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Developing an Instrument of Assessing the Middle School Students' Perceptions of Mathematics Teachers' PCK

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Many researches proposed different models and concepts for the PCK. It is important to understand its composition. Most studies investigated the development of PCK and its influence on students' learning from the teachers' perspectives. We developed an instrument for assessing middle school students' perceptions of mathematics teachers' PCK (SPOMTPCK) to investigate the nature of PCK. Theoretical claims and empirical research in PCK were used to design questions and sub-scales for the SPOMTPCK. The face validity of the instrument was established by the expert mathematics teachers and students. A questionnaire consisting of 38 items on a five-point Likert-type scale was used for data collection from 799 middle school students. The exploratory factor analyses resulted in the development of a three-factor scale of 17 items that was proved valid and reliable, that is, pedagogical representation, understanding students and curriculum, and encouraging students' engagement. The Cronbach α coefficients of the scale was 0.935, and the Cronbach α coefficient of three factors were ranged from 0.721 to 0.912. The confirmatory factor analysis showed that the questionnaire has good construct validity and the fit indexes are good. MANOVA analysis of variance revealed that the differences in mathematics teachers' PCK identified by students of different school types and grades were statistically significant. It is a validate measurement to evaluate the perceived mathematics teachers' PCK for middle school students.

Keywords: mathematics teacher, middle school student, students' perceptions, PCK, scale development.
MESC Classification: B50, B59
MSC2010 Classification: 97B50, 97D20, 97D60

I. INTRODUCTION

Pedagogical content knowledge (PCK) was first proposed by Shulman in 1986. PCK proposed by Shulman (1986) is a unique contribution to teacher education research, because PCK solves how to teach content and how to understand students' thinking. It guides people to pay attention to the two most important factors (knowledge of subjects and knowledge of pedagogy) of teacher education, which are separated from each other for a long time. Shulman (1987) points out that PCK is a knowledge that transcends the knowledge of the subject itself. PCK involves the dimension of subject knowledge in teaching, It is a mixture of subject content and teaching. PCK proposed by Shulman gives new vitality to research of teacher knowledge. PCK provides a more complete perspective of studying teacher knowledge and classroom practice. It is a special knowledge of teachers, and It is essential to understand what knowledge the effective teacher needs. Teachers need PCK to organize content in the classroom, develop the fully understood representations of the topic to be taught, and understand the difficulties that students may encounter when learning specific topics. It is the particular form of content knowledge that embodies the aspects of content most germane to its teachability (Shulman, 1986).

PCK since its introduction in 1986 has permeated the scholarship that deals with teacher education and the subject matter of education (see, for example, Cochran, King, & DeRuiter, 1991; Grossman ,1990; Shulman,1987; Wilson, Shulman & Richert, 1987). But PCK is a vague and complicated concept. It is difficult to clearly define the exact meaning of PCK. It's hard to see PCK as an independent concept, and a great deal of explanation is needed. It is quite vague even for Shulman himself about the interpretation of PCK. PCK is closely related to subject knowledge. In the seven knowledge bases of teachers described by Shulman (1986), Subject knowledge is independent of PCK. But Gudmundsdottir & Shulman (1987) suggest PCK should include subject knowledge, general pedagogical knowledge and knowledge of learners. Wilson, Shulman & Richert (1987) subdivide PCK into six categories, that is, knowledge of teaching representation, knowledge of teaching reasoning, knowledge related to learners, curriculum knowledge, knowledge of pedagogy, and knowledge of teaching situations, which don't include subject knowledge.

Different researchers had different definitions and components of PCK. "What are the components of PCK" was one of the fundamental questions that researchers tried to figure out. Although PCK is theoretically an integrated and whole concept, the ingredients of which cannot be separated, it is of practical significance of clarifying its components. A large volume of studies have been conducted by using the key elements of PCK. However, the components of PCK in these studies often varied from one to another. The subsequent researchers had further expanded the components and their sub-components of PCK. Some clarification of components of PCK was reviewed in the following section.

II. LITERATURE REVIEW

1. THE COMPONENTS OF PCK

In Shulman's 1986 article, a general description of PCK components was made as follows: the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations-in a word, the ways of representing and formulating the subject that make it comprehensible to others. Gudmundsdottir and Shulman (1987) divided PCK into three categories, which were (1) knowledge of the central topics, concepts, and areas of the subject matter that can be and are taught to students and knowledge of analogies, similes, examples and metaphors by which to explain the subject matter to students, which is influenced by content knowledge, (2) knowledge of the different ways topics can be taught, and the pros and cons of each approach, which is influenced by general pedagogical knowledge, and (3) knowledge of students' preconceptions or misconceptions about the topics they learn, and knowledge of the topics students find interesting, difficult or easy to learn, which is influenced by knowledge of students. Elaborating on Shulman's work, other scholars have adopted the two key elements of PCK (i.e., knowledge of comprehensible representations of subject matter and understanding of content-related learning difficulties). Moreover, each of them has extended the concept by including in PCK some of the categories of knowledge distinct in Shulman's knowledge base for teaching. For example, Tamir (1988) considered that the elements of PCK include: (1)orientation to teaching, (2) knowledge about students' understandings, (3) curriculum knowledge, (4) knowledge of assessment, (5) knowledge of teaching strategies. Smith and Neale (1989) described PCK as having three components: knowledge of typical student errors, knowledge of particular teaching strategies, and knowledge of content elaboration.

