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A NOTE ON INEXTENSIBLE FLOWS OF CURVES WITH FERMI-WALKER DERIVATIVE IN GALILEAN SPACE G_3

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Abstract. In this paper, Fermi-Walker derivative for inextensible flows of curves are researched in 3-dimensional Galilean space G_3. Firstly using Frenet and Darboux frame with the help of Fermi-Walker derivative a new approach for these flows are expressed, then some results are obtained for these flows to be Fermi-Walker transported in G_3.

1. Introduction

The flow of a curve is called inextensible if its arc length is preserved. Physically, the inextensible curve flows give rise to motions in which no strain energy is induced. The flows of inextensible curve and surface are used to solve many problems in computer vision [8, 14], computer animation [3] and even structural mechanics [20]. The methods used in the present study are developed by Gage and Hamilton [5] and Grayson [6]. A general formulation for inextensible flows of curves and developable surfaces in $\mathbb{R}^3$ is revealed by Kwon in [12]. After that many studies have been done on this subject, such that inextensible flows of curves are investigated in Minkowskian 3-space by [13]. On the other hand inextensible flows of curves in the 3-dimensional Galilean space G_3 and in the 4-dimensional Galilean space G_4 are studied in [15] and [16], respectively.
There are different transport laws for a tensor along a curve. One of them is Fermi-Walker's law. The Fermi-Walker transport of the tensor along a curve is determined as the law that makes the Fermi-Walker

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derivative along the curve be zero [2]. Construction of Fermi-Walker transported frames is expressed in [4, 10]. According to this frame a new characterization for inextensible flows are given by [1, 11].
In the present study, inextensible flows of curves according to Fermi-Walker derivative are examined in the 3-dimensional Galilean Space G_3. Besides, new characterization in terms of inextensible flows of curves according to this frame are revealed in G_3. Correspondingly, some results are obtained for these flows of curves to be Fermi-Walker transported in G_3.

2. Preliminaries

The Galilean space is a Cayley-Klein space equipped with the projective metric of signature (0,0,+,+). In [17] for the vectors $v_1 = (x_1, y_1, z_1)$ and $v_2 = (x_2, y_2, z_2)$ the Galilean scalar product is defined by

$$\langle v_1, v_2 \rangle = \begin{cases} x_1 x_2 & , if \;\; x_1 \neq 0 \;\; \vee \;\; x_2 \neq 0 \\ y_1 y_2 + z_1 z_2 & , if \;\; x_1 = 0 \;\; \wedge \;\; x_2 = 0 \end{cases} \tag{1}$$

and for the vector $v = (x, y, z)$ the Galilean norm is determined by

$$\|v\| = \begin{cases} |x| & , \; if \;\; x \neq 0 \\ \sqrt{y^2 + z^2} & , \; if \;\; x = 0 \end{cases} . \tag{2}$$

A vector $v = (v_1, v_2, v_3)$ is called non-isotropic vector if the first component of a vector is not zero, that is $v_1 \neq 0$ otherwise i.e., $v_1 = 0$ it is called isotropic vector. If a curve C of the class $C^r (r \geq 3)$ is given by the parametrization

$$r = r\left(x, y\left(x\right), z\left(x\right)\right) \tag{3}$$

where x is a Galilean invariant the arc length on C. Then, the curvature and torsion of this curve are given by

$$\kappa\left(x\right) = \sqrt{y''^2 + z''^2} \text{ and } \tau\left(x\right) = \frac{1}{\kappa^2\left(x\right)} \det\left(r'\left(x\right), r''\left(x\right), r'''\left(x\right)\right). \tag{4}$$

In Galilean 3-space the orthonormal trihedron is expressed by

$$\begin{aligned} T\left(x\right) &= \left(1, y'\left(x\right), z'\left(x\right)\right) \\ N\left(x\right) &= \frac{1}{\kappa\left(x\right)}\left(0, y''\left(x\right), z''\left(x\right)\right) \\ B\left(x\right) &= \frac{1}{\kappa\left(x\right)}\left(0, -z''\left(x\right), y''\left(x\right)\right). \end{aligned} \tag{5}$$

Also, in this space the following Frenet formulas hold:

$$
\begin{aligned}
T'(x) &= \kappa(x)\,N(x) \\
N'(x) &= \tau(x)\,B(x) \\
B'(x) &= -\tau(x)\,N(x)
\end{aligned} \tag{6}
$$

where T, N, B are called the vectors of tangent, principal normal and binormal, respectively [9]. For a Darboux frame in G_3, the following definition can be given,

