Research Paper
International Journal of Aerospace System Engineering
Vol.7, No.2, pp.6-12 (2020)

ISSN 2383-4986
EISSN 2508-7169
http://dx.doi.org/10.20910/IJASE.2020.7.2.6

Differential Evolution for Regular Orbit Determination

Pratik V. Dedhia†, R V. Ramanan
Aerospace Engineering, Indian Institute of Space Science and Technology, Thiruvananthapuram, INDIA
†E-mail: pratik.dedhia91@gmail.com

Abstract

The precise prediction of future position of satellite depends on the accurate determination of orbit, which is also helpful in performing orbit maneuvers and trajectory correction maneuvers. For estimating the orbit of satellite many methods are being used. Some of the conventional methods are based on (i) Differential Correction (DC) (ii) Extended Kalman Filter (EKF). In this paper, Differential Evolution (DE) is used to determine the orbit. Orbit Determination using DC and EKF requires some initial guess of the state vector to initiate the algorithm, whereas DE does not require an initial guess since a wide range of bounds for the design unknown variables (orbital elements) is sufficient. This technique is uniformly valid for all orbits viz. circular, elliptic or hyperbolic. Simulated observations have been used to demonstrate the performance of the method. The observations are generated by including random noise. The simulation model that generates the observations includes the perturbation due to non-spherical earth up to second zonal harmonic term.

Key Words : Observation, random noise, Orbit Determination, Differential Evolution, Perturbation

1. Introduction

Regular Orbit determination (ROD) is carried out to determine the state vector i.e. position and velocity at any time of a satellite. The orbit is determined using observations obtained from ground based tracking station. The observation typically includes range ρ, azimuth β, elevation h, and range-rate $\dot{\rho}$.

The classical methods like Gauss, Gibbs and Herrick-Gibbs give closed form solution for orbit determination problems [1]. Since perturbations and noise are not accounted while deriving these analytical solutions, these methods are valid only for unperturbed orbit with uncorrupted observation data. The iterative techniques based on Differential Correction [2, 3, 4] and Extended Kalman Filter [2, 3, 5] use the actual observations which takes into account the perturbation effects. These methods employ weight matrix to compensate for the noisy observation data. These methods require an initial guess of state to initiate the algorithm. So, alternatively an evolutionary algorithm, Differential Evolution can be used without any initial guess. In this paper, Differential Evolution [6, 7, 8, 9] is employed successfully for ROD.

Differential Evolution is an Evolutionary optimization technique which uses mutation, cross over and random selection, the three principles that has led to evolution of every living species on Earth. The technique begins with setting bounds for each of the unknown design variables. In this ROD problem there are six unknowns, the orbital elements. The strategy to fix these bounds are different for different problems. A detailed discussion on how to fix the bounds for this problem is included in next section.

Received: Oct 13, 2018 Revised: Feb 29, 2020 Accepted: June 09, 2020
† Corresponding Author
Tel: +91-99-6956-4991, E-mail: pratik.dedhia91@gmail.com

From these bounds each unknown element is searched randomly from their respective domains and are used to evaluate the objective function.

The conventional methods require good initial guess on the state vector. Because the state vector does not have any physical meaning it becomes difficult to guess on the state vector. On the contrary, Differential Evolution requires only bounds on the unknown orbital elements which are easy to visualize. Also, the objective function does not require to be continuous and differentiable for differential evolution technique unlike in the cases of gradient based methods which require the objective function to be continuous and differentiable. Further, the differential evolution results in globally optimal solution. This is shown in results section.

2. PROBLEM FORMULATION

The observations obtained from ground based tracking station typically includes range ρ, azimuth β, elevation h and range-rate $\dot{\rho}$. The parameter range is the distance between the satellite and the tracking station and usually measured in km. The parameter azimuth is the angle measured clockwise from the local north to the projection of the range vector on the horizon plane. The parameter elevation is the angle measured in degrees between the range vector and its projection on to the horizon plane.

