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# On Power of Correlated Superposition Coding in NOMA

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## Abstract

We present the power of the correlated superposition coding (SC) in non-orthogonal multiple access (NOMA). This paper derives closed-form expressions for the total allocated power with the constant total transmitted power. It is shown that the total allocated power is the function of a correlation coefficient. In result, the correlated SC NOMA should be designed with consideration of the correlation coefficient.

*Key words : Non-orthogonal multiple access, superposition coding, successive interference cancellation, power allocation, correlation coefficient.*

## Ⅰ. Introduction

With the world's first launching of the fifth generation (5G) mobile communication in Korea, April 3, 2019, the commercialization of 5G new radio (NR) wireless networks is in progress all over the world. One of the state-of-the-art techniques for 5G and beyond mobile radio access networks is non-orthogonal multiple access (NOMA) [1-6]. Recently, in the orthogonal NOMA (O NOMA), the correlated superposition coding (SC) not only achieves the near-perfect successive interference cancellation (SIC) bit-error rate (BER) performance for the stronger channel user, but also fixes the performance crash for the weaker channel user [5], [6]. For the correlated SC NOMA, this paper derives the analytical expression of the total allocated power, with the correlation coefficient.

## Ⅱ. System Model (Conventional)

Assume that the total transmitted power is $P$, the power allocation factor is $\alpha$ with $0 \leq \alpha \leq 1$, and the channel gains are $h_1$ and $h_2$, with $|h_1| > |h_2|$. Then $\alpha P$ is allocated to the user-1 signal $s_1$ and $(1-\alpha)P$ is allocated to the user-2 signal $s_2$, with $E[|s_1|^2] = E[|s_2|^2] = 1$. The superimposed signal is expressed by

$$x = \sqrt{\alpha P}\, s_1 + \sqrt{(1-\alpha)P}\, s_2 \tag{1}$$

Before the SIC is performed on the user-1 with the better channel condition, the received signals of the user-1 and user-2 are represented as

$$\begin{aligned} r_1 &= |h_1|\sqrt{\alpha P}\, s_1 + (|h_1|\sqrt{(1-\alpha)P}\, s_2 + n_1) \\ r_2 &= |h_2|\sqrt{(1-\alpha)P}\, s_2 + (|h_2|\sqrt{\alpha P}\, s_1 + n_2) \end{aligned} \tag{2}$$

where $n_1$ and $n_2 \sim N(0, N_0/2)$ are additive white Gaussian noise (AWGN) and $N_0$ is one-sided power

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spectral density. Note that the total allocated power $\alpha P+(1-\alpha)P$ is the same as the total transmitted power $P$, due to the independent SC in the standard NOMA.

## III. System Model (Generalized)

First, we start a simple example, and then generalize the result.

$$\begin{aligned} &3\text{V},\ 3\text{V}, \quad 6\text{V}(=3\text{V}+3\text{V}) \\ &9\text{W},\ 9\text{W}, \quad 36\text{W}(\neq 9\text{W} \quad 9\text{W}) \end{aligned} \tag{3}$$

In the above example, if the superimposed signal of the amplitude, 6V, is transmitted, the base station consumes 36W. However, the two modulators (two isolated power sources) supply only 18W. Thus, the two modulators must deliver the additional power, 18W, "mathematically invisibly" [7], [8].

Now, we derive the expression of the total allocated power, given the constant total transmitted power. As the first step, we calculate the total transmitted power in the previous section, when the two signals are correlated;

$$\begin{aligned} E[|x|^2] &= E\left[\left|\sqrt{\alpha P}s_1+\sqrt{(1-\alpha)P}s_2\right|^2\right] \\ &= P+2Re\{\rho\}\sqrt{\alpha}\sqrt{1-\alpha}P \end{aligned} \tag{4}$$

where the correlation coefficient $\rho=E\left[s_1 s_2^*\right]$. We want the total transmitted power to be a constant $P$. Therefore, we scale both sides by

$$\frac{P}{P+2Re\{\rho\}\sqrt{\alpha}\sqrt{1-\alpha}P} \tag{5}$$

Then we have

$$E\left[\left|\frac{\sqrt{\alpha P}}{\sqrt{1+2Re\{\rho\}\sqrt{\alpha}\sqrt{1-\alpha}}}s_1+\frac{\sqrt{(1-\alpha)P}}{\sqrt{1+2Re\{\rho\}\sqrt{\alpha}\sqrt{1-\alpha}}}s_2\right|^2\right]=P \tag{6}$$