Definition 2.1. *Let S be a surface in G_3 and α be a curve on S. If T denotes the unit tangent vector to α, n denotes the unit normal vector of S at the point $\alpha(x)$ of α, and $Q = n \times T$ denotes the tangential normal. Then $\{T, Q\}$ is the basis for the vectors tangent to S at $\alpha(x)$ and $\{T, Q, n\}$ is the orthonormal basis for all vectors at $\alpha(x)$ in G_3. Thus there is a new frame at every point of a curve on a surface S in G_3 is constructed other than the Frenet-Serret frame. This frame is called Galilean Darboux frame or tangent-normal frame* [18].

Theorem 2.2. *Let $\alpha : I \subset \mathbb{R} \longrightarrow M \subset G_3$ be a unit speed curve and let $\{T, Q, n\}$ be the Darboux frame field of α with respect to M,then*

$$
\begin{bmatrix} T \\ Q \\ n \end{bmatrix}' = \begin{bmatrix} 0 & \kappa_g & \kappa_n \\ 0 & 0 & \tau_g \\ 0 & -\tau_g & 0 \end{bmatrix} \begin{bmatrix} T \\ Q \\ n \end{bmatrix}. \tag{7}
$$

where κ_g, κ_n and τ_g are geodesic curvature, normal curvature and geodesic torsion, respectively [18].

Definition 2.3. *Suppose that X be any vector field and α is any unit speed curve in G_3, then*

$$
\widetilde{\nabla}_T X = \nabla_T X - \langle T, X \rangle A + \langle A, X \rangle T \tag{8}
$$

defined as Fermi-Walker derivative of the vector field X in G_3. Where T is the unit tangent vector of α and $A = \nabla_T T$ [19].

Definition 2.4. *Suppose that X be any vector field and α is any unit speed curve in G_3, if the Fermi-Walker derivative of the vector field X in G_3 is vanishes, i.e., $\widetilde{\nabla}_T X = 0$, then X is called Fermi-Walker transported vector field along the curve* [19].

In [19] using definition 2.3 and considering the equation (1) the following lemmas are given.

Lemma 2.5. *Let $\alpha : I \subset \mathbb{R} \longrightarrow G_3$ be a curve in Galilean 3-space, X is any vector field along the curve $\alpha(x)$, then Fermi-Walker derivative with respect to the Frenet frame can be expressed as*

- *If X is an isotropic vector field along the curve $\alpha(x)$, then Fermi-Walker derivative of X is determined by*

$$\widetilde{\nabla}_T X = \nabla_T X + \kappa \langle N, X \rangle T, \tag{9}$$

- *If X is a non-isotropic vector field along the curve $\alpha(x)$, then Fermi-Walker derivative of X is determined by*

$$\widetilde{\nabla}_T X = \nabla_T X - \kappa \langle T, X \rangle N. \tag{10}$$

Lemma 2.6. *Let $\alpha : I \subset \mathbb{R} \longrightarrow G_3$ be a curve in Galilean 3-space, X is any vector field along the curve $\alpha(x)$, then Fermi-Walker derivative in view of Darboux frame is determined as*

- *If X is an isotropic vector field along the curve $\alpha(x)$, then Fermi-Walker derivative of X is defined by*

$$\widetilde{\nabla}_T X = \nabla_T X + (\kappa_g \langle Q, X \rangle + \kappa_n \langle n, X \rangle) T, \tag{11}$$

- *If X is a non-isotropic vector field along the curve $\alpha(x)$, then Fermi-Walker derivative of X is defined by*

$$\widetilde{\nabla}_T X = \nabla_T X - (\kappa_g Q + \kappa_n n) \langle T, X \rangle . \tag{12}$$

3. Fermi-Walker Derivative for Inextensible Flows of Curves in Galilean Space G_3

Let $\gamma : I \subset \mathbb{R} \longrightarrow G_3$ be a curve in G_3 and V be a vector field along the curve γ. In this study, the same method in [11, 1] is used to construct the Fermi-Walker derivative for inextensible flows of curves according to Frenet and Darboux frame, respectively, in G_3.