Using the observations obtained at different instants of time from a particular tracking station, the orbit of the satellite is determined by minimizing an objective function J.

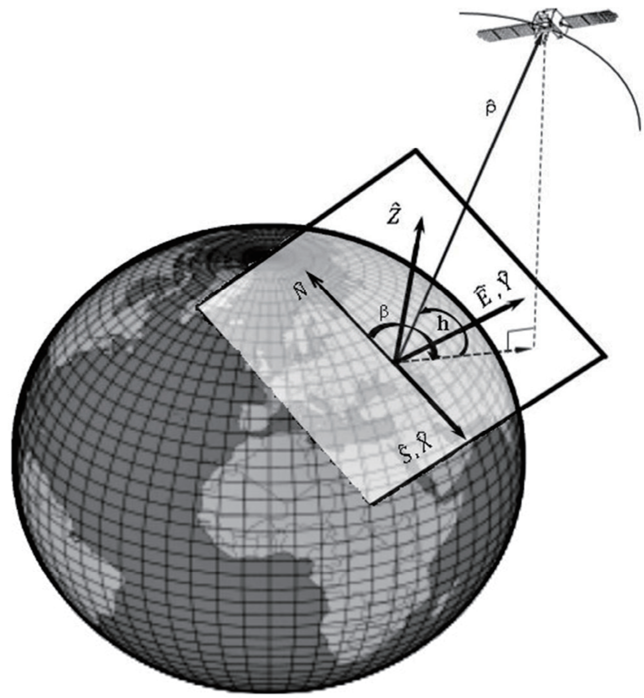


Fig. 1 Geometry of observation showing range, azimuth and elevation.

The objective function for this problem is the sum of the square of the differences between the actual and computed observations at all-time instants of the possible candidate orbits.

$$J = \sum_{i=1}^{n} [w_1.(\rho_{det} - \rho_{obs})^2{}_i + w_2.(\beta_{det} - \beta_{obs})^2{}_i + w_3.(h_{det} - h_{obs})^2{}_i + w_4.(\dot{\rho}_{det} - \dot{\rho}_{obs})^2{}_i] \quad (1)$$

Here the subscript *det* denotes determined and *obs* denotes Observed. The parameters w_1, w_2, w_3, w_4 are the weights that corresponds to each observation and they are introduced to normalize the units of the different terms of the objective function. The subscript i denotes different time instants.

2.1. THE UNKNOWN DESIGN VARIABLES

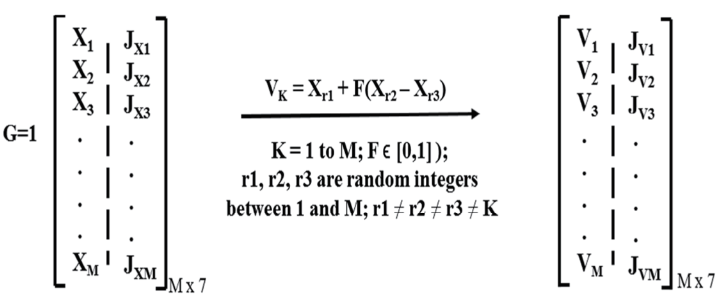
The orbit determination requires six unknown parameters to be determined to completely define the orbit of the satellite. The six parameters can either be the state vector $X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}$ in geocentric co-ordinate frame or the orbital elements, namely Semi-major axis (a), Eccentricity (e), Inclination (i), Right Ascension of Ascending Node (RAAN/Ω), Argument of Perigee (AoP/ω) and True anomaly (υ).

These two sets of unknown orbit parameters are equivalent and one can be obtained from other using basic transformations. The unknown parameters are obtained using an algorithm based on differential evolution technique.