Marks (1990) considered that pedagogical content knowledge consist of four components: subject matter for instructional purposes, students' understanding of the subject matter, media for instruction in the subject matter, and instructional processes for the subject matter. Grossman (1990) thinks that PCK includes four central components: (1) conception of teaching purposes, (2) knowledge of students' understanding and (mis)conceptions, (3) curricular knowledget, (4) knowledge of instructional strategies and representations for teaching particular topics. Cochran, King and DeRuiter (1991) devided PCK into four components: knowledge of subject matter, knowledge of students, knowledge of environmental contexts, and knowledge of pedagogy. Reynold (1992) believed PCK consists of teaching methods, content organization, knowledge of students' content learning, content representation, and assessment knowledge. Fernandez-Balboa (1995) believed that five generic PCK components emerged: knowledge about (1) the subject matter, (2) the students, (3) numerous instructional strategies, (4) the teaching context, and (5) one's teaching purposes. Tuan (1996) claimed that pedagogical content knowledge integrated seven domains of knowledge: pedagogical knowledge, representational knowledge, subject matter knowledge, curriculum knowledge, assessment knowledge, student knowledge, and context and social knowledge. Geddis and Wood (1997) considered PCK included the learner's prior concepts, subject matter representations, instructional strategies, curriculum materials, and curricular saliency. Morine-Dershimer and Kent (1999) presented a model that showed the PCK consists of six knowledge: (1) knowledge of the purposes and educational objectives linked directly to knowledge of assessment procedures, (2) pedagogical knowledge, (3) curriculum knowledge, (4) content knowledge, (5) knowledge of specific contexts, and (6) knowledge of learners and learning. Magnusson, Krajcik, and Borko (1999) conceptualized pedagogical content knowledge for science teaching as consisting of five components: (1) orientations toward science teaching, (2) knowledge and beliefs about science curriculum, (3) knowledge and beliefs about students' understanding of specific science topics, (4) knowledge and beliefs about assessment in science, and (5) knowledge and beliefs about instructional strategies for teaching science. Carlsen (1999) emphasized the importance of understanding students' misconceptions, and proposed that understanding students' misconceptions should be regarded as a component of PCK.

Veal and Makinster (2000) considered that the elements of PCK included: (1) subject matter knowledge, (2) knowledge about students, (3) knowledge of contexts, (4)knowledge of environment, (5) knowledge about the nature of discipline, (6) knowledge of classroom management, (7) knowledge of society and culture, (8) knowledge of evaluation, (9) knowledge of pedagogy, (10) curriculum knowledge. An et al. (2004) believed that PCK included content knowledge, curriculum knowledge and knowledge of pedagogy, and emphasized the connection among the three kinds of knowledge. Lee and Luft (2008) thought PCK included: knowledge of science, knowledge of goals, knowledge of students, knowledge of teaching, knowledge of curriculum organization, knowledge of assessment, and knowledge of resources. Rollnick, Bennett, Rhemtula, Dharsey, and Ndlovu (2008) stated subject matter knowledge should be put into PCK, together with other knowledge (students, pedagogy, contexts) to form teachers' PCK, and PCK's specific forms include representation, evaluation, teaching strategies and curriculum characteristics. Ball, Thames, and Phelps (2008) believed PCK mainly included: (1) knowledge of content and students (KCS), (2) knowledge of content and teaching (KCT), (3) knowledge of content and curriculum. Along with the working definition of PCK, Park & Oliver (2008) identified five components of PCK for science teaching: (1) orientations to science teaching, (2) knowledge of students' understanding in science, (3) knowledge of science curriculum, (4) knowledge of instructional strategies and representations for teaching science, and (5) knowledge of assessments of science learning. On the compositions of PCK in mathematics, Wong & Xu (2009) argued that PCK consisted of knowledge of mathematical subjects (MK), pedagogical knowledge (PK) and content knowledge. Jong (2009) believed PCK's most basic constituents were (1) knowledge of students' conceptions of specifific topics including knowledge of students' diffificulties in understanding these topics, (2) knowledge of instructional strategies including knowledge of representations (e.g., models, metaphors) and activities (e.g., explications, experiments) for teaching specifific topics, and (3) knowledge of subject matter.

Tong (2010) pointed out PCK was composed of two core and five components, that is, (1) knowledge of specific mathematical content and teaching, including: (a) How do we organize and present mathematical content in a meaningful order? (b) In order to promote students' understanding, which form of expression should we provide (interpretation, graphics, symbols, situations and operations, etc.)? (2) knowledge of specific mathematical content and students, including: (a) What difficulties will students encounter in learning specific mathematics content? (b) What views or misunderstandings do students have? (c) How do teachers use the teaching methods to deal with students' difficulties and misunderstandings? Bukova-Güzel (2010) developed a comprehensive framework of PCK consisting of three components: (1) knowledge of teaching strategies and multiple representations, (2) knowledge of learner, (3) knowledge of curriculum. Li & Ning (2011) believed that PCK consisted of mathematics knowledge (MK), pedagogical knowledge (PK), content knowledge (CK) and technology knowledge (TK).