Finally, for the constant total transmitted power $P$, we obtain the total allocated power

$$\frac{P}{1+2Re\{\rho\}\sqrt{\alpha}\sqrt{1-\alpha}} \tag{7}$$

If we apply the result to the equation (1), the generalized superimposed signal is given by

$$x=\frac{\sqrt{\alpha P}}{\sqrt{1+2Re\{\rho\}\sqrt{\alpha}\sqrt{1-\alpha}}}s_1+\frac{\sqrt{(1-\alpha)P}}{\sqrt{1+2Re\{\rho\}\sqrt{\alpha}\sqrt{1-\alpha}}}s_2 \tag{8}$$

## IV. Application to Orthogonal NOMA

The correlation coefficient of the correlated SC in the O NOMA [5], [6] is calculated by

$$\rho=\frac{1}{\sqrt{2}}\simeq 0.707 \tag{9}$$

Consider Rayleigh fading channels with $E[|h_1|^2]=\Sigma_1$ and $E[|h_2|^2]=\Sigma_2$. For the simplification of Rayleigh fading BER performance, we define the notation as

$$F(\gamma_b)=\frac{1}{2}\left(1-\sqrt{\frac{\gamma_b}{1+\gamma_b}}\right) \tag{10}$$

First, for the constant total allocated power $P$, the O NOMA BER performances for the user-1 and user-2 are given by, with the conditional BER performance in [5]

$$\begin{aligned} &P_e^{(1;O\ NOMA;\ ML\ practical;\ with\ P\ allocated)} \\ &=\frac{1}{2}F\left(\frac{\alpha P\Sigma_1}{N_o}\right)+\frac{1}{2}F\left(\frac{P(\sqrt{2(1-\alpha)}+\sqrt{\alpha})^2\Sigma_1}{N_o}\right) \end{aligned} \tag{11}$$

and

$$\begin{aligned} &P_e^{(2;\ O\ NOMA;\ ML\ practical;\ with\ P\ allocated)} \simeq F\left(\frac{(1-\alpha)\Sigma_2 P/2}{N_0}\right) \\ &-\frac{1}{2}F\left(\frac{P(2\sqrt{\alpha}+3\sqrt{(1-\alpha)/2})^2\Sigma_2}{N_0}\right) \\ &+\frac{1}{2}F\left(\frac{P(2\sqrt{\alpha}+\sqrt{(1-\alpha)/2})^2\Sigma_2}{N_0}\right) \end{aligned} \tag{12}$$

(Remark: in [5], actually the total transmitted power, i.e., $P+\sqrt{2}\sqrt{\alpha}\sqrt{1-\alpha}P$, is greater than the total allocated power $P$ by $\sqrt{2}\sqrt{\alpha}\sqrt{1-\alpha}P$.)

Second, we present the BER performances for the constant total transmitted power $P$; the O NOMA BER performances for the user-1 and user-2 are given by, with the equation (8)

$$P_e^{(1;\ O\ NOMA;\ ML\ practical;\ with\ P\ transmitted)} = -\frac{1}{2}F\left(\frac{\alpha P \Sigma_1}{(1+\sqrt{2}\sqrt{\alpha}\sqrt{1-\alpha})N_0}\right) + \frac{1}{2}F\left(\frac{P(\sqrt{2(1-\alpha)}+\sqrt{\alpha})^2\Sigma_1}{(1+\sqrt{2}\sqrt{\alpha}\sqrt{1-\alpha})N_0}\right) \quad (13)$$

and

$$P_e^{(2\ O\ NOMA;\ ML\ practical;\ with\ P\ transmitted)} \simeq F\left(\frac{(1-\alpha)\Sigma_2 P/2}{(1+\sqrt{2}\sqrt{\alpha}\sqrt{1-\alpha})N_0}\right) - \frac{1}{2}F\left(\frac{P(2\sqrt{\alpha}+3\sqrt{(1-\alpha)/2})^2\Sigma_2}{(1+\sqrt{2}\sqrt{\alpha}\sqrt{1-\alpha})N_0}\right) + \frac{1}{2}F\left(\frac{P(2\sqrt{\alpha}+\sqrt{(1-\alpha)/2})^2\Sigma_2}{(1+\sqrt{2}\sqrt{\alpha}\sqrt{1-\alpha})N_0}\right) \quad (14)$$

