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Numerical Switching Performances of Cumulative Sum Chart for Dispersion Matrix

Duk-Joon Chang†

Abstract

In many cases, the quality of a product is determined by several correlated quality variables. Control charts have been used for a long time widely to control the production process and to quickly detect the assignable causes that may produce any deterioration in the quality of a product. Numerical switching performances of multivariate cumulative sum control chart for simultaneous monitoring all components in the dispersion matrix Σ under multivariate normal process $N_p(\underline{\mu}, \Sigma)$ are considered. Numerical performances were evaluated for various shifts of the values of variances and/or correlation coefficients in Σ. Our computational results show that if one wants to quick detect the small shifts in a process, CUSUM control chart with small reference value k is more efficient than large k in terms of average run length (ARL), average time to signal (ATS), average number of switches (ANSW).

Keywords : ARL, ANSW, LRT Statistic, Sequential Probability Ratio Test (SPRT)

1. Introduction

The main purpose of statistical process control (SPC) is to improve the quality of a product and the productivity of the production process. The quality of a product is generally determined by several correlated quality variables of the process, not by single quality variable. To improve and maintain the quality of a product, the variation of the production process must be continuously monitored, analyzed and improved by tracking the changes of the process environment. Whenever the process variation exceeds the boundaries of control limits of the process, we judge that the process change was occurred by some special or assignable causes. Therefore, quality engineers in the process wish to detect any departure from in control state as quickly as possible, and identify which attributes caused the deviation, and eliminate them. And it is general way to detect the special or assignable causes using the control chart procedure.

In traditional control chart procedure, the power of a control chart is determined by the following two properties; one is the length of time required for the chart to signal when the process parameters of the production process have changed, and the other is the false alarm rate when the process is in control state. That is, a process change occurs under in control state, the longer the required time to signal the better (so called, large the better characteristics, or the-larger-the-better characteristics), and the less the false alarm rate the better (so called, the smaller the better characteristics, or the-smaller-the-better characteristics).

The Shewhart chart, proposed first by Shewhart[1], is simple to apply and implement for monitoring the quality parameters of the production process. But it uses the information only the current sample information at any sampling time, and ignores all of the past sample informations. So, Shewhart control chart is known as being inefficient to detect small process changes. When the detection of the small changes of process parameter is important, the cumulative sum (CUSUM) control chart, proposed first by Page[2], is a good alternative to the

Department of statistics, Changwon National University, Changwon
†Corresponding author : djchang@changwon.ac.kr
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Shewhart chart.

Reynolds et al.[3] showed that in his study for controlling process mean μ, VSI CUSUM chart is more efficient than standard FSI CUSUM chart in terms of the average time to signal when the process has changed.

Vargas et al.[4] compared between the performances of the cumulative sum (CUSUM) charts and the exponentially weighted moving average (EWMA) charts. The EWMA control chart, presented by Lucas and Saccucci[5], is one of the charting methods aimed at correction of the deficiency of the Shewhart chart's insensitivity to small shifts.

Ryu and Wan[6] studied the optimal design of a CUSUM chart about a mean shift of unknown size. Champ and Woodall[7] presented a method using Markov chains for approximating the ARL of Shewhart charts with supplementary runs rules, and compared basic Shewhart chart and CUSUM chart which are monitoring mean μ. ARL of EWMA chart is studied by Crower[8], Robinson and Ho[9]. Jo and Cho[10] studied GLR control charts for the detection of a shift in mean vector and/or dispersion matrix.

2. Control Statistic for Dispersion Matrix

To control the process dispersion of a single quality variable, most quality engineers are recommended to use either R chart or S chart for moderate-to-large sample sizes. Meanwhile, some practitioners recommend to use S^2 control chart which is based on the sample variance S^2. For multivariate normal $N_p(\underline{\mu}, \Sigma)$ process, shifts in the components of target dispersion matrix Σ_0 for the correlated quality variables are often considere as bing important. Here we consider a multivariate control statistic for monitoring Σ under multivariate normal process. In this study, we assume that the p quality variables are jointly distributed as a p-variate normal $N_p(\underline{\mu}, \Sigma)$ process and that a random sample of size n is available from the multivariate normal process.