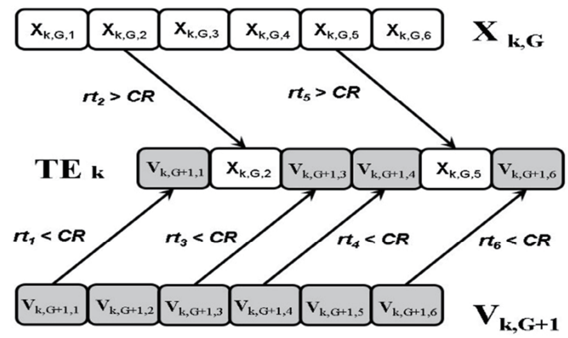
2.2. ALGORITHM FOR ORBIT DETERMINATION USING DIFFERENTIAL EVOLUTION

1. Get the actual observation data ρ_i, β_i, h_i and if possible $\dot{\rho}_I$ at t_i $(i = 1$ to $N)$. Set upper and lower bounds for orbital elements a, e, i, Ω, ω, υ.
2. Select M (population size) combinations of a, e, i, Ω, ω, υ from their respective bounds randomly. The random selection must be uniformly distributed.
 Let $X_j = \{a_j, e_j, i_j, \Omega_j, \omega_j, \upsilon_j\}$, where $j = 1$ to M.
3. Convert orbital elements into their corresponding state vectors.
4. For each of the M combinations, propagate the state vector to subsequent instants t_i $(i = 1$ to $N)$ and calculate corresponding observation. Here, t_i are the time instants at which actual observation data are recorded. N is number of observation.
5. Evaluate the objective function (J). Refer equation 1.
6. Generate M x 7 Matrix P, having orbital elements in the first six columns and the objective function J in the seventh column. P is generation one (G1) matrix
7. Generate mutant vectors for each of the M combinations of current population such that,

$$V_{k,G+1} = X_{r1,G} + F.(X_{r2,G} - X_{r3,G}) \qquad (2)$$



Where, $r1$, $r2$, $r3$ are randomly generated integers between 1 and M such that $r1 \neq r2 \neq r3 \neq k$, where $k = 1$ to M and $F \in [0,1]$ is mutant scale factor.
8. Form trial element TE_k (1 x 6 matrix) using elements of $X_{k,G}$ and the mutant vector $V_{k,G+1}$ as follows:
   8a. Generate six uniformly distributed random numbers rt_q (q = 1 to 6) between 0 and 1. Each random number corresponds to one of the six orbital elements.
   8b. Select a value for crossover ($CR \in [0,1]$). If ($rt_q < CR$) replace the element $X_{k,G,q}$ with corresponding element of $V_{k,G+1,q}$.



9. Evaluate the objective function for the trial element based on steps 4 and 5.
10. If the objective function of trial element is less than that of current value then replace the entire row k in P with the trial element vector.
11. Carry out this process for all the elements of the population till a new population Q is generated. Q is the second generation matrix. The cycle then continues to produce the next generation matrix and so on.
12. REPEAT the steps 3 to 12 until the minimum of objective function in Q matrix is less than prefixed tolerance value in case of uncorrupted observations. For noisy observations the convergence criteria is fixed based on (Max- Min) less than a prefixed tolerance value.
13. The row corresponding to the lowest objective function is then the required orbital elements/state vector for uncorrupted observation data and any row in the matrix is the solution for the noisy observation data.
14. The error in position and velocity of the determined state vector is obtained using,

$$\Delta r = ||(\vec{r}_{det} - \vec{r}_{actual})||;\ \Delta v = ||(\vec{v}_{det} - \vec{v}_{actual})||$$

Where, $\vec{r} = [X, Y, Z]^T$ and $\vec{v} = [\dot{X}, \dot{Y}, \dot{Z}]^T$

3. STRATEGY FOR BOUNDS SELECTION

Regular orbit determination happens on regular basis and the orbital elements of the satellite are roughly known. The first five orbital elements vary very slowly with time and can be treated almost as constant. Hence the bounds with +/- 30 degrees for inclination, RAAN and AoP are used. For semi-major axis, +/- of 1000 km and +/- 0.1 for eccentricity is found to be adequate. The parameter true anomaly is a rapidly changing element of these six and setting its bounds is the major task. An estimation of true anomaly can be made using range, elevation, azimuth and conic equations. The bounds with +/- 30 degrees on the estimated value of true anomaly are fixed.

