innovative approaches. In fact, the teaching and learning culture implores the teachers, researchers, learners, administrators, policymakers, and publishers to continue to think outside the prescribed "box"—i.e., the broadly accepted norms. For example, Ludorowska (2018) advocates the benefits of illustrative examples that include simple tasks involving guided action statements and complex logical puzzles necessitating cyclic algorithms. Carroll and Porter (1998) promote student-generated algorithms and empowering students to incorporate their prior experiences.

An analysis of the above literature review reveals that much focus has been devoted to the pedagogical methods in the development of professional competencies needed to support students' use of the ICTs. Yet, there lacks any meaningful examination of the issues of teaching and learning of the basic concepts of programming through the Edumatrix set at the preschool and elementary school levels. In the era when educators are competing for students' time, focus, and priorities and when information technologies have a significant impact on socio-economic development, a pressing problem arises: when and how to introduce students to programming and continue to foster their problem solving and logical reasoning abilities. The subsequent sections of this study investigate the didactic capabilities.

III. DIDACTIC CAPABILITIES AND EXPECTED LEARNING OUTCOMES

Considering a pedagogical justification in using the Edumatrix set, we advocate Vygotsky's social learning theory (Vygotsky, 1991). Within the subject-professional and social contexts, the teacher serves as the specialist. Applying the social development model provides ample opportunities for collaborative work in small group settings, and the participants include the students and teacher—and ideally, parents. Kim and his colleagues (2012) observe that students learn mathematics effectively when the teacher guides lessons so that students have occasions to interact with each other. Moreover, Light (2001) notes that students, who learned in collaborative group settings, showed significantly higher academic performances than those who learned individually.

When facilitating the Edumatrix exercises, the teacher's primary focus is to plan and organize interactive moments within groups as well as to establish a productive classroom atmosphere. Specifically, the teacher serves in the following roles: encourager, coordinator, counselor, and evaluator. Throughout this envisioned facilitation, the teacher is mindful to group students into mixed ability grouping to maximize peer-to-peer teaching and learning moments. Intricacies of interactions are new experiences for students due primarily to playing and social connections.

1. ILLUSTRATIVE EXERCISES AND DIDACTIC CAPABILITIES

In this section, we provide several illustrative exercises involving the Edumatrix set. To solve some exercises, basic and advanced prerequisite mathematical knowledge is expected. In other exercises, students can reinforce their prior mathematical understanding, and in some instances, they will investigate new mathematical concepts and algorithmic reasoning.

Before we examine the exercises, let us inspect the below Edumatrix boards and their arrangements (Figure 1). Readers can attest that the left board is similar to the Microsoft Excel spreadsheet in that we see the column designations with the capital English letters, A, B, C, ..., J, and the row designations with the Hindu-Arabic numbers, 1, 2, 3, …, 10. In a spreadsheet or a matrix arrangement with the 10 by 10 dimensions, one could interpret the left board to represent a flower. In short, some Edumatrix blocks have bright colors. On the other hand, as the below right board indicates, the grid resembles the first quadrant of the Cartesian Coordinate System. Along with some colored blocks, the right board contains various symbols and numbers.