As it is clear from the above-mentioned studies, researchers have defined the components of PCK differently. We also see that there is no universally accepted the components of PCK. The differences among the scholars occurred with respect to the components they integrate in PCK, and to specific labels or descriptions of these components. However, most scholars agreed on Shulman's (1986) two key components of PCK : (1) knowledge of instructional strategies incorporating representations of subject matter and responses to specific learning difficulties and (2) student conceptions with respect to that subject matter. We have commented on the composition of PCK by different scholars. It is very difficult to determine the specific components of PCK in practice. As Loughran, Gunstone, Berry, Milroy and Mulhall (2000) pointed out, we contended that to "see" PCK is not to see PCK at all but rather to see a mixture of interacting elements which, when combined, help to give insights into PCK. However, as the mixture of elements invariably vary, PCK itself is variable in that the different mixtures of elements influence the richness of the PCK and changes in any of the elements inevitably influence the nature of the PCK that is being portrayed. Rather, the boundaries that exist between domains are "fuzzy" (Marks, 1990). Few studies can really grasp the true meaning of PCK and the specific components it contain. It is very difficult and time-consuming to measure teachers' PCK. However, Since PCK is considered one of the cornerstones of teacher knowledge, it is important to understand its composition. If it is possible to describe and model its formation, it should be the target of improvement in teacher education.

2. STUDENTS' PERCEPTIONS OF TEACHERS' PCK

PCK is a special knowledge of teachers. Most studies investigated the development of PCK and its influence on students' learning from the teachers' perspectives. Only a limited number of studies have investigated the components of teachers' PCK from the perspective of students (Halim, Abdullah, & Meerah, 2014). Knight and Waxman (1991) advocated the importance of investigating students' perceptions of teachers' knowledge. Tuan et al. (2000) proposed students' perceptions of teachers' knowledge may provide rich information about students' cognition and classroom processes. Jang (2009) also pointed out students' perceptions was the important factors of developing science teachers' PCK. Using students' perceptions will enable teachers to appreciate the perceived instructional influences on students' learning processes (Senocak, 2009). It can also help teachers improve their teaching based on the students' perceptions. Students' views on what are needed of their teachers in promoting science learning could also provide information on the quality of their teachers' PCK. From the students' perspective, a good teacher knows the subject matter knowledge well, explains things clearly, makes the subject interesting, gives regular feedback, and gives extra help to students (Olson and Moore, 1984). One can understand effective PCK from students' perspectives which in turn can also help teachers to develop and refine their PCK for students' learning (Halim et al., 2014).

Students' perceptions are one of the sources that can be used to capture teachers' PCK (Jang, Tsai, and Chen 2013; Tuan et al. 2000). Students' perceptions of teachers' PCK have great importance in determining inadequate points of teachers' PCK. If these points can be detected, this information may be used about the PCK components that need to be helped for development. It may help to develop a theoretical perspective on the PCK development which is still very important for education. Halim et al. (2014) argued that it was important to draw from the students' perspectives of what constitutes of an effective PCK. It is highly important to develop a scale that determines students' perceptions of the types of knowledge teachers have, since it will allow teachers to organize their teaching according to students' views. It also helps determine what needs to be done for students' expectations and teachers' needs to coincide with each other. Knight and Waxman (1991) identified three areas of students' perceptions of classroom processes, namely, specific strategy of instruction, generic teachers' behaviour and the classroom learning environment. Tuan et al. (2000) designed an instrument to determine students' perceptions of teachers' various kinds of knowledge which could be performed in classroom teaching, and found that only four distinct aspects of teachers' knowledge were identified as instruction, representation, subject-matter knowledge, and knowledge of students' understanding. Bukova-Güzel E et al. (2013) developed a scale to determine pre-service mathematics teachers' perceptions related to their pedagogical content knowledge, and found that a five-factor scale composed of 17 items was developed. The factors of the scale are as follows: a) Knowledge of teaching strategies, b) Knowledge of mathematical language and symbols, c) Knowledge of misconceptions, d) Knowledge of learners, and e) Knowledge of curriculum. Uner & Akkus (2019) developed a scale to determine secondary students' perceptions of their teachers' PCK. The factors of the scale contained perceptions about knowledge of students, knowledge of curriculum, knowledge of instructional strategies, and knowledge of assessment.

However, no studies on PCK appear to have examined the middle school students' perceptions of mathematics teachers' knowledge. The main purpose of this study is to develop an instrument as a tool that assesses middle school students' perceptions of mathematics teachers' PCK. In doing so, we first re-examined the construct of PCK based on the above-mentioned studies, designed a questionnaire to investigate the middle school students' perceptions of mathematics teachers' PCK, and hoped to gain a better understanding of components of PCK from the students' perspectives. It is worthwhile to develop an instrument of addressing middle school students' perceptions of mathematics teachers' PCK that can help mathematics teachers understand how their knowledge may be recognized by their teaching and how their teaching can be improved based on the students' perceptions.

III. METHOD

In this study, a scale entitled "Scale for middle school students' perceptions of mathematics teachers' PCK" was developed, and then its validity and reliability were investigated. The process of developing the scale included the following steps, that is, constructing the scale's items, consulting for expert opinion for content validity, and running exploratory factor analysis, confirmatory factor analysis and reliability analysis for construct validity.