Even if initial information about the orbit of satellite is unknown wide bounds can be given and DE algorithm can be used to get the solution at the cost of higher computation time. It can be observed that except for semi-major axis and eccentricity, the remaining parameters are angles which, by conic geometry, are bounded. For the earth bound satellites e can be set between 0 and 1 and for interplanetary bound satellites e can be set between 1 and 10. A rough idea for semi-major axis can be obtained from range observation. These initial bounds are enough to initiate the DE algorithm.

4. Results

4.1. DIFFERENTIAL EVOLUTION PARAMETERS

The important parameters of Differential Evolution technique are population size M, cross-over ratio CR and mutant scale Factor F. The population size M depends on number of unknown elements and hence differs from problem to problem. The M should be such that it tries to capture most of the possible combinations in uniform manner for generation of initial population which results in lower and consistent computation time irrespective of seed for a particular problem. If the population size is small, the number of generations required to attain the solution will be high and if M is high, though the number of generations will be less but the computation per generation will be significantly high. Hence an optimum value of M should be selected for lowest computation time.

Differential algorithm mutates the original population to produce a set of trial vectors. The mutation adds randomly scaled vectors to the third vector as shown in the equation 2. The mutant scale factor F is positive real number (between 0-1) controlling the rate at which the population evolves. For optimal performance F should be selected suitably.

Some elements of each vector from original population are replaced by the corresponding elements in the mutant vector. Which all elements will get replaced is decided by CR. The value of CR is between 0-1. For each element in the given set, uniform random number is generated between 0 - 1 and if the value is less than or equal to CR, then the element from original population is retained otherwise it gets replaced by corresponding element from mutant vector to form an intermediary population of trial vectors.

The computation time for a given number of observations is dependent on all three parameters. The parameters are so chosen such that the computation time is minimum and consistent for different seeds. A lot of trials were carried out, few of which are shown in Table 1, 2 and 3, to obtain a set of M, F and CR which results in minimum and consistent computation time. Fixing population size of 50 and mutation factor at 0.7, the variation in computation time with CR is analyzed for Molniya orbit for 80 number of observation and is shown in Table 1.

Table 1 Error and computation time for F=0.7, M=50 and different crossover ratios

CR	Δr (km)	Δv (km/s)	Computation time (sec)
0.5	8.26E-02	6.65E-04	5514
0.6	8.26E-02	6.65E-04	3165
0.7	8.26E-02	6.65E-04	1933
0.8	8.26E-02	6.65E-04	1179
0.9	8.26E-02	6.65E-04	734

Computation time is minimum for CR value of 0.9. Adopting this CR value and same population size, the computation time is checked for different mutation factors and is shown in Table 2.

Table 2 Error and computation time for CR=0.9, M=50 and different mutation factors

F	Δr (km)	Δv (km/s)	Computation time (sec)
0.9	8.26E-02	6.65E-04	1299
0.8	8.26E-02	6.65E-04	1014
0.7	8.26E-02	6.65E-04	734
0.6	8.26E-02	6.65E-04	469
0.5	8.26E-02	6.65E-04	367
0.4	8.26E-02	6.65E-04	1134

Mutation factor of 0.5 reduces the computation time further. Table 3 shows the variation in computation time with different population sizes for CR = 0.9 and F = 0.5. It is observed that the computation time is minimum for the population size of 40 for above F, CR.

Table 3 Error and computation time for F=0.5, CR=0.9 and different population size

M	Δr (km)	Δv (km/s)	Computation time (sec)
30	8.26E-02	6.65E-04	704
40	8.26E-02	6.65E-04	294
50	8.26E-02	6.65E-04	367
60	8.26E-02	6.65E-04	478

The combination of population size, mutation factor and crossover-ratio for minimum computation time need not be unique. For some other combinations it is possible to achieve the above minimum computation time.