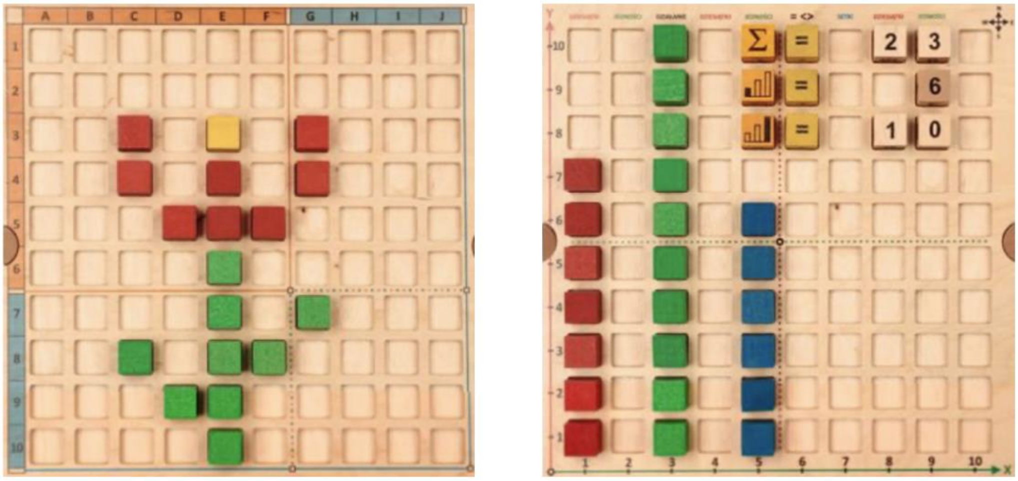

Figure 1. Sample Edumatrix boards

As the above images demonstrate, the Edumatrix blocks are movable, tangible objects. The Edumatrix set supports preschool and elementary school children in particular to brainstorm and express their reasoning easily when compared to drawing or notating on a piece of paper or even imitating the boards in a virtual environment. Afterall, there is nothing like playing chess against a real person with real game pieces.

<u>Exercise 1: Kasia Plants Flower Bulbs</u>

Ludorowska (2018, p. 19) shares the following exercise:

> *On Monday, Kasia planted 3 red tulip bulbs. On Tuesday, she did not plant anything. On Wednesday, she planted 5 daffodil bulbs. On Thursday, she did not plant anything. Finally, on Friday, she planted 2 green tulip bulbs. (1) How many flower bulbs did Kasia plant during the whole week? (2) On which day of the week did Kasia plant the fewest number of bulbs? (3) On which day of the week did Kasia plant the most bulbs? (4) How many more bulbs did Kasia plant on Wednesday than on Monday? (5) How many fewer bulbs did Kasia plant on Friday than on Monday?*

To answer the above questions, elementary school children will benefit by "visualizing" Kasia's planting throughout the week. With guidance, children can create the below Edumatrix board (Figure 2):

Figure 2. Data representing Kasia's planting flower bulbs

From this particular student's work, we can make the following observations:

On the x-axis, 1 represents Monday, 2 represents Tuesday, 3 represents Wednesday, 4 represents Thursday, and 5 represents Friday. We see the matched colored blocks to represent the planted tulip and daffodil bulbs. The upper right hand portion of the board provides the subsequent statistical information: the total number of bulbs planted during the whole week (indicated by the Greek capital letter sigma, Σ) equals 10; the minimum number of bulbs planted on any given day (excluding the no planting days, Tuesday and Thursday) equals 2; and the maximum number of bulbs planted on any given day equals 5.

Having organized the given information in this manner, the student can answer the posed questions more informatively:

(1) How many flower bulbs did Kasia plant during the whole week? **10**

(2) On which day of the week did Kasia plant the fewest number of bulbs? **Friday**

(3) On which day of the week did Kasia plant the most bulbs? **Wednesday**

(4) How many more bulbs did Kasia plant on Wednesday than on Monday? 5 - 3 = **2**

(5) How many fewer bulbs did Kasia plant on Friday than on Monday? 3 - 2 = **1**

This exercise helps young students to understand the meaning of a situation by representing the data as a bar graph. With this formulated board, students find the answers to the posed questions. In short, students make sense of the quantities in the given context due to the utility of the Edumatrix set.

<u>Exercise 2: The Reverse Polish Notation</u>

We are familiar with the meaning behind an arithmetic expression 2+3. We place the operator (+) between the operands (or terms) (2 and 3), and we state that 2+3 is in the "infix notation." In comparison, the Polish Notation (PN), or sometimes referred to as the "prefix notation," is when the placement of the operator is before the operands. In other words, the above expression, 2+3, is written as +23 in PN. Lastly, in the Reverse Polish Notation (RPN), or sometimes referred to as the "postfix notation," the placement of the operator follows the operands. In the RPN, we write the original expression as 23+.

Using the RPN, rewrite the expression, A+B×C. Without looking at the next board, take a moment to solve this task in earnest. As the blocks in row 2 of Figure 3 reveal, the operands are sequenced from left to right, ABC, and the operators are written from right to left, ×+. Putting these two conventions together, we conclude that ABC×+ in RPN represents the original expression, A+B×C. Could another student also express A+B×C as AB+C×? In short, one could make a strong case that multiple expressions in the RPN are equivalent.