The dispersion matrix Σ of multivariate normal process is symmetric, and the parameters being controlled in the process are the p variance components σ_i $(i = 1, 2, \cdots, p)$ and the $p(p-1)/2$ correlation coefficients ρ_{ij} $(i = 2, 3, \cdots, p : i < j)$. Therefore, the number of process parameters to be monitored is $p(p+1)/2$ in total.

In this situation, the multivariate control chart is more efficient than the separate control chart, because multivariate control chart operates once while the separate control chart operates $p(p+1)/2$ times. Especially, when p quality variables are correlated, multivariate control procedure is better then the separate control chart in terms of easy operating and interpreting the process. Woodall and Ncube[11] proposed the multivariate CUSUM scheme based on p-variate normal distribution to monitoring mean vector $\underline{\mu}$ instead of existing the single multivariate CUSUM scheme.

The general multivariate statistical quality control procedure can be considered as a series of consecutive hypothesis tests where p quality variables $X_1, X_2, \cdots, X_p$ are observed at each sampling time i. The the joint distribution of p quality variables $\underline{X} = (X_1\, X_2\, \cdots\, X_p)'$ at each sampling time i is given by

$$f(\underline{X}) = \frac{1}{(2\pi)^{p/2}\sqrt{|\Sigma|}} \exp\left[-\frac{(\underline{X}-\underline{\mu})'\Sigma^{-1}(\underline{X}-\underline{\mu})}{2}\right], \quad (1)$$

where dispersion matrix Σ is positive definite matrix.

For consecutive random samples, control statistic for Σ can be derived from the repeated hypothesis tests about $H_0 : \Sigma = \Sigma_0$ against $H_1 : \Sigma \neq \Sigma_0$ where the process mean vector $\underline{\mu}$ is known. Theoretically, the likelihood ratio test (LRT) about $H_0 : \underline{\theta} \in \Omega$ against $H_1 : \underline{\theta} \in \Omega - \omega$ is conducted based on the likelihood ratio test statistic

$$\lambda = \left(sup_{\underline{\theta} \in \omega} f(\underline{X}, \underline{\theta})\right) / \left(sup_{\underline{\theta} \in \Omega} f(\underline{X}, \underline{\theta})\right).$$

If we take natural log to the likelihood ratio test statistic λ, then through some mathematical simplifying calculation, we can obtain the following control statistic for each random sample i

$$W_i = -pn + pn\ln n - n\ln(|A_i|/|\Sigma_0|) + tr(\Sigma_0^{-1} A_i) \quad (2)$$

where square matrix A_i denotes the sum of squares and cross-product matrix and tr is the trace operator. Note that $A_i = (n-1)S_i$, where S_i is the $p \times p$ sample variance-covariance matrix for the ith random sample. Thus the LRT statistic W_i in (2) for the ith sample can be used as the control statistic for monitoring Σ.

3. FSI and VSI Control Charts

It is general way to maintain the control chart using the fixed sampling time interval. In other words, the samples are selected by the equal time interval, and the control chart using the fixed sampling time is called the fixed sampling interval (FSI) chart. In FSI chart, the average run length (ARL) is an important criterion, and so ARL is frequently used to design the proposed control charts and compare their efficiencies. However, as mentioned before the efficiency of a proposed control chart is traditionally estimated by the way how quickly a control chart detects the changes in the process. Through many researches until now, it is known that when the quick detection of small shifts is important, the CUSUM chart or the EWMA chart are more efficient than the traditional Shewhart chart.

Meanwhile, if a chart statistic for a process parameter has values close to the control limit, then we can think that there can be a change in the process, and it will better to make the sampling interval shorter than FSI chart. On the other hand, if the chart statistic has values close to the center line (CL), then there can be no significant change in the process, and it will be efficient to make the sampling interval longer than FSI chart. There are many studies about the VSI procedure which adopts this intuitive idea. That is, VSI procedure selects samples with different time interval according to the values of control statistic. Therefore, there are many studies to make small ARL for the FSI control chart, and average time to signal (ATS) for the VSI control chart

Arnold[12] first investigated the problem of determining sampling plans with the VSI. Reynolds and Arnold[13] evaluated that when using two different sampling intervals d_1 and d_2 $(d_1 < d_2)$, it is optimal for the two sampling interval of the VSI chart to be as far apart as possible. Many studies about the VSI show that the ARL in the FSI and the ATS in VSI procedures are set to be equal under in control state, but the ATS in the VSI chart decreases more fast than the ARL in the FSI chart when the process is turned to be out of state. Therefore, the VSI chart is more efficient than the FSI chart in terms of the ATS when process are changed.