4.2. CHECK FOR GLOBAL OPTIMALITY OF THE SOLUTION

The differential evolution technique produces globally optimal solution. This is demonstrated by reproducing the same solution for different bounds and seeds. Three cases of bounds are selected as shown in Table 4 and the solution obtained for each case is shown in Table 5. Number of observation used are 80 for obtaining the below results.

Table 4 Bounds for global optimality check of differential evolution

Cases	Case I		Case II		Case III	
Orbital Elements	Upper	Lower	Upper	Lower	Upper	Lower
a (km)	25000	27000	24000	28000	20000	30000
e	0.65	0.75	0.6	0.8	0.3	0.9
i (degree)	50	70	20	80	3	88
Ω (degree)	30	60	20	170	10	350
ω (degree)	250	290	200	350	10	350
υ (degree)	20	50	10	100	5	350

Table 5. Comparison of results for different bounds

	Case I	Case II	Case III
a (km)	26633.75218	26633.74957	26633.74950
e	0.730755	0.730754	0.730754
i (degree)	63.4368	63.4368	63.4368
Ω (degree)	44.98326	44.98326	44.98325
ω (degree)	269.9794	269.9794	269.9794
υ (degree)	34.4722	34.4722	34.4722
Δr (km)	8.2596E-02	8.2623E-02	8.2623E-02
Δv (km/s)	6.6487E-04	6.6486E-04	6.6486E-04
Computation time (sec)	241	294	1322

For different seeds, the variation of the results is shown below in Table 6. The bounds used for this case are case II bounds.

Table 6. Comparison of results for different seeds for case II bounds

	Seed 1	Seed 2	Seed 3
a (km)	26633.75119	26633.74916	26633.74901
e	0.730754	0.730754	0.730754
i (degree)	63.4368	63.4368	63.4368
Ω (degree)	44.98326	44.98326	44.98326
ω (degree)	269.9794	269.9794	269.9794
υ (degree)	34.47222	34.47222	34.47222
Δr (km)	8.2592E-02	8.2627E-02	8.2610E-02
Δv (km/s)	6.6489E-04	6.6490E-04	6.6494E-04
Computation time (sec)	319	270	295

4.3. DIFFERENT NUMBER OF OBSERVATIONS

The accuracy of the determined orbit and computational time is greatly affected by the number of observations used for estimating the orbit. It can be observed from the Table 7 that the accuracy of the estimated orbit increases with number of observations at the expense of computational time. The improvement in the accuracy after particular number of observations is marginal. The suitable number of observations for this elliptical orbit is found to be 90.

4.4. IMPLICATION OF RANGE–RATE DATA

In differential evolution same level of accuracy can be reached without using the range-rate data as can be seen in Table 8. It is observed that for a particular number of observation data, the accuracy of the estimated orbit obtained without range-rate data is comparable to the accuracy of the estimated orbit obtained with range-rate data. Without the range-rate data it requires more computational time for the solution to converge against the computational time when range-rate data is used. Hence it is possible to achieve the same accuracy even if the range-rate data is not available. It can be noted that the accuracy of estimated orbit is improved by one order in DC and EKF after inclusion of range-rate data.

Table 7. Accuracy of orbit for different number of observations.

No. of Observations	Δr (km)	Δv (km/s)	Computation
30	1.20516	1.51E-02	105
50	0.8737	7.68E-03	193
70	0.30799	1.38E-03	230
90	8.58E-02	6.57E-04	339
150	0.3493	5.85E-04	554
170	0.3578	5.48E-04	614

Table 8. Performance of DE with and without range-rate data.

	Observations	40	60	80	100
DE With Range-rate	Δr (km)	0.80768	0.64797	8.26E-02	0.27329
	Δv (km/s)	8.34E-03	3.99E-03	6.65E-04	7.07E-04
	Comp. time(sec)	170	204	270	351
DE Without Range-rate	Δr (km)	0.78991	0.23245	0.15412	0.21644
	Δv (km/s)	7.73E-03	1.78E-03	6.87E-04	4.57E-04
	Comp. time(sec)	573	588	682	908

4.5. COMPARISON BETWEEN DIFFERENT METHODS FOR DIFFERENT ORBITS

A comparison is made between results obtained using Differential correction, Extended Kalman Filter and Differential Evolution for circular, elliptic and hyperbolic orbit in Table 9.