## V. Results and Discussions

Assume $\Sigma_1=(2.0)^2$ and $\Sigma_2=(0.9)^2$, and the constant total allocated or transmitted signal power to noise power ratio $P/N_o=40\text{dB}$, for the equation (11) and (12), or for the equation (13) and (14), respectively. In the standard NOMA, the ideal perfect SIC BER performance of the user-1 is simply the BER performance of the BPSK modulation

$$P_e^{(1;\ NOMA;\ perfect\ SIC;\ ideal)} = F\left(\frac{\alpha P \Sigma_1}{N_0}\right) \quad (15)$$

and the optimal maximum likelihood (ML) BER performance $P_e^{(2;\ NOMA;\ ML;\ optimal)}$ of the user-2 is given in [9]. As shown in Fig. 1, the O NOMA user-1 BER performance of the constant total transmitted power $P$ is slightly worse than that of the constant total allocated power $P$. This is reasonable, because the O NOMA of the constant total allocated power $P$ uses (i.e., consumes, transmits) more power than the O NOMA of the constant total transmitted power $P$ by $\sqrt{2}\sqrt{\alpha}\sqrt{1-\alpha}P$ from the equation (4). As shown in Fig. 2, even though the O NOMA user-2 BER performance degrades with such correction, the O NOMA still fixes the performance crash of the standard NOMA.

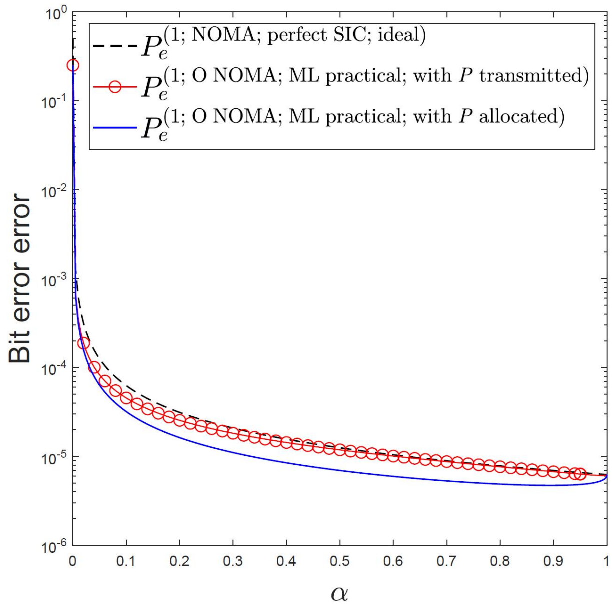


Fig. 1. Comparison of BERs for standard NOMA, and O NOMA with constant total transmitted or allocated power $P$ for user-1.

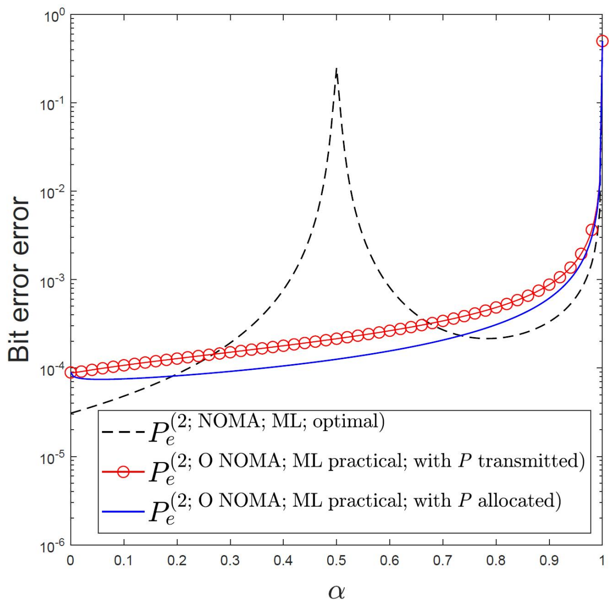


Fig. 2. Comparison of BERs for standard NOMA, and O NOMA with constant total transmitted or allocated power $P$ for user-2.

## Ⅵ. Conclusion

We derived the analytical expression of the total allocated power with the constant total transmitted power for the correlated SC in NOMA. It was shown that the total allocated power is the function of a correlation coefficient. As a consequence, the correlated SC NOMA should be designed with consideration of the correlation coefficient.

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