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Estimation of Hardness of Indentation Made with a Conical Indenter Using Numerical Slip-Line Field Technique

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Abstract

When a rigid wedge is indented in to a semi-infinite block, the material is bulged up around the wedge that is generally called lip. The previous works in this filed considered the outer profile of the lip to be linear. But, present authors observed both experimentally and with the aid of finite element analysis that the profile of the lip is not always linear, and it depends on the angle of the wedge and friction parameters. So, in this work, attempts have been made to calculate hardness of indentation for different wedge angles and friction parameters. As hardness is intrinsic property of material, consideration of either linear or parabolic lip will not be affected much. A comparative study of hardness for linear and parabolic free surface profiles of the piled up material around the cone is analyzed in this work.

Key Words: Hardness, Indentation, Conical indenter, Numerical slip-line field, Lip Profile

1. Introduction

Hardness of a material is its resistance to plastic deformation. Hence, it is intrinsic property of a matter for a set of particular conditions viz. angle of the indenter, radius of the indenter, and the frictional prosperities of mating surfaces and also sometimes, according to some researchers, it also depends on the loading rate. Hence, for a specific set of conditions, the hardness should bear the same value irrespective of the bulged-up lip around the indenter in linear or parabolic lip formation.

Some research works have been conducted in the field of material hardness. The pioneering work of Tabor, 1970 [1] is important as it discussed several aspects of material hardness under different conditions. The work of Yu and Blanchard in 1996 focused [2] on the development of analytical models for hardness. They considered conical, spherical, wedge shaped and cylindrical shaped indenters. In the work by Cheng and Li in 2000, the relationship between cone angle and hardness was found out [3] by FEM technique. The results obtained were found to have good matching with the classical slip-line field theory. The work of Mata and Alcalá [4] examined the frictional effect on sharp indenter of strain hardened solids by FEM technique. The work was correlated with the classical slip-line field theory. Hardness was expressed as a function of the lip profile of the material. The problem was mathematically formulated and solved by FEM technique. Mukhopadhyay et al. [5] used micro and nano-indentation techniques for evaluation of material properties in their investigation. They introduced the existing models available in this field. They concluded that the hardness obtained by micro

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and nano-indentation techniques had a large mismatch. They also inferred that the issue of mechanical deformation using indentation techniques in new materials is worth pursuing for resolving many unsolved problems. Poon et al. analysed [6] nano-indentation in elasto-plastic solids. Mathematical procedure for finding hardness in nano-indentation and finite element analysis of the problem were carried out. For different values of elastic modulus, Poisson's ratio and friction coefficient, different values of hardness have been given in tabular form. Alcalá and Ojos in 2011 depicted [7] the spherical micro-indentation experiments in FCC single crystal. They compared their work with Tabor's hardness index. Finite element modeling was also performed. They concluded that Peirce, Asaro and Needleman (PAN) model was violated in fully-plastic regimes and in their work, Tabor's model rules were strengthened. Ghosh et al. investigated [8] hardness-strength relationship for cubic-corner indentation under quasi-static and dynamic testing regimes. The theoretical proportionality constant value for the cube-corner tip, 1.81, has been formulated based on the two-dimensional wedge and slip-line models and a cone equivalent angle of 42°. The value was verified experimentally. In a recent work in the year of 2017, Hamada et al. reexamined [9] the relation between strength and hardness of a material by numerical analysis. Finite element studies were carried out and compared with the classical slip-line model and good results were obtained.

The present work considers the results taken from slip-lines network generated for rigid wedge indentation to a semi-infinite ductile plate. Originally, this problem was initiated by Hill et al. way back in 1947 with the aid of slip-line field theory [10]. The present authors also consider the slip-line field network formulation for this problem. The indenting force is calculated from slip-line field theory for some depth of indentation. It is told earlier that previous investigations in this field considered lip around the wedge as linear. But, the earlier works of the present authors predicted that the lip is not always linear and the detail of experimental, Finite element and numerical slip-line field network were discussed in those works [11][12][13][14][15][19]. The investigations of present authors in this field revealed that profile of lip depends on several other factors.