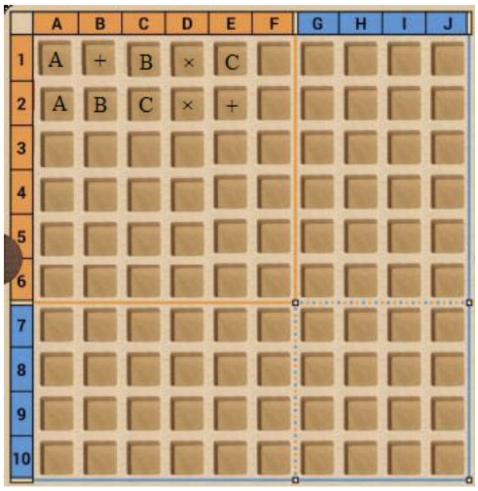

Figure 3. The infix notation versus the RPN

Next, attempt to use the Edumatrix blocks to express the below expressions in RPN:

(a) C×B+A (b) A×B+C×D (c) (A+B)/(C-D)

Solutions: (a) CB×A+ (b) AB×CD×+ (c) AB+CD-/

As one can see from the final case (c), the Reverse Polish Notation permits us to omit the grouping symbols (parentheses)—hence, uses four fewer blocks. Finally, test your partner by providing an expression in the infix notation and ask her to provide an equivalent RPN. Note that formulating thought-provoking questions can be just as challenging as solving them.

Through this exercise, students use their prior knowledge of the order of operations to minimize the number of blocks. Teachers should "be aware of learners' prior knowledge about particular topics and how that knowledge is organized and structured" while observing their arrangements of the Edumatrix blocks (Borko & Putnam, 1995, p. 42). As researchers have shown, "learning is enhanced when teachers pay attention to the knowledge and beliefs that learners bring to a learning task, use this knowledge as a starting point for new instruction, and monitor students' changing conceptions as instruction proceeds" (Bransford et al. 1999, p. 11).

Exercise 3: The Braille Alphabets

Learning languages can benefit one's professional opportunities, life's outlook, and appreciation of distinctive cultures. Learning a sign language or how to read Braille leads to an enhanced empathy toward those who have auditory, speech, or vision impediments. This exercise explores how to communicate using the Braille alphabet. Students can formulate words using the Edumatrix blocks and challenge each other to decipher the words or even messages that they have created.

First, let us examine the Braille alphabet below as shown in Figure 4. Each letter can be formed by placing raised "dots" within the three rows by two columns matrix block. For example, we can create the letter "r" by placing the Edumatrix blocks into A1, A2, A3, and B2.

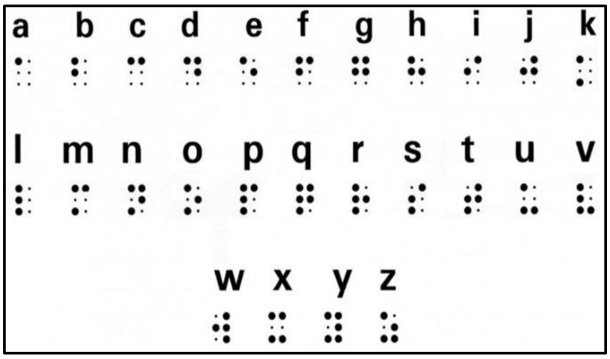

Figure 4. The Braille alphabets

Before creating words, students might benefit more by examining the actual Braille texts. For example, Figure 5 below denotes "Spring!" in the context of a poem and its Edumatrix representation.

First, we notice that there are six, three rows by two columns matrix blocks. Comparing the text to the Braille alphabets, we can match up the second, third, and fourth matrix blocks as the letters, s, p, r. So, what does the first matrix block indicate? After some search on the Internet or maybe some time to ponder about the differences among the letters, s, p, r, one can conclude that the first matrix block, represented by B3, indicates that a capital letter will follow—hence, combing the first two matrix blocks, (B3) and (A2, A3, B1), we get the capital S.