However, from the standpoint of quality engineers who are operating the VSI chart, the large number of switches between different sampling intervals are cumbersome to operate, especially when the process is in control state than when out of control state. Amin and Letsinger[14] studied and stated the problems of switches between different sampling intervals, and defined average number of switches (ANSW), the probability of switches for the different interval $\Pr(\text{switch})$, and so on, to estimate the properties of the VSI chart.

For the two sampling interval VSI chart, we use two different sampling intervals d_1 and d_2 $(d_1 < d_2)$ when the chart is started at time 0 and the first sampling interval $d_0 = 1$. The number of switches (NSW) is the number of switches from the start of the process until the chart gives a signal, and the average number of switches (ANSW) is the expected value of NSW, and $\Pr(\text{switch})$ is obtained as follows

$$ANSW = (ARL - d_0) \cdot \Pr(\text{switch}) \tag{3}$$

and

$$\Pr(\text{switch}) = \frac{\mathrm{N}(\mathrm{d}_2 \to \mathrm{d}_1) + \mathrm{N}(\mathrm{d}_1 \to \mathrm{d}_2)}{\mathrm{ARL} - 1} \tag{4}$$

where $N(d_2 \to d_1)$ is the number of switches that the sampling interval changes from d_2 to d_1 and $N(d_1 \to d_2)$ is the number switching from d_1 to d_2.

In this study, we evaluate $N(d_2 \to d_1)$, $N(d_1 \to d_2)$, ANSW and $\Pr(\text{switch})$ by simulation with 10,000 iteration.

4. Cumulative Sum Control Chart

The Cumulative Sum (CUSUM) control chart has received a lot of attention in quality literatures owing to its simplicity and efficiency for small or moderate shifts on the process. The CUSUM chart used for monitoring a process mean and/or variation are often called as the CUSUM location charts and/or CUSUM dispersion charts, respectively.

Since we can consider control procedures at each sampling occasion as a sequence of independent hypothesis tests for process parameter θ where each test is a sequential probability ratio test (SPRT) for testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$. Tests are applied sequentially until a test statistic larger than c (corresponding to reject H_0) which means that a signal is given. This sequence of SPRTs is equivalent to using the CUSUM statistic

$$\hat{Y}_j = \max\{\hat{Y}_{j-1} + (Z_j - k), 0\}, \quad (5)$$

where $Z_j = \ln\{f(\underline{X}_j \mid \theta_1)/f(\underline{X}_j \mid \theta_0)\}$. This FSI CUSUM chart signals whenever $\hat{Y}_j > c$.

Reynolds et al.[3]suggested the CUSUM statistic Y_j in (6), which is a modified CUSUM statistic $\hat{Y}_j$ in (5), to apply the VSI CUSUM chart as follows

$$Y_j = \max\{Y_{j-1}, 0\} + (Z_j - k) \quad (6)$$

where $Y_0 = \omega\ (\omega \geq 0)$ is a specified constant.

A multivariate CUSUM chart with dispersion matrix Σ based on the LRT control statistic W_i $(i = 1, 2, 3, \cdots)$ in (2) at the ith sampling time is given by

$$Y_{W,i} = \max\{Y_{W,i-1}, 0\} + (W_i - k), \quad (7)$$

where $Y_{W,0} = \omega_W \cdot I_{\omega_W \geq 0}$. This CUSUM chart signals whenever $Y_{W,i} > h_W$.

For the VSI multivariate CUSUM chart, suppose the sampling interval as follows

$$\begin{aligned} &d_1 \text{ is used when } Y_{W,i} \in (g_W, h_W], \\ &d_2 \text{ is used when } Y_{W,i} \in (-k_W, g_W], \end{aligned} \quad (8)$$

where $-k_W < g_W < h_W$ and $d_1 < d_2$.