2. Problem Formulation

The indenting force has been calculated here with the aid of slip-line field theory. Slip line field is composed of two orthogonal lines called α-lines and β-lines. Any plastic deformation problem, in general problems on plane strain, can be solved with the help of this slip line field theory. The compressive stress in the plastically deformed material, the velocities of flow and the total force required for plastic deformation can be determined with help of this theory. This theory has immense applications in metal forming processes.

The present work deals with the slip line field associated with the micro-indentation of a cone to a semi-infinite block. The description of slip-line field has been shown in Fig. 1. The slip line field has three zones I, II and III. Zone I is DEFD, zone II is GDFG, called centre fan zone and zone III is DOGD. The numerical development of these zones with different boundary conditions has been discussed in detail by Chakraborty [16]. Now, if the semi-apex of the cone is less than 45°, the periphery of the plastic line EFGO meets at O. Otherwise, the line EFG meets at A. AD is a line 45° to the axis of the cone. This variation has been discussed by Haddow et al. [17]. Most of the previous works considered that the line DE is linear. But, the work of Biswas et al. [11][12][13][14][15][19], considering the profile of DE as parabolic found good agreement with the experimental result of Mulhern [18].

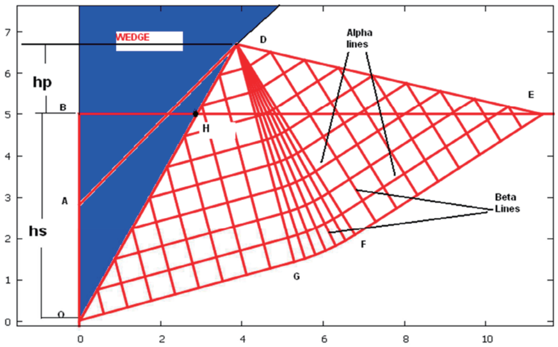

Fig. 1 Description of slip-lines field for micro-indentation of a cone to a semi-infinite metal plate

The present work deals with the comparison of hardness values for different cone angles of indenters and co-efficient of friction to establish the parabolic model of the lip DE as shown in Fig. 1. In Fig. 1, BHE is the undeformed original surface of the metal. Now, as cone indents, the material below the cone is pushed upward and it is piled up above the original surface. The cone indents to a depth h_s and the height of the lip is h_p and thus total height $h = h_s + h_p$. Hence, (h/h_s) is a factor greater than unity. Mata et al. [4] defines hardness for this type of indentation problems as $H = {}^{P}/_{A}$, where P is the indenting normal load and A is constant defined by, $A = fh^2 = \alpha f h_s^2$, where $\sqrt{\alpha} = (h/h_s) = (a/a_s)$ in which a is the area of the cone at a section through D and a_s is the area of the cone at a section through BH. Therefore, $\sqrt{\alpha} \geq 0$ always indicates pile up of material. f is a constant generally taken as 24.5. The present work considers the shear yield stress value as 150MPa.

3. Results and Discussions

The detail expression of the indenting force, P is available in the work of Biswas et al. [13][19]. The hardness, H is found out by the method described in Problem Formulation Section of this paper. Fig. 2 shows the variation of force for increasing semi-apex angles of wedge. Two cases are described here, first, the force for smooth surfaces i.e. considering coefficient of friction, $\mu = 0.0$, and secondly, for surfaces with $\mu = 0.2$. The value of μ in second case is not always exactly equal to 0.2. As this μ value is dependent on another angle λ which is nothing but ODG (as described in Fig. 1). The exact values of μ for different values of λ can be found the work of Biswas [19]. As it is seen form Fig. 2 that there is an increase of value of force with increment of semi-apex angle of wedge for smooth surfaces. The increase is quite sharp after 45°. Again, below 45°, the indenting force for $\mu = 0.2$ is less than that for $\mu = 0.0$. But, at a semi apex angle beyond 45°, the indenting force is higher for $\mu = 0.2$ than that for $\mu = 0.0$. Previous works and the work of the present authors suggested that at high value of semi-apex angle, friction reduces to nearly equal to zero. That is why there are no values of forces for semi-apex angle 70° and 80° at $\mu = 0.2$. Once the force is found out with the application of slip-line field theory, the hardness can be determined.