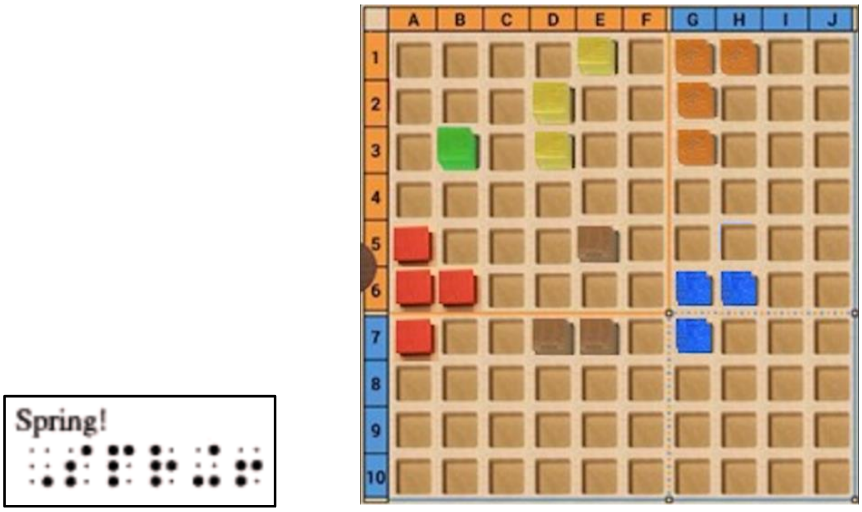

Figure 5. The Braille word, "Spring!" and its Edumatrix representation

Now, how do we match up the remaining letters, i, n, g, and the exclamation symbol (!), with the two unknown matrix blocks? As expected, Braille does not "spell out" words using every letter. Since "ing" is so often used, the fifth matrix block, (A3, B1, B3), represents the three remaining letters. Moreover, the final matrix block, (A2, A3, B2), represents the exclamation symbol, !. Finally, it would be exceptionally gratifying for students to actually close their eyes and read some of their words or some of the words created by their classmates and be able to relate more with the people of the Braille world.

Some may argue the usefulness of this exercise in developing students' mathematical skills and concepts. Contrarily, this exercise can enhance students' generalization skill by representing the Braille words and analyzing the patterns by using Edumatrix blocks. Many teachers use patterns to promote generalization in a pre-algebraic context (e.g. Mason, 1996).

Exercise 4: Binary Numbers

With the advent of computers and programming in the 20th century, scientists have utilized binary numbers. For prospective elementary school teachers, examining numbers in different bases (for example, base 2, base 20, and base 60) provides a greater understanding and much appreciation of the base-10 (decimal), place-value system.

As a convention, we could let a block in a cell to denote "1" and an empty cell to denote "0." For example, if we place the three blocks into E1, F1, and J1 (see Figure 6), the corresponding binary number is 110001_2. Additionally, we can convert this value into the equivalent base-10 place-value number by carrying out the following computations:

$$110001_2 = 1\times2^5 + 1\times2^4 + 0\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0$$

$= 32 + 16 + 0 + 0 + 0 + 1$

$= 49$

In short, 110001_2 is equivalent to 49.

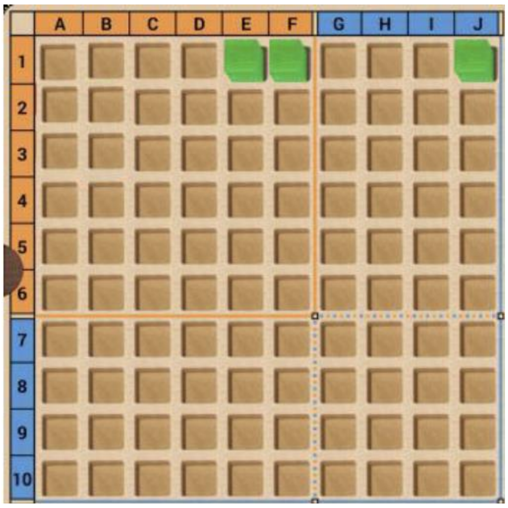

Figure 6. Blocks representing the binary number 110001_2

Using the Edumatrix blocks, try the following exercises:

- Represent 1011100_2. Did you start at D1 with a block? Why starting at C1 with a block can be confusing?
- Formulate 1011100_2 on the board and ask your partner to find the equivalent base-10, place-value number.
- Given 987, determine, with your partner, the equivalent binary number and display this on the board.
- Using only one row, what is the largest binary number can you form? What is its equivalent base-10 place-value number?

This exercise helps students to compare and contrast between the decimal system and the binary system. By converting binary numbers into decimals and vice versa, students learn how to represent base-10 numbers differently. Furthermore, teachers can expand upon this activity by guiding students to create simple programs that uses binary counting.

<u>Exercise 5: Simple Programming</u>

The Edumatrix set includes blocks to create a set of instructions for programming. For instance, compare and contrast the below two boards (Figure 7). Can you make sense of the instructional steps found on the left board? Of course, it would be helpful if you compare them to the output found on the right board. It appears the unique block in A1 indicates the command, "Starting cell." In short, we can interpret row 1 instruction as "Starting cell equals A2." Row 2 provides the following instructional steps:

- place a red block in A2,
- move one space right and place a green block,
- move one space down and place a blue block, and
- move one space left and place a yellow block.