1. CONSTRUCTING THE ITEMS OF THE SCALE

The scale development studies began with reviewing the literature on PCK. The instrument was revised based on the questionnaire designed by Tuan. et al. (2000), in which PCK included 'Instructional Repertoire', 'Representational Repertoire', 'Subject Matter

Knowledge', and 'Knowledge of Students' Understanding'. Since the items in the PCK questionnaire designed by Tuan. Et al. (2000) are suited for science, many items and their expressions are not suitable for mathematics. We revised the questionnaire by combining the composition of PCK in the literature review according to the characteristics of mathematics. The components of SPOMTPCK were initially determined. For example, Examples of items in 'Instructional Repertoire' category in the questionnaire designed by Tuan. et al. (2000) are: 1. My teacher's teaching methods keep me interested in science. 7. My teacher uses a variety of teaching approaches to teach different topics. Combining the characteristics of mathematics, we rewrote them as: 3. My teacher's teaching methods keep me interested in mathematics. 23. My math teacher uses different approaches (questions, discussion, etc.) to find out whether I understand. After constructing the preliminary scale, we consulted with two mathematics teachers who are experts on PCK and six middle school students with respect to the appropriateness of the items in the preliminary scale of mathematics teachers' PCK. Following the expert teachers' and students' suggestions, we analyzed and revised the project repeatedly. We removed the ambiguous and erroneous items and determined the predictive questionnaire.

2. CONTENT VALIDITY

Content validity indicates whether the items constituting the scale are quantitatively and qualitatively adequate for measuring the property that is intended to be measured. To ensure the content validity, two mathematics educators and two mathematics teachers were asked to express their opinions with regard to the items in the scale and the appropriateness of the scale for the subject to be measured. Two mathematics educators and two mathematics teachers who analyzed the scale's content validity are all experts on PCK. If there is a discrepancy in identifying a suitable category for the response, a discussion is conducted among them to agree on a consensus. An agreement of 95.4 % was reached for the categories identifified. In the light of the suggestions received, the scale was given its final form by omitting and revising some items which are unable to reach an agreement. The preliminary scale initially included 38 items. The questionnaire was structured as a Likert type five-choice scale, with choices being "1: Almost Never", "2: Seldom", "3: Sometimes", "4: Often", "5: Almost Always".

3. PARTICIPANTS

Select one key city junior middle school and one ordinary rural junior middle school in Wenzhou city Zhejiang province in China respectively. In China, teachers in key schools are relatively excellent, and students' performance is generally better than that of ordinary rural schools. According to class as a unit and cluster random sampling, extract 409 students in grade 7 and grade 8 in two junior middle schools. Similarly, select one key city high school and one ordinary rural high school in Wenzhou city respectively. According to class as a unit and cluster random sampling, extract 390 students in grade 10 and grade 11 in two high schools. The sample included final 799 participants, 51.8% of them (n = 414) were in key city middle school, and 48.2% (n = 385) were in ordinary rural middle school. By checking and verifying students' questionnaires, If there are students who do not answer the questions in accordance with the rules or only one single answer (For example, all questions are ticked "sometimes"), this questionnaire is discarded. The effective rate of the whole questionnaire is 98%.

4. QUESTIONNAIRE SURVEY

By adopting a unified guide, researchers used group measurement in class as a unit. All of surveys were tested by researchers. The questionnaire was recovered on the spot. Test time is about 15 minutes.

5. ITEM ANALYSIS

First, P value and total item correlation were used to carry out Item analysis of division of scale items. Excluded items with low critical ratio and total correlation coefficient less than 0.20. Then we deleted item 9. Secondly, Adopt the method of "gradual exploration" to carry out factor analysis. The first step. The maximum orthogonal rotation method was used to extract the factors from 37 items, results were extracted to 7 factors. But from 7 factors, each item was confusing and the attribution was not clear enough. In addition, in 7 factors whose eigenvalues were greater than 1, and because the difference between factor loadings of the item 12, 13, 17, 23, 25, 32, 35 in different factors was less than 0.1, the 7 items were eliminated from the scale. The second step. The maximum orthogonal rotation method was used to extract the factors from the remaining 30 topics, results were extracted to 6 factors whose eigenvalues were greater than 1. Because the difference between factor loadings of the item 7, 14, 28, 36 in different factors was less than 0.1, the 4 items were eliminated from the scale. The third step. The maximum orthogonal rotation method was used to extract the factors from the remaining 26 topics, results were extracted to 5 factors whose eigenvalues were greater than 1. Because the difference between factor loadings of the item 15 in different factors was less than 0.1, item 15 was eliminated from the scale. The fourth step. The maximum orthogonal rotation method was used to extract the factors from the remaining 25 topics, results were extracted to 5 factors whose eigenvalues were greater than 1. Because the difference between factor loadings of the item 16 in different factors was less than 0.1, and factor loading in each factor is relatively small (less than 0.1) , item 16 was eliminated from the scale. The fifth step. The maximum orthogonal rotation method was used to extract the factors from the remaining 24 topics, results were extracted to 5 factors whose eigenvalues were greater than 1. By homogeneity test, we found that items 10, 21 and 22 were not homogeneous with other items. Therefore, we deleted items 10, 21 and 22. Because items 11 and 18 were inconsistent with the content of the factors, we deleted items 11 and 18. Therefore, the fourth factor contained only two items. According to the suggestions there were at least three questions in each factor, we deleted items 19 and 20 in factor 4.