As it is stated earlier, the novelty in this work is consideration of parabolic Lip profile. Fig. 3 is showing the comparison of hardness for parabolic and straight lips. Again, beyond 45° values of semi-apex angle of the wedge, the work of Haddow [17] prevails. So after 45°, two cases of straight and parabolic lips are described considering Haddow's Approximation.

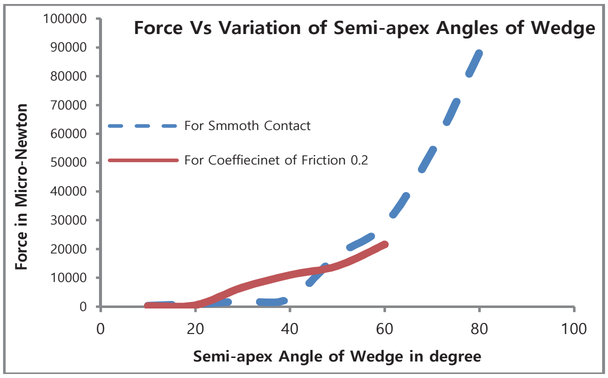

Fig. 2 Variation of Force Vs Semi-apex angle of Wedge for 0.15μm Indentation

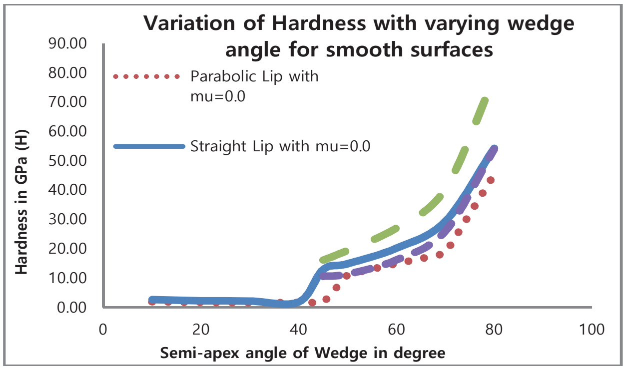

Fig. 3 Variation of Hardness for different semi-apex angles of the wedge

Fig. 3 shows that the hardness values are very close to each other for parabolic and straight lip up to 40° value of Semi-apex angle. For semi-apex angle beyond 40°, the hardness values for straight lip are more than that for parabolic one. Again considering Haddow's work, the hardness values for parabolic lip are less than those for straight lip. So, from the work of Biswas et al. [11][12][13][14][15][19], it can be stated that consideration of parabolic lip can be the substitute for the straight lip as the hardness values are not affected much by considering parabolic lip. Again, consideration of parabolic lip with Haddow's approximation work gives better fit with old straight lip approach.

Fig. 4 represents the variations of f_1 values with increasing semi-apex angle of the wedge. Here, $f_1 = \sqrt{\alpha} - 1$, the values of α is described in Problem Formulation section.