Since it is difficult to obtain the exact distribution of the CUSUM chart in (8), the performances and properties of this multivariate VSI CUSUM chart can be evaluated by simulation when the production process is in control or out of control state. We assume that all variances and covariances are $V(X_i) = \sigma_1^2 = 1.0 \quad (i = 1, 2, \cdots, p)$ and $Cov(X_i, X_j) = \rho_{ij}\sigma_i\sigma_j = 0.3 \quad (i, j = 1, 2, \cdots, p, i \neq j)$ when the process is in control state. Note that since covariance and variance are fixed, if variances are changed then the correlation coefficients are also changed.

In order to obtain numerical performances of the proposed CUSUM chart based on the control statistic W_i in (2), we consider the following types of shifts of the components in Σ. That is, we consider the following four typical types of shifts for numerical comparison in the process parameters: σ_1^2 in Σ_0 is increased, ρ_{12} and ρ_{21} in Σ_0 are changed, both σ_1^2 and ρ_{12} are changed at the same time, and dispersion matrix Σ_0 under in control state is changed to $c\,\Sigma_0\ (c > 1)$.

5. Numerical Performances and Concluding Remarks

Suppose that a single quality variable is normally distributed $N(\mu, \sigma^2)$, and also that we are interested in the process mean μ. Then $\bar{x}$ chart is usually applied in practice for monitoring process mean μ. In 3σ $\bar{x}$ chart, which is a control chart with control limit of 3σ away from CL, the number of samples to signal is distributed as a geometric distribution and the ARL is given by $ARL = 1/\{\Pr(\bar{X} > 3\sigma_{\bar{x}})\}$. Therefore, the ARL under in control state is $1/\{\Pr(\text{signal is given})\} = 370.4$ in univariate $\bar{x}$ chart. In this case, when the fixed sampling time interval is $d = 1$ (hour) and the process is in control state, the ATS of FSI Shewhart $\bar{x}$ chart is $370.4 \times d = 370.4$(hour).

In order to evaluate and compare the numerical performances and switching behaviors of the proposed multivariate charts, in this article we set the fixed sampling interval $d = 1$ in FSI chart and $(d_1, d_2) = (0.1, 1.9)$ in two sampling intervals VSI chart. We also set the ARL and ATS of the proposed charts as 370.4 each when the process is in control state, and the sample size n for each variable as 5.

Once the reference values $k's$ in (6) of proposed multivariate CUSUM chart had been determined, the values of g_W and h_W in (8) were obtained through simulation work with 10,000 iteration. Numerical performances and switching properties of the proposed charts were also calculated through simulation under both in control and out of control state.

Table 1 through Table 8 show that numerical results of proposed charts when $p=3$ and the ATS under in control is 370.4 with different reference value k. We had also applied the proposed charts to several $p(\neq 3)$ to check if there are any differences in numerical performances, and we found the similar results as $p=3$ case.

If one is interested in the small shifts of a process, he/she can be recommended to use small k because the values of ARL, ATS, ANSW show a tendency to be smaller for the small reference values k than for large k. Also, $N(d_2 \to d_1)$ and $N(d_1 \to d_2)$ are small for small k.

And when correlation coefficient ρ_{ij} of two quality variables X_i and X_j is shifted, the proposed chart gives signals less sensitive (slowly) for the changes of the process comparing to when the variance components, both variances and correlation coefficients, and dispersion matrix Σ_0 are changed, respectively. If one wants to monitor only correlation coefficients between two quality variables in multivariate normal process, then he/she needs to search for any control charts reducing the ARL, ATS, and ANSW.

Table 1. Performances of CUSUM chart based on W_i when σ_1^2 in Σ_0 is changed. $(k=9.0)$

σ_1	ARL	ATS	ANSW	N(d2→d1)	N(d1→d2)	N(d2→d2)	N(d1→d1)	Pr(switch)
in-control	370.3	370.4	72.98	36.57	36.40	148.30	148.07	0.198
$\sigma_1=1.1$	302.2	289.5	58.87	29.52	29.35	114.21	128.17	0.195
$\sigma_1=1.3$	83.1	57.4	14.25	7.24	7.01	19.74	48.10	0.174
$\sigma_1=1.5$	24.5	12.3	3.92	2.12	1.80	3.17	16.46	0.166
$\sigma_1=1.7$	11.7	5.6	2.26	1.34	0.91	1.04	7.38	0.211
$\sigma_1=1.9$	7.2	3.5	1.71	1.12	0.59	0.44	4.03	0.277
$\sigma_1=2.1$	5.0	2.5	1.41	1.00	0.41	0.21	2.38	0.353