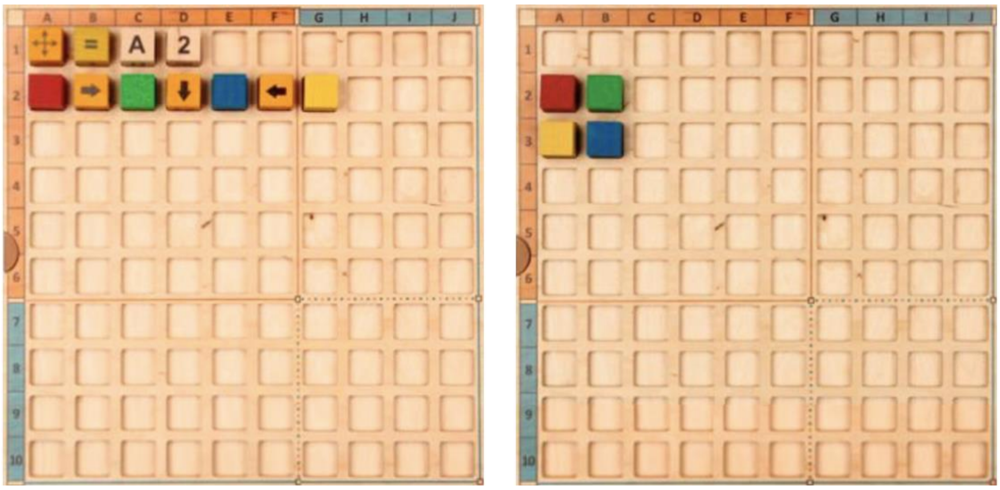

Figure 7. Programming instructions and the output

Having carried out all instructional steps, students should have created the output shown on the right board in Figure 7.

In related exercises, educators could explore a wealth of possible tasks: (1) given a board arrangement, students, in small groups, formulate the programming steps; (2) conversely, given a set of programming steps, students articulate the output. In particular, the first task could generate students' varying perspectives and their distinctive programming steps.

<u>Exercise 6: Area Calculation</u>

This exercise involves the task: Given three randomly placed noncollinear blocks that represent three vertices of a triangle, determine its area. Let us say that three green blocks were randomly placed on the Edumatrix board (see Figure 8). Considering the blocks' absolute references (A1, B4, and D3), we can transform them into their relative references as vertices of the triangle in a modified Cartesian Coordinate System. In other words, A1, B4, and D3 blocks correspond to the coordinates (1, 1), (2, 4) and (4, 3), respectively.

Next, applying the Gaussian Area Formula (or the Shoelace Algorithm), we get:

$$\begin{aligned} \text{Area} &= 0.5|(a_1 \cdot b_2 + a_2 \cdot b_3 + a_3 \cdot b_1) - (b_1 \cdot a_2 + b_2 \cdot a_3 + b_3 \cdot a_1)| \\ &= 0.5|(1 \cdot 4 + 2 \cdot 3 + 4 \cdot 1) - (1 \cdot 2 + 4 \cdot 4 + 3 \cdot 1)| \end{aligned}$$

$= 0.5|(4 + 6 + 4) - (2 + 16 + 3)|$
$= 0.5|(14) - (21)|$
$= 0.5|7|$
$= 3.5$

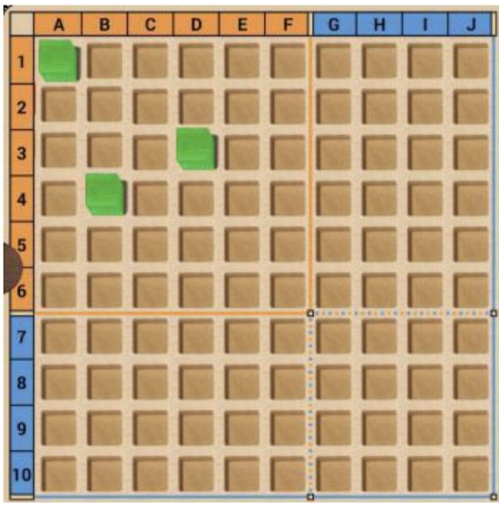

Figure 8. Random blocks (vertices) and area calculation

We should note two aspects about this exercise: (1) The polygons are not restricted to triangles. Students can calculate the areas of quadrilaterals, pentagons, hexagons, etc. (2) Mathematically, this exercise is quite challenging. While proving the formula is beyond the scope of a typical high school curriculum, applying the theorem has positive benefits. Similarly, the Heron's Formula to calculate the area of a triangle based on the side lengths is quite ingenious even though the proof is beyond the school mathematics.