Throughout this paper, we assume that $\gamma(u,t)$ is a one parameter family of smooth curves in 3-dimensional Galilean space G_3. The arc length of γ is given by

$$s(u) = \int_0^u \left| \frac{\partial \gamma}{\partial u} \right| du \,,$$

where

$$\left| \frac{\partial \gamma}{\partial u} \right| = \left| \left\langle \frac{\partial \gamma}{\partial u}, \frac{\partial \gamma}{\partial u} \right\rangle \right|^{\frac{1}{2}} .$$

The operator $\frac{\partial}{\partial s}$ is given in terms of u by

$$\frac{\partial}{\partial s} = \frac{1}{v}\frac{\partial}{\partial u},$$

where $v = \left|\frac{\partial \gamma}{\partial u}\right|$ and the arc length parameter is $ds = vdu$.

Definition 3.1. *If γ is a one parameter family of smooth curves in Galilean space G_3, then any flow of γ can be represented as*

$$\frac{\partial \gamma}{\partial t} = fT + gN + hB \tag{13}$$

where $\{T, N, B\}$ is Frenet frame in G_3.

Moreover, for inextensible flows of curves in the 3-dimensional Galilean space, the following theorem is hold [15].

Theorem 3.2. *Let $\frac{\partial \gamma}{\partial t} = fT + gN + hB$ be a smooth flow of the curve γ in G_3. Then the flow is inextensible if and only if f is constant, that is,*

$$\frac{\partial f}{\partial s} = 0. \tag{14}$$

Furthermore, for inextensible flows of curves in the 3-dimensional Galilean space, the following equations are hold.

Theorem 3.3. *Let $\frac{\partial \gamma}{\partial t} = fT + gN + hB$ be a smooth flow of the curve γ in G_3. Then,*

$$\begin{aligned} \frac{\partial T}{\partial t} &= \left(\frac{\partial g}{\partial s} + f\kappa - h\tau\right) N + \left(\frac{\partial h}{\partial s} + g\tau\right) B \\ \frac{\partial N}{\partial t} &= \left(-\frac{\partial g}{\partial s} - f\kappa + h\tau\right) T \\ \frac{\partial B}{\partial t} &= \left(-\frac{\partial h}{\partial s} - g\tau\right) T \end{aligned} \tag{15}$$

Proof. If γ be a curve flow then it can be written that

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial t}\frac{\partial \gamma}{\partial s} = \frac{\partial}{\partial s}\left(fT + gN + hB\right)$$

Thus, it is seen that

$$\frac{\partial T}{\partial t} = \frac{\partial f}{\partial s} T + \left(\frac{\partial g}{\partial s} + f\kappa - h\tau\right) N + \left(\frac{\partial h}{\partial s} + g\tau\right) B \tag{16}$$

On the other hand substituting theorem 3.2 in (16), it is obtained that

$$\frac{\partial T}{\partial t}=\left(\frac{\partial g}{\partial s}+f\kappa-h\tau\right)N+\left(\frac{\partial h}{\partial s}+g\tau\right)B.$$

The differentiation of the Frenet frame with respect to t is as follows:

$$\begin{aligned}0&=\frac{\partial}{\partial t}\langle T,N\rangle=\left(\frac{\partial g}{\partial s}+f\kappa-h\tau\right)+\left\langle T,\frac{\partial N}{\partial t}\right\rangle,\\0&=\frac{\partial}{\partial t}\langle T,B\rangle=\left(\frac{\partial h}{\partial s}+g\tau\right)+\left\langle T,\frac{\partial B}{\partial t}\right\rangle,\\0&=\frac{\partial}{\partial t}\langle B,N\rangle=\left\langle\frac{\partial B}{\partial t},N\right\rangle+\left\langle B,\frac{\partial N}{\partial t}\right\rangle\end{aligned}$$

Using the above equation, Galilean inner product and the following statement

$$\left\langle\frac{\partial B}{\partial t},B\right\rangle=\left\langle\frac{\partial N}{\partial t},N\right\rangle=0,$$

it is concluded that

$$\begin{aligned}\frac{\partial N}{\partial t}&=\left(-\frac{\partial g}{\partial s}-f\kappa+h\tau\right)T\\\frac{\partial B}{\partial t}&=\left(-\frac{\partial h}{\partial s}-g\tau\right)T\end{aligned}$$

□

For the Fermi-Walker derivative of inextensible flows in G_3, the following theorem can be obtained.