IV. RESULTS

To determine the structure of the SPOMTPCK instrument, we randomly split the data into two equal groups based on the overall SPOMTPCK summation score of Likert scale items so that the distribution between the two groups was the same. We used an exploratory factor analysis (EFA) with the first group of 391 student surveys to determine which items should be retained for the final instrument. We used a confirmatory factor analysis on the second group of 392 student surveys to verify the structure obtained through the exploratory factor analysis. This method gives more strength and validity to the research method and results (Fink, 2003; Litwin, 1995; Thompson, 2004).

1. EXPLORATORY FACTOR ANALYSIS

After the removal of the items 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 28, 32, 35, 36 from the analysis, we completed an exploratory factor analysis on the remaining 17 items with three factors. The three-factor analysis matches the data well (see Table 1). Before conducting factor analysis, we first examined whether or not the sample was appropriate for factor analysis. To determine this, we calculated the Kaiser-Meyer-Olkin (KMO) index. For the sample of this study, the KMO index was found to be 0.949, which suggests that the sample is sufficient for conducting factor analysis. Furthermore, Bartlett's Spherecity Test was run to check whether the data represent a multivariate normal distribution. The test resulted in Approx. Chi-square: 4010.350 and $p = 0.000 < 0.001$,which shows that the results are significant. Hence, in its final form, the scale includes 17 items. The results of exploratory factor analysis are shown in Table 1.

Table 1. Factor loadings obtained from the factor analysis

Item No	Item content	Eigen-value	Factor loadings	Factor common variances	Var (%)
	Factor 1	8.627			50.745
27	My math teacher uses analogies with which I am familiar to help me understand mathematics concepts		0.689	0.675	
29	My math teacher uses real objects to help me understand concepts		0.778	0.682	
30	My math teacher uses appropriate examples to explain concepts related to subject matter		0.777	0.703	
31	My math teacher will demonstrate and explain the concept		0.657	0.571	
33	My math teacher uses my daily life experience to illustrate the concept		0.720	0.669	
34	My math teacher uses appropriate metaphors to explain the concepts in textbooks		0.764	0.721	
	Factor 2	1.319			7.758
1	Math teacher's teaching methods make me think hard		0.751	0.748	
2	My teacher uses interesting methods to teach mathematics topics		0.773	0.733	
3	My teacher's teaching methods keep me interested in mathematics		0.842	0.811	
4	My teacher uses appropriate models to help me understand mathematics concepts		0.683	0.730	
5	Math teacher uses different teaching activities to promote my interest in learning		0.695	0.731	
6	Math teacher provides opportunities for me to express my point of view		0.570	0.609	
	Factor 3	1.106			6.504
8	My teacher uses multimedia or technology (e.g. Geometer's Sketchpad) to express mathematics concept		0.659	0.486	
24	My math teacher's questions evaluate my understanding of a topic		0.581	0.543	
26	My math teacher knows students' learning difficulties before class		0.644	0.597	
37	My math teacher adjusts the textbook content according to the actual situation of the student		0.669	0.542	
38	My math teacher knows my misunderstandings when I study mathematical concepts		0.677	0.501	
	Cumulatively Var (%)				65.008

As seen in Table 1, the first factor explains 50.745% of total variance concerning the scale, the second does 7.758%, the third does 6.504%. In total, the three factors together explain 65.008% of the variability. By and large, the factor loadings of the items in the dimensions are found above the accepted limit and the total variance that the factors explain are found satisfactory. After the rotation, it is determined that the first factor consists of six items (27, 29, 30, 31, 33, 34), the second factor consists of six items (1, 2, 3, 4, 5, 6), the third factor consists of five items (8, 24, 26, 37, 38). According to the meaning of items in each factor, we can see that factor 1 reflects teachers' teaching representation, factor 1 is named "teaching representation". Factor 2 reflects teachers' encouraging students to participate in classroom teaching, we name it "encouraging students to participate". Factor 3 reflects teachers' understanding students and curriculum, factor 3 is called "understanding students and curriculum".

2. CONFIRMATORY FACTOR ANALYSIS

We continued the study by modeling the relations between the factors identified through exploratory factor analysis and by considering the theoretical structure and their related items. To test the fit of the created model, confirmatory factor analysis of questionnaires for 392 secondary school students was conducted by using AMOS 7.0. For confirmatory factor analysis, various fit indexes are examined to evaluate the goodness of fit of the proposed factorial structure model. There are several criteria which considered as an indication of good fit of the factorial structure (Hooper, Coughlan & Mullen, 2008). That is, (a) higher than .90 value for Comparative Fit Index (CFI) and Goodness of Fit Index (GFI) and (b) .08 or less value for Root Mean Square Error of Approximation (RMSEA). However, GFI > .85, RMSEA < .10 are also considered acceptable criteria for evaluating the goodness of the fit of the model (Cole, 1987). We conducted confirmatory factor analysis of the 3 subscales and the total tables respectively. We used the observed variables in each subscale and total table to constitute the correlation matrix of the samples, which acted as a basis of model checking. We used maximum likelihood estimation to test the degree of fitting, the results are shown in Table 2.