2. EXPECTED LEARNING OUTCOMES

In this section, we discuss students' expected learning outcomes. First, we align each exercise to the CCSSM content standards that define the domains grade-level students should understand and be able to do. Next, we examine which Standards for Mathematical Practices (SMP) the proposed exercises promote.

Several illustrative exercises align well with the mathematical concepts advocated by the Common Core State Standards for Mathematics:

- Exercise 1: Kasia Plants Flower Bulbs
  CCSS.MATH.CONTENT.1.OA.A.1

Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
CCSS.MATH.CONTENT.3.MD.B.3
Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets*.

- Exercise 2: The Reverse Polish Notation
  CCSS.MATH.CONTENT.5.OA.A.2
  Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.
  CCSS.MATH.CONTENT.6.EE.A.2.C
  Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
- Exercise 3: The Braille Alphabets
  CCSS.MATH.CONTENT.4.OA.C.5
  Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.
- Exercise 4: Binary Numbers
  CCSS.MATH.CONTENT.4.OA.C.5
  Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.
- Exercise 5: Simple Programming
  CCSS.MATH.CONTENT.4.OA.C.5
  Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.
- Exercise 6: Area Calculation
  CCSS.MATH.CONTENT.5.G.A.1
  Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis,

with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

CCSS.MATH.CONTENT.6.EE.A.2.C

Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.*

Utilizing the Edumatrix set, a teacher could plan lessons relating to the proposed exercises. The expected learning outcomes associated with the SMP are:

- to make sense of problems (Exercises 1, 2, 3, 4, 5, 6);
- to reason abstractly and quantitatively (Exercises 1, 4, 5, 6);
- to construct viable arguments and critique the reasoning of others (Exercises 2, 3, 4, 5, 6);
- to model with mathematics (Exercises 1, 5, 6);
- to attend to precision (Exercises 1, 2, 3, 4, 5, 6);
- to look for and make use of structure (Exercises 2, 4, 6);
- to look for and express regularity in repeated reasoning (Exercises 2, 4, 5, 6).

Most importantly, in the exercises, the teacher will have introduced and students will have used the appropriate tool (Edumatrix) strategically.

IV. DISCUSSION

Implementing Edumatrix, teachers could envision limitless didactic capabilities and learning outcomes. As we stated at the onset and demonstrated through the exercises, students reinforced their prior mathematical understanding and investigated new mathematical concepts and algorithmic reasoning. Moreover, as expounded within this paper, educators should value: (1) students to make sense of the quantities in the given context and if needed, to utilize appropriate tools; (2) students to formulate thought-provoking questions which can be just as challenging as solving them; (3) students to generalize based on the given parameters; (4) students to ponder deeply about the complex and yet elegant decimal system by examining the other number systems, such as the binary system, the Mayan base-20 system, and the Babylonian base-60 system; (5) students to articulate and have ownership of their newly invented programming languages; (6) students to visualize rich mathematical concepts. In particular, as “Kasia’s Planting Flower Bulbs” and "Simple Programming" exercises demonstrate, we believe the greatest promise lies in students' expressing their original thoughts in collaborative settings.

Educators could explore future research involving the use of the Edumatrix set within a specific content domain. For example, the CCSSM (NGA & CCSSO, 2010) promotes the following sixth-grade, statistics and probability standard:

> *Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (CCSS.MATH.CONTENT.6.SP.A.2)*

Research studies, such as the proposed one, can validate the prior studies and provide the explicit benefits and any shortcomings in the use of Edumatrix.

In an era that has witnessed a proliferation of "screenagers"—children who spend hours each day texting, twitting, facetiming, browsing, shopping in virtual settings—it would be beneficial for younger children to reclaim the benefits of seeing, holding, and manipulating tactile objects, such as the Edumatrix cubes on the wooden boards. While there is much evidence to support that technology can enhance students' reasoning in mathematics, we should reaffirm that much can be learned with paper and pencils, blackboard and chalks, and compass and straightedge. The authors advocate adding the Edumatrix set to the venerable list.

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