Theorem 3.4. *The Fermi-Walker derivatives of the* $\frac{\partial T}{\partial t},\frac{\partial N}{\partial t}$ *and* $\frac{\partial N}{\partial t}$ *vector fields as follows*

$$\begin{aligned}\widetilde{\nabla}_T\frac{\partial T}{\partial t}&=\kappa\left(\frac{\partial g}{\partial s}+f\kappa-h\tau\right)T\\&+\left[\frac{\partial}{\partial s}\left(\frac{\partial g}{\partial s}+f\kappa-h\tau\right)-\tau\left(\frac{\partial h}{\partial s}+g\tau\right)\right]N\\&+\left[\frac{\partial}{\partial s}\left(\frac{\partial h}{\partial s}+g\tau\right)+\tau\left(\frac{\partial g}{\partial s}+f\kappa-h\tau\right)\right]B\end{aligned}\tag{17}$$

$$\widetilde{\nabla}_T\frac{\partial N}{\partial t}=\left[\frac{\partial}{\partial s}\left(-\frac{\partial g}{\partial s}-f\kappa+h\tau\right)\right]T\tag{18}$$

$$\widetilde{\nabla}_T\frac{\partial B}{\partial t}=\left[\frac{\partial}{\partial s}\left(-\frac{\partial h}{\partial s}-g\tau\right)\right]T\tag{19}$$

Proof. According to the theorem 3.3. it is seen that $\frac{\partial T}{\partial t}$ is isotropic. So using the equation (9), the following equation can be written

$$\begin{aligned}
\widetilde{\nabla}_T \frac{\partial T}{\partial t} &= \nabla_T \frac{\partial T}{\partial t} + \kappa \left\langle N, \frac{\partial T}{\partial t} \right\rangle T \\
&= \frac{\partial}{\partial s}\left[\left(\frac{\partial g}{\partial s} + f\kappa - h\tau\right) N + \left(\frac{\partial h}{\partial s} + g\tau\right) B\right] \\
&+ \kappa \left\langle N, \frac{\partial T}{\partial t} \right\rangle T.
\end{aligned}$$

In terms of the theorem 3.3. it is said that $\frac{\partial N}{\partial t}$ and $\frac{\partial B}{\partial t}$ is non-isotropic, then

$$\begin{aligned}
\widetilde{\nabla}_T \frac{\partial N}{\partial t} &= \nabla_T \frac{\partial N}{\partial t} - \kappa \left\langle T, \frac{\partial N}{\partial t} \right\rangle N \\
&= \frac{\partial}{\partial s}\left[\left(-\frac{\partial g}{\partial s} - f\kappa + h\tau\right) T\right] - \kappa \left\langle T, \frac{\partial N}{\partial t} \right\rangle N \\
&= \left[\frac{\partial}{\partial s}\left(-\frac{\partial g}{\partial s} - f\kappa + h\tau\right)\right] T\ , \\
\widetilde{\nabla}_T \frac{\partial B}{\partial t} &= \nabla_T \frac{\partial B}{\partial t} - \kappa \left\langle T, \frac{\partial B}{\partial t} \right\rangle N \\
&= \frac{\partial}{\partial s}\left[\left(-\frac{\partial h}{\partial s} - g\tau\right) T\right] - \kappa \left\langle T, \frac{\partial B}{\partial t} \right\rangle N \\
&= \left[\frac{\partial}{\partial s}\left(-\frac{\partial h}{\partial s} - g\tau\right)\right] T\ .
\end{aligned}$$

equations are hold. After the necessary calculations the proof is completed. □

Besides, for inextensible flows of curves in the 3-dimensional Galilean space according to Darboux frame the following equations are hold [7].

Theorem 3.5. *Let* $\frac{\partial \gamma}{\partial t} = f_1 T + f_2 Q + f_3 n$ *be a smooth flow of the curve* γ *in* G_3*. Then the flow is inextensible if and only if*

$$\frac{\partial f_1}{\partial s} = 0. \tag{20}$$

Moreover, for inextensible flows of curves in the 3-dimensional Galilean space according to Darboux frame the following equations are hold.