Table 2. Fitting index of model

Scale	χ^2	df	χ^2/df	GFI	NFI	CFI	TLI	RMSEA
Teaching representation	22.588	8	2.823	0.981	0.982	0.988	0.978	0.068
Encouraging students to participate	39.097	9	4.344	0.964	0.973	0.979	0.965	0.092
Understanding students and curriculum	23.944	5	4.789	0.975	0.948	0.958	0.916	0.098
Total	339.714	116	2.929	0.904	0.915	0.942	0.932	0.070

From table 2, We can see that the RMSEA of the total scale and each subscale is less than 0.1, $2<\chi^2/df<5$, the GFI, NFI, CFI and TLI of the whole scale and each subscale are all above 0.9. This indicates that the fitting effect of the model is satisfactory.

With confirmatory factor analysis, we investigated the power of the items to represent and to what extend the sub-dimensions of the model relate to PCK, by examining the relations between the sub-dimensions and the relation of each sub-dimension to the PCK construct. Confirmatory factor analysis (CFA) is presented in examining first order and second order CFA. The proposed model was tested with second order confirmatory factor analysis. The model generated in this part illustrates a second-level model in which three sub-dimensions represent the PCK construct with significant relations. Confirmation of this model would indicate that the characteristics of pedagogical content knowledge can be measured in multiple ways and that the measured characteristics would be related to a higher-level factor (PCK). With this in mind, we ran the second order confirmatory factor analysis. The results are presented in Figure 1. In the figure, the circles represent the latent constructs, and the rectangles represent the measured variables. One-way arrows give information about how well each item represents its implicit variable. The path coefficients (standardized regression coefficients) for the first-level factors vary between .81 and 1.13. In addition, these three sub-dimensions found to represent the overall dimension of PCK at a high level (.57–.66).

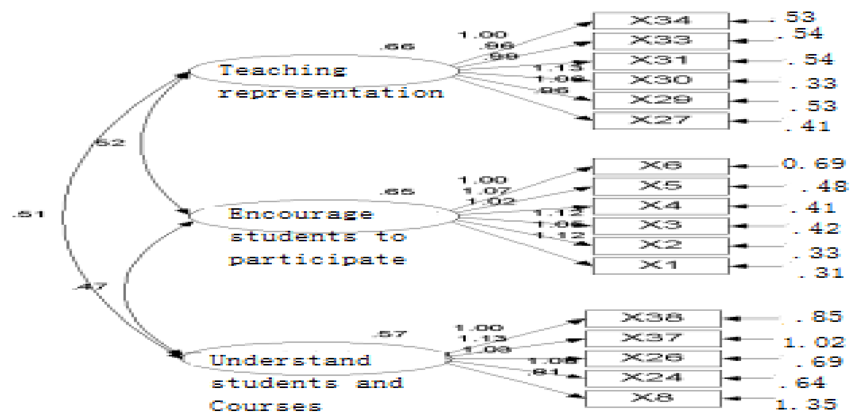

Figure 1. A structure model of mathematics teachers' PCK by middle school students' perceiving

3. RELIABILITY

After exploratory and confirmative factor analysis, the reliability of each subscale and the whole scale were examined by calculating the Split-Halves reliability and internal consistency coefficient. The results of the reliability test for each subscale and the whole scale are presented in Table 3. The test results show that the reliability of the measurement tools has reached the requirements for assessment. The questionnaire is reliable as a tool to measure middle school students' perception of mathematics teachers' PCK.

Table 3. Cronbach alpha coefficients of the factors（n=783）

	Teaching representation	Encouraging students to participate	Understanding students and curriculum	Total
Split-Halves reliability	0.892	0.869	0.775	0.873
Internal consistency coefficient	0.896	0.912	0.770	0.935

4. CONSTRUCT VALIDITY

Construct validity is used to examine whether a test measured previous theoretical conceptions and assess the explanatory power of tests. Construct validity focus[es] on the assessment of whether a particular measure relates to other measures consistent with a theoretically anticipated way' and this is usually done by factor analysis (Anastasi, 1988). As shown in Table 1, the results of exploratory factor analysis show that the scale is clear, the correlation coefficient among factors is 0.630~0.699, and reach a significant level. Scale meets the requirements for correlation between factors. Each item's factor loading is greater than 0.50, and the meaning of each item is clear and highly explanatory. This shows that the division of all dimensions is reasonable. The results of confirmatory factor analysis also show that the structural model is satisfactory. Therefore, The scale has good construct validity.

Another way to validate the validity of the scale is to compare whether there are any differences of the scales and the subscales among different classes, then we do ANOVA analysis. The analysis of variance should be consistent with the assumption of independence, normality, variability homogeneity, additivity and sphericity of the observer. Results of Item analysis show that the scale is suitable for variance analysis. As shown in Table 4, There is a significant difference ($p<0.001$) in the scores of the scale and the subscales among different classes. Thus, we can see there is really different in math teachers' PCK perceived by students in different classes. It also shows that the scale can be used to evaluate mathematics teachers' PCK through students' views.