Table 9. Performance of DC and EKF with different initial guesses and DE for different Orbits.

Types of Orbits	Methods	DC	EKF	DE
				CaseII bounds
Circular orbit	Δr (km)	0.11308	0.08290	0.12434
	Δv (km/s)	2.49E-04	4.05E-04	5.64E-04
	Comp. Time	265	2	198
Elliptic Orbit	Δr (km)	9.09E-02	0.25339	8.26E-02
	Δv (km/s)	4.59E-04	5.44E-04	6.65E-04
	Comp. Time	177	2	270
Hyperbolic Orbit	Δr (km)	0.1725	0.3890	0.3166
	Δv (km/s)	1.12E-03	1.03E-03	1.54E-03
	Comp. Time	91	1	155

Case II Bounds: The bounds for elliptic orbit is as mentioned in table 4. For circular and hyperbolic orbit the bounds are selected according to the strategy discussed in section 3.
Number of observation used here for circular, elliptic and hyperbolic orbit are 60, 80 and 40 respectively.

It is observed that if the initial guess of state is not good enough EKF requires more number of observations and DC requires longer time to obtain accurate results. Though EKF computes faster, it gives the solution state vector at time tn rather than at time t0 like in DE and DC. The accuracy of solution obtained from DE is comparable with the solutions of DC and EKF for all orbits.

5 CONCLUSIONS

The successful implementation of Differential Evolution algorithm for regular orbit determination was tested with uncorrupted data which resulted in the position and velocity accuracy of order 10-8 to 10-10. The accuracy achieved using Differential Evolution with noisy data is comparable to the accuracy obtained using conventional methods like Differential Correction and Extended Kalman Filter. A wide range of bounds on orbital elements is sufficient to initiate the computation process for DE and this technique is uniformly valid for all types of orbits. Results obtained using DE technique always results in globally optimal solution irrespective of bounds selected or seeds used for given number of observations. The accuracy of estimated orbit improves with number of observations but after particular number of observations, the improvement in accuracy is marginal. Same level of accuracy can be achieved using DE technique even without range-rate data but at the expense of computation time. This is beneficial when sensors for obtaining range-rate is not available at the tracking station.

References

[1] Escobal, P. R., "Methods of Orbit Determination", 1st ed.,Wiley, New York, 1965, Chaps. 6, 7, 8.

[2] Vallado, David A. 1997. "Fundamentals of Astrodynamics and Applications". New York: McGraw-Hill.

[3] Tapley, B., Schutz B., Born, G., "Statistical Orbit Determination", Elsevier Inc, 2004.

[4] Montenbruck, O., and Gill, E., "Satellite Orbits. Models, Methods, Applications", 1st ed., Springer, Berlin, 2000.

[5] David Hobbs and Preben Bohn, "Precise Orbit Determination for Low Earth Orbit Satellites",Terma A/S, Space, Vasekaer 12, 2730 Herlev, Denmark.

[6] K. Price, R. Storn, J. Lampinen, "Differential Evolution-A practical Approach to Global Optimization", Springer, 2005, Germany.

[7] Pranav Nath, R.V. Ramanan, "Precise halo orbit design and optimal transfer to halo orbits from earth using differential evolution", Elsevier, Article in Advances in Space Research 57(2016) 202 - 217, DOI:0.1016/j.asr.2015.10.03

[8] Olds, Aaron D., Kluever, Craig A., Cupples, Michael L., 2007. Interplanetary mission design using differential evolution. J. Spacecraft Rockets 44 (5), 1060-1070.

[9] Rainer S., Kenneth P." Differential Evolution - A Simple and Efficient Heuristic for Global Optimization over Continuous Spaces", Journal of Global Optimization 11: 341-359, 1997.