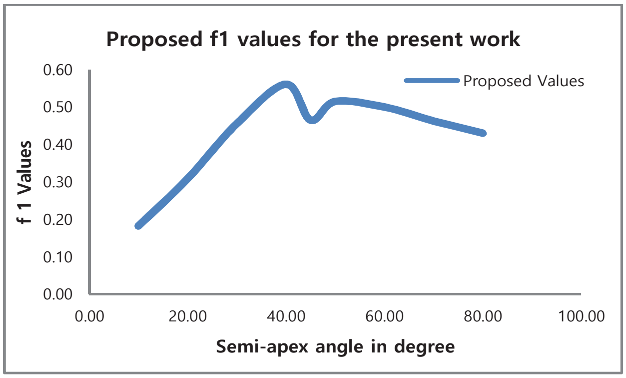

Fig. 4 Proposed Variation of f_1-values for different semi-apex angles of wedge with parabolic lip

Apparently, it seems that this figure may not have any relevance to the work of hardness. But, f_1 is an important parameter for slip-line field. As the generation of slip line network is completely dependent on the trail values of f_1. Proper choice of f_1 actually gives the convergence of the numerical program of slip-line field. This can well be understood by analyzing works of Biswas et al. [11][12[13[14][15][19]. Actually, there are four models of Biswas et al. The first two models are the slip-line field for straight and parabolic lips up to semi-apex angle of wedge of 80°. The other two, the slip-line field for straight and parabolic lips considering Haddow's Approximation from 45° to 80°. For these four models, several values of f_1 are generated. Now, with careful observation of hardness values, an attempt has been made here to predict a unique set of values of f_1 for the whole domain from 10° to 80°. That set of values of f_1 has been depicted in Fig 4.

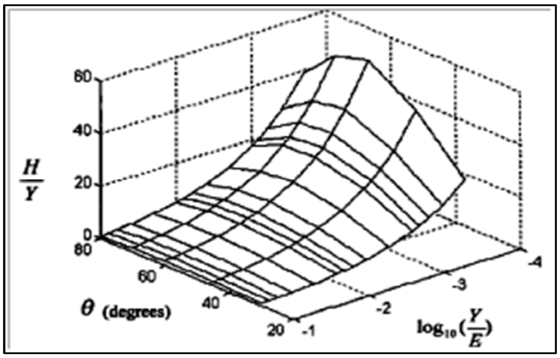

Fig. 5 Plot of H/Y with semi-apex angle after Cheng and Li, 2000 [3]

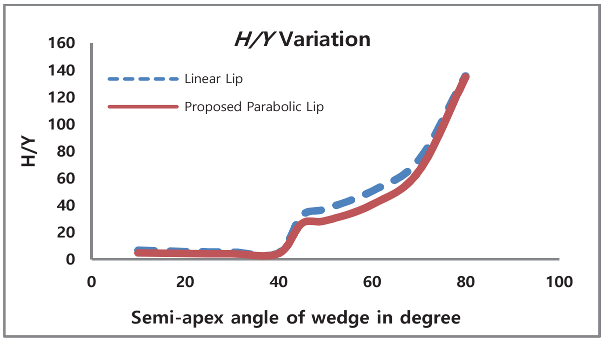

Fig. 6 Plot of variation of H/Y with semi-apex angle for the present work

Fig. 5 shows the variation of (H/Y) for different semi-apex angle of the wedge from the work of Cheng and Li, 2000 [3]. Here, Y is the yield strength value of the material. For, the present work Y value is taken as 300MPa. Fig. 6 also shows the variation of (H/Y) versus increasing value of semi-apex angle of the wedge for the present work. From, Fig. 5 and Fig. 6, it is observed that the variation of the hardness follows the same nature.

4. Conclusions

Hardness is an intrinsic property of material. This is unaffected by type of deformation of the metals. The previous work considered straight profile of the lip for wedge indentation problems. This is unfavourable one. Present work considers parabolic lip profile. The results obtained in the present work show that the hardness values are quite close to each other whether straight or parabolic profile of lip is taken. Again, experimental and Finite Elements results obtained by the present authors showed that the lip profile is parabolic. Therefore, the straight lip can be superseded by the parabolic lip. Hence, hardness results presented in this work validate that favorability of the parabolic-lip model over straight-lip model as previously predicted by the present authors.

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