Table 4. Single factor ANOVA analysis of 17 classes

Scale		Sum of Squares	df	Mean Square	F	Sig.
Teaching representation	Between Groups	7 261.242	16	453.828	24.108	0.000
	Within Groups	14 419.678	766	18.825		
	Total	21 680.920	782			
Encouraging students to participate	Between Groups	10 442.256	16	652.641	37.586	0.000
	Within Groups	13 300.819	766	17.364		
	Total	23 743.075	782			
Understanding students and curriculum	Between Groups	5 836.687	16	364.793	28.305	0.000
	Within Groups	9 872.210	766	12.888		
	Total	15 708.897	782			
Total Scale	Between Groups	63 898.107	16	3993.632	38.396	0.000
	Within Groups	79 672.478	766	104.011		
	Total	143 570.600	782			

5. DIFFERENCE OF MATHEMATICS TEACHERS' PCK BY MIDDLE SCHOOL STUDENTS' PERCEIVING

According to school type, grade and gender, We calculated the mean and standard deviation of the whole scale and all subscales. All data obtained from the survey were analyzed by using Multivariate ANOVA. The results of various data were summarized in Table 5.

Table 5. A comparison of difference ofmathematics teachers' PCK by middle school students' perceiving

		Teaching representation	Encouraging students to participate	Understanding students and curriculum	Total
School differences	key	23.24±5.57	23.90±5.518	19.45±4.104	66.26±12.916
	ordinary	19.53±4.24	19.53±4.872	15.09±3.922	54.07±11.187
	F	11.893***	18.372***	42.153***	14.955***
Grade differences	Grade 7	20.32±4.860	20.35±5.261	15.80±4.275	56.63±12.455
	Grade 8	22.42±5.304	22.49±6.005	16.93±5.417	61.88±15.223
	Grade 10	22.08±6.059	22.22±6.114	18.36±4.045	61.99±14.286
	Grade 11	20.85±4.678	21.98±4.922	18.29±3.813	60.57±11.058
	F	2.683***	3.536***	6.436***	4.120***
Gender differences	male	21.25±5.410	21.82±5.821	16.93±4.857	59.61±13.736
	female	21.66±5.207	21.78±5.492	17.77±4.235	61.07±13.338
	F	0.636	1.748	1.193	1.554
School×Grade	*F*	6.150***	5.285***	6.010***	4.811***

* $p<0.05$; *** $p<0.001$

According to school type, grade and gender, Table 5 showed that the average scores and standard deviation of the whole scale and all subscales of math teachers' PCK by middle school students' perceiving. In order to compare the differences among different schools, grades and sexes students' perceptions of mathematics teachers' PCK, We made MANOVA analysis on the two variables of school type, grade and gender. As shown in Table 5, In school type and grade variables, There are significant differences in the 3 dimensions of middle school students' perceptions of mathematics teachers' PCK. However, There is basically no significant difference in gender variables. There is a significant difference in the interaction effect between school and grade.

V. DISCUSSION AND CONCLUSION

1. KEY FINDINGS AND EXTENDING THE PACK LITERATURE

In this paper, we present our efforts for developing a scale to determine middle school students' perceptions of mathematics teachers' PCK. Following the statistical analyses, a three-factor scale composed of 17 items was developed. The factors of the scale are as follows: a) teaching representation, b) encouraging students to participate, c) understanding students and curriculum.

After constructing the scale, we proceeded to the validity and reliability analyses. As a result, The Cronbach α coefficient of each subscale of the questionnaire is 0.770 - 0.912, the split half reliability is 0.775~0.892. The Cronbach α coefficient of whole scale of the questionnaire is 0.935, the split half reliability is 0.873. The results of exploratory factor analysis show that the structure of each subscale is clear. Each item's factor loading is greater than 0.50, and the meaning of each item is clear and highly explanatory. The total variance interpretation rate is 65.008%.

Confirmatory factor analysis showed that the RMSEA of the total scale and each subscale is less than 0.1, $2<\chi^2/df<5$, the GFI, NFI, CFI and TLI of the scale and each subscale are all above 0.9. This indicates that the fitting effect of the model is satisfactory. This shows that the division of all dimensions is reasonable. In sum, the findings of the study showed that the generated scale is a valid and reliable instrument for determining middle school students' perceptions of mathematics teachers' PCK.

2. LIMITATIONS OF THIS STUDY

The limitations of this study are in two ways. On the one hand, the sample students in this study are from four different kinds of public middle schools, and their abilities and characteristics may not have the greatest variation. Also, the sample students are selected in convenience and not strictly random sampling, the numbers of samples selected are somewhat less, which will affect the effectiveness of the scale. In order to eliminate this disadvantageous situation, a large number of participants were included in the study. In future studies using the SPOMTPCK scale, this limitation can be eliminated by using the random sampling method. On the other hand, we strictly follow the procedures of constructing the scale. that is, based on an extensive literature review especially the questionnaire designed by Tuan. et al. (2000), by combining the characteristics of mathematics, the items of the scale were determined. But the items of the scale are not also comprehensive and further need to be improved. We acknowledge that these are limiting in terms of the use of our survey, yet we see unearthing such a limitation as a key finding in expanding the PCK literature domain.

3. IMPLICATIONS OF THIS STUDY

The implication of the study is mainly to help teachers improve their own PCK. Our conceptualization of the components of PCK provides an important conceptual tool for helping mathematics teachers construct the specific knowledge they need to be effective teachers. This scale is students' perceptions of teachers' PCK. Teachers can use this scale to collect students' opinions, which can be seen as the most specific reference material for teachers to reflect on their own PCK. By using the SPOMTPCK scale, teachers may obtain data from their students. They can monitor themselves by reaching more students in less time compared with interviewing students or observing them, and try to develop their PCK, in terms of pedagogical representation, understanding students and curriculum, and encouraging students' engagement according to the SPOMTPCK scale's results. Moreover, examining students' perceptions of their teachers' PCK at difffferent times by using the scale may be used to see the development of teachers' PCK and determine the critical points of improvement in teachers' PCK. In addition, this study is signifificant since it may be an example to the researchers who want to develop a scale related to PCK.