Table 2. Performances of CUSUM chart based on W_i when ρ_{12} in Σ_0 is changed. $(k=9.0)$

ρ_{12}	ARL	ATS	ANSW	N(d2→d1)	N(d1→d2)	N(d2→d2)	N(d1→d1)	Pr(switch)
in-control	370.3	370.4	72.98	36.57	36.40	148.30	148.07	0.198
$\rho_{12}=0.4$	321.6	311.7	62.89	31.53	31.36	123.46	134.23	0.196
$\rho_{12}=0.5$	209.1	182.1	39.35	19.77	19.58	69.46	99.31	0.189
$\rho_{12}=0.6$	103.5	72.7	17.55	8.88	8.66	25.48	59.43	0.171
$\rho_{12}=0.7$	43.1	22.4	6.28	3.28	3.00	6.56	29.28	0.149
$\rho_{12}=0.8$	18.5	7.6	2.66	1.52	1.14	1.53	13.36	0.151
$\rho_{12}=0.9$	8.4	3.0	1.53	1.06	0.46	0.23	5.68	0.205

Table 3. Performances of CUSUM chart based on W_i when (σ_1, ρ_{12}) in Σ_0 are changed. $(k=9.0)$

(σ_1, ρ_{12})	ARL	ATS	ANSW	N(d2→d1)	N(d1→d2)	N(d2→d2)	N(d1→d1)	Pr(switch)
in-control	370.3	370.4	72.98	36.57	36.40	148.30	148.07	0.198
$(1.1, 0.4)$	274.5	257.6	53.16	26.67	26.49	100.90	119.44	0.194
$(1.3, 0.5)$	69.0	44.9	11.52	5.88	5.64	14.97	41.55	0.169
$(1.5, 0.6)$	20.3	9.7	3.27	1.81	1.46	2.32	13.69	0.169
$(1.7, 0.7)$	9.6	4.3	1.94	1.21	0.73	0.65	6.00	0.226
$(1.9, 0.8)$	5.7	2.6	1.46	1.04	0.42	0.20	3.01	0.311
$(2.1, 0.9)$	3.6	1.7	1.13	0.94	0.19	0.04	1.44	0.434

Table 4. Performances of CUSUM chart based on W_i when Σ_0 is changed to $c\Sigma_0$. $(k=9.0)$

$c\Sigma_0$	ARL	ATS	ANSW	N(d2→d1)	N(d1→d2)	N(d2→d2)	N(d1→d1)	Pr(switch)
in-control	370.3	370.4	72.98	36.57	36.40	148.30	148.07	0.198
$1.1\times\Sigma_0$	213.8	189.0	40.57	20.38	20.19	72.42	99.77	0.191
$1.2\times\Sigma_0$	65.5	41.1	10.69	5.47	5.22	13.45	40.38	0.166
$1.3\times\Sigma_0$	23.8	11.4	3.72	2.03	1.69	2.84	16.20	0.163
$1.4\times\Sigma_0$	12.3	5.5	2.22	1.33	0.89	0.97	8.15	0.196
$1.5\times\Sigma_0$	7.8	3.5	1.71	1.13	0.58	0.41	4.71	0.250
$1.6\times\Sigma_0$	5.5	2.5	1.44	1.04	0.39	0.17	2.91	0.318
$1.7\times\Sigma_0$	4.2	1.9	1.25	0.98	0.27	0.08	1.85	0.393
$1.8\times\Sigma_0$	3.4	1.6	1.12	0.93	0.19	0.03	1.20	0.476
$1.9\times\Sigma_0$	2.8	1.4	1.00	0.87	0.13	0.02	0.77	0.560

Table 5. Performances of CUSUM chart based on W_i when σ_1^2 in Σ_0 is changed. $(k=10.0)$