Theorem 3.6. *Let* $\frac{\partial \gamma}{\partial t} = f_1 T + f_2 Q + f_3 n$ *be a smooth flow of the curve* γ *in* G_3. *Then,*

$$
\begin{aligned}
\frac{\partial T}{\partial t} &= \left(\frac{\partial f_2}{\partial s} + f_1\kappa_g - f_3\tau_g\right) Q + \left(\frac{\partial f_3}{\partial s} + f_1\kappa_n + f_2\tau_g\right) n \\
\frac{\partial Q}{\partial t} &= \left(-\frac{\partial f_2}{\partial s} - f_1\kappa_g + f_3\tau_g\right) T \\
\frac{\partial n}{\partial t} &= \left(-\frac{\partial f_3}{\partial s} - f_1\kappa_n - f_2\tau_g\right) T
\end{aligned} \tag{21}
$$

where $\{T, Q, n\}$ *is Darboux frame in* G_3.

Proof. For any arbitrary flow γ it is known that

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial t}\frac{\partial \gamma}{\partial s} = \frac{\partial}{\partial s}\left(f_1 T + f_2 Q + f_3 n\right)$$

So, the following equation is hold,

$$\frac{\partial T}{\partial t} = \frac{\partial f_1}{\partial s} T + \left(\frac{\partial f_2}{\partial s} + f_1\kappa_g - f_3\tau_g\right) Q + \left(\frac{\partial f_3}{\partial s} + f_1\kappa_n + f_2\tau_g\right) n \tag{22}$$

On the other hand substituting theorem 3.5 in (22), it is obtained that

$$\frac{\partial T}{\partial t} = \left(\frac{\partial f_2}{\partial s} + f_1\kappa_g - f_3\tau_g\right) Q + \left(\frac{\partial f_3}{\partial s} + f_1\kappa_n + f_2\tau_g\right) n.$$

The differentiation of the Darboux frame with respect to t is as follows:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial t}\langle T, Q\rangle = \left(\frac{\partial f_2}{\partial s} + f_1\kappa_g - f_3\tau_g\right) + \left\langle T, \frac{\partial Q}{\partial t}\right\rangle, \\
0 &= \frac{\partial}{\partial t}\langle T, n\rangle = \left(\frac{\partial f_3}{\partial s} + f_1\kappa_n + f_2\tau_g\right) + \left\langle T, \frac{\partial n}{\partial t}\right\rangle, \\
0 &= \frac{\partial}{\partial t}\langle Q, n\rangle = \left\langle \frac{\partial Q}{\partial t}, n\right\rangle + \left\langle Q, \frac{\partial n}{\partial t}\right\rangle
\end{aligned}
$$

Using the above equation and the following statement

$$\left\langle \frac{\partial Q}{\partial t}, Q\right\rangle = \left\langle \frac{\partial n}{\partial t}, n\right\rangle = 0,$$

it is concluded that

$$
\begin{aligned}
\frac{\partial Q}{\partial t} &= \left(-\frac{\partial f_2}{\partial s} - f_1\kappa_g + f_3\tau_g\right) T\ , \\
\frac{\partial n}{\partial t} &= \left(-\frac{\partial f_3}{\partial s} - f_1\kappa_n - f_2\tau_g\right) T.
\end{aligned}
$$

□

In view of the theorem 3.6., for the Fermi-Walker derivative for inextensible flows in G_3 the following theorem can be obtained.

Theorem 3.7. *The Fermi-Walker derivatives of the* $\frac{\partial T}{\partial t}$, $\frac{\partial Q}{\partial t}$ *and* $\frac{\partial n}{\partial t}$ *are as follows,*

$$\begin{aligned} \widetilde{\nabla}_T \frac{\partial T}{\partial t} &= \left[\kappa_g\left(\frac{\partial f_2}{\partial s}+f_1\kappa_g-f_3\tau_g\right)+\kappa_n\left(\frac{\partial f_3}{\partial s}+f_1\kappa_n+f_2\tau_g\right)\right]T \\ &+\left[\frac{\partial}{\partial s}\left(\frac{\partial f_2}{\partial s}+f_1\kappa_g-f_3\tau_g\right)-\tau_g\left(\frac{\partial f_3}{\partial s}+f_1\kappa_n+f_2\tau_g\right)\right]Q \\ &+\left[\frac{\partial}{\partial s}\left(\frac{\partial f_3}{\partial s}+f_1\kappa_n+f_2\tau_g\right)+\tau_g\left(\frac{\partial f_2}{\partial s}+f_1\kappa_g-f_3\tau_g\right)\right]n \end{aligned} \tag{23}$$