4. FUTURE STUDY

To establish the new instrument's usefulness, future research is needed to provide more specific analysis of concerning the relationship between this instrument and students' responses to the items along with students' different achieving ability. We will further find whether the results obtained from the SPOMTPCK scale were in accordance with the results obtained from interviews with teachers and teachers' lessons observations. Further studies in which the scale is administered to different samples and the dissemination of the related findings are recommended, as those would greatly contribute to the applicability of the scale. Taking different PCK frameworks into account, the scale can be elaborated further by increasing the number of items. Other research is needed to examine whether teachers with acknowledged weak knowledge on pedagogical representation, encouraging students' engagement, understanding students and curriculum would be scored lower on the three scales of PCK compared to teachers with strong knowledge on these three areas.

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Appendix

Students' perceptions of teachers' knowledge

Directions for students:

This questionnaire contains statements about practices which could take place in this class. You will be asked how often each practice takes place. There are no `right' or `wrong' answers. Your opinion is what is wanted. Think about how well each statement describes what this class is like for you.

Draw a circle around:

1. if the practice takes place	Almost Never
2. if the practice takes place	Seldom
3. if the practice takes place	Sometimes
4. if the practice takes place	Often
5. if the practice takes place	Almost Always

Be sure to give an answer for all questions. If you change your mind about an answer, just cross it out and circle another.Some statements in this questionnaire are fairly similar to other statements. Don't worry about this. Simply give your opinion about all statements.

Your School______________________Grade_____________Gender： ________ （1.Male 2. Female）

	Almost Never	Seldom	Some-times	Often	Almost Always
1. Math teacher's teaching methods make me think hard	1	2	3	4	5
2. My teacher uses interesting methods to teach mathematics topics	1	2	3	4	5
3. My teacher's teaching methods keep me interested in mathematics	1	2	3	4	5
4. My teacher uses appropriate models to help me understand mathematics concepts	1	2	3	4	5
5. Math teacher uses different teaching activities to promote my interest in learning	1	2	3	4	5
6. Math teacher provides opportunities for me to express my point of view	1	2	3	4	5
7. Math teacher praises my good behavior in classroom	1	2	3	4	5
8. My teacher uses multimedia or technology (e.g. Geometer's Sketchpad) to express mathematics concept	1	2	3	4	5
9. Math teacher creates a classroom circumstance to promote my interest for learning	1	2	3	4	5
10. My teacher tells me to go to the library to check relevant information for teaching mathematics	1	2	3	4	5
11. My teacher designs many different teaching aids to teach mathematics	1	2	3	4	5
12. My math teacher uses appropriate diagrams and graphs to explain math concepts	1	2	3	4	5

13. My math teacher tells me how to apply mathematics knowledge to life	1	2	3	4	5
14. My math teacher reminds me of what I have learned before	1	2	3	4	5
15. My math teacher tells me why I should learn this unit	1	2	3	4	5
16. Math teacher explains similar concepts I learned in other subjects in class	1	2	3	4	5
17. My teacher uses familiar examples to explain mathematics concepts	1	2	3	4	5
18. My teacher explains the impact of mathematics on society	1	2	3	4	5
19. My teacher asks me evaluate myself whether I have learned	1	2	3	4	5
20. My math teacher asks me know the criteria of evaluation	1	2	3	4	5
21. Math teacher uses tests to check whether I understand what I have learned	1	2	3	4	5
22. Math teacher uses classmates to evaluate each other to see if I have learned	1	2	3	4	5
23. My math teacher uses different approaches (questions, discussion, etc.) to find out whether I understand	1	2	3	4	5
24. My math teacher's questions evaluate my understanding of a topic	1	2	3	4	5
25. When math teacher teaches new concepts, he reviews previous concepts	1	2	3	4	5
26. My math teacher knows students' learning difficulties before class	1	2	3	4	5
27. My math teacher uses analogies with which I am familiar to help me understand mathematics concepts	1	2	3	4	5
28. My math teacher knows in advance how I may react when I am learning	1	2	3	4	5
29. My math teacher uses real objects to help me understand concepts	1	2	3	4	5
30. My math teacher uses appropriate examples to explain mathematics concepts	1	2	3	4	5
31. My math teacher will demonstrate and explain the concept	1	2	3	4	5
32. My math teacher asks questions to help me understand the concept	1	2	3	4	5
33. My math teacher uses my daily life experience to illustrate the concept	1	2	3	4	5
34. My math teacher uses appropriate metaphors to explain the concepts in textbooks	1	2	3	4	5
35. My math teacher explains the concepts in textbooks with the form I can understand	1	2	3	4	5
36. My teacher realizes students' prior knowledge before class	1	2	3	4	5
37. My math teacher adjusts the textbook content according to the actual situation of the student	1	2	3	4	5
38. My math teacher knows my misunderstandings when I study mathematical concepts	1	2	3	4	5

Thanks for filling in this questionnaire