σ_1	ARL	ATS	ANSW	N(d2→d1)	N(d1→d2)	N(d2→d2)	N(d1→d1)	Pr(switch)
in-control	370.3	370.4	115.99	58.13	57.86	126.85	126.50	0.314
$\sigma_1=1.1$	318.0	308.3	98.76	49.53	49.23	103.86	114.35	0.312
$\sigma_1=1.3$	102.4	77.8	28.78	14.57	14.21	22.81	49.84	0.284
$\sigma_1=1.5$	26.4	14.2	6.16	3.31	2.85	3.06	16.21	0.242
$\sigma_1=1.7$	11.0	5.1	2.57	1.56	1.01	0.73	6.71	0.257
$\sigma_1=1.9$	6.3	3.0	1.68	1.14	0.53	0.27	3.36	0.316
$\sigma_1=2.1$	4.3	2.1	1.32	0.97	0.34	0.11	1.90	0.397

Table 6. Performances of CUSUM chart based on W_i when ρ_{12} in Σ_0 is changed. $(k=10.0)$

ρ_{12}	ARL	ATS	ANSW	N(d2→d1)	N(d1→d2)	N(d2→d2)	N(d1→d1)	Pr(switch)
in-control	370.3	370.4	115.99	58.13	57.86	126.85	126.50	0.314
$\rho_{12}=0.4$	334.2	326.9	104.06	52.17	51.89	110.68	118.47	0.312
$\rho_{12}=0.5$	241.3	218.8	73.45	36.88	36.57	71.07	95.79	0.306
$\rho_{12}=0.6$	136.4	105.5	38.70	19.53	19.17	31.35	65.36	0.286
$\rho_{12}=0.7$	57.9	33.5	13.73	7.08	6.65	8.22	34.97	0.241
$\rho_{12}=0.8$	20.9	8.1	3.78	2.17	1.61	1.24	14.93	0.189
$\rho_{12}=0.9$	7.8	2.4	1.43	1.11	0.32	0.09	5.26	0.211

Table 7. Performances of CUSUM chart based on W_i when (σ_1, ρ_{12}) in Σ_0 are changed. $(k=10.0)$

(σ_1, ρ_{12})	ARL	ATS	ANSW	N(d2→d1)	N(d1→d2)	N(d2→d2)	N(d1→d1)	Pr(switch)
in-control	370.3	370.4	115.99	58.13	57.86	126.85	126.50	0.314
$(1.1, 0.4)$	296.8	283.6	91.88	46.09	45.79	94.80	109.12	0.311
$(1.3, 0.5)$	85.3	61.4	23.30	11.84	11.46	17.42	43.56	0.276
$(1.5, 0.6)$	21.3	10.6	4.76	2.62	2.14	2.08	13.44	0.235
$(1.7, 0.7)$	8.9	3.9	2.06	1.33	0.73	0.42	5.38	0.262
$(1.9, 0.8)$	4.9	2.2	1.36	1.02	0.33	0.10	2.46	0.346
$(2.1, 0.9)$	3.1	1.4	0.98	0.87	0.12	0.01	1.06	0.478

Table 8. Performances of CUSUM chart based on W_i when Σ_0 is changed to $c\Sigma_0$. $(k=10.0)$

$c\Sigma_0$	ARL	ATS	ANSW	N(d2→d1)	N(d1→d2)	N(d2→d2)	N(d1→d1)	Pr(switch)
in-control	370.3	370.4	115.99	58.13	57.86	126.85	126.50	0.314
$1.1\times\Sigma_0$	242.0	221.7	73.82	37.07	36.76	72.45	94.73	0.306
$1.2\times\Sigma_0$	82.1	57.5	22.09	11.24	10.85	16.01	43.03	0.272
$1.3\times\Sigma_0$	26.2	13.2	5.81	3.14	2.66	2.71	16.69	0.230
$1.4\times\Sigma_0$	11.9	5.1	2.55	1.57	0.98	0.67	7.66	0.234
$1.5\times\Sigma_0$	7.0	2.9	1.68	1.18	0.50	0.24	4.09	0.280
$1.6\times\Sigma_0$	4.7	2.1	1.32	1.02	0.30	0.08	2.34	0.353
$1.7\times\Sigma_0$	3.5	1.6	1.11	0.93	0.19	0.03	1.40	0.438
$1.8\times\Sigma_0$	2.8	1.4	0.96	0.84	0.12	0.01	0.86	0.525
$1.9\times\Sigma_0$	2.3	1.3	0.82	0.75	0.08	0.01	0.51	0.617

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