(24)
$$\widetilde{\nabla}_T \frac{\partial Q}{\partial t} = \left[\frac{\partial}{\partial s}\left(-\frac{\partial f_2}{\partial s}-f_1\kappa_g+f_3\tau_g\right)\right]T$$

(25)
$$\widetilde{\nabla}_T \frac{\partial n}{\partial t} = \left[\frac{\partial}{\partial s}\left(-\frac{\partial f_3}{\partial s}-f_1\kappa_n-f_2\tau_g\right)\right]T$$

Proof. By using the theorem 3.6 and considering the equation (11),

$$\begin{aligned} \widetilde{\nabla}_T \frac{\partial T}{\partial t} &= \nabla_T \frac{\partial T}{\partial t} \\ &+ \left[\kappa_g\left\langle Q,\frac{\partial T}{\partial t}\right\rangle+\kappa_n\left\langle n,\frac{\partial T}{\partial t}\right\rangle\right]T \\ &= \frac{\partial}{\partial s}\left[\left(\frac{\partial f_2}{\partial s}+f_1\kappa_g-f_3\tau_g\right)Q+\tau_g\left(\frac{\partial f_3}{\partial s}+f_1\kappa_n+f_2\tau_g\right)n\right] \\ &+ \left[\kappa_g\left\langle Q,\frac{\partial T}{\partial t}\right\rangle+\kappa_n\left\langle n,\frac{\partial T}{\partial t}\right\rangle\right]T \end{aligned}$$

is obtained. After that using the equation (12) the following equations are hold:

$$\begin{aligned}
\widetilde{\nabla}_T \frac{\partial Q}{\partial t} &= \nabla_T \frac{\partial Q}{\partial t} - (\kappa_g Q + \kappa_n n)\left\langle T, \frac{\partial Q}{\partial t}\right\rangle \\
&= \frac{\partial}{\partial s}\left[\left(-\frac{\partial f_2}{\partial s} - f_1\kappa_g + f_3\tau_g\right)T\right] \\
&+ \kappa_g\left(-\frac{\partial f_2}{\partial s} - f_1\kappa_g + f_3\tau_g\right)Q + \kappa_n\left(-\frac{\partial f_2}{\partial s} - f_1\kappa_g + f_3\tau_g\right) \\
&- (\kappa_g Q + \kappa_n n)\left(-\frac{\partial f_2}{\partial s} - f_1\kappa_g + f_3\tau_g\right),
\end{aligned}$$

$$\begin{aligned}
\widetilde{\nabla}_T \frac{\partial n}{\partial t} &= \nabla_T \frac{\partial n}{\partial t} - (\kappa_g Q + \kappa_n n)\left\langle T, \frac{\partial n}{\partial t}\right\rangle \\
&= \frac{\partial}{\partial s}\left[\left(-\frac{\partial f_3}{\partial s} - f_1\kappa_n - f_2\tau_g\right)T\right] \\
&+ \kappa_g\left(-\frac{\partial f_3}{\partial s} - f_1\kappa_n - f_2\tau_g\right)Q + \kappa_n\left(-\frac{\partial f_3}{\partial s} - f_1\kappa_n - f_2\tau_g\right) \\
&- (\kappa_g Q + \kappa_n n)\left(-\frac{\partial f_3}{\partial s} - f_1\kappa_n - f_2\tau_g\right),
\end{aligned}$$

So, the proof is completed. □

Corollary 3.8. *In view of Theorem 3.4 it is seen that, if* $\frac{\partial T}{\partial t}, \frac{\partial N}{\partial t}, \frac{\partial B}{\partial t}$ *along the curve is parallel to the Fermi-Walker terms (Fermi-Walker transported), then the following equalities are hold,*

$$\begin{aligned}
\left(\frac{\partial g}{\partial s} + f\kappa - h\tau\right) &= 0 \\
\left(\frac{\partial h}{\partial s} + g\tau\right) &= 0
\end{aligned}$$

Corollary 3.9. *In view of Theorem 3.7 it is seen that, if* $\frac{\partial T}{\partial t}, \frac{\partial Q}{\partial t}, \frac{\partial n}{\partial t}$ *along the curve is parallel to the Fermi-Walker terms (Fermi-Walker transported), then the following equalities are hold,*

$$\left(\frac{\partial f_2}{\partial s} + f_1\kappa_g - f_3\tau_g\right) = 0$$
$$\left(\frac{\partial f_3}{\partial s} + f_1\kappa_n + f_2\tau_g\right) = 0$$

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