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An analysis of U.S. pre-service teachers' modeling and explaining 0.14m^2

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넓이 0.14m^2에 대한 미국 예비교사들의 모델링과 설명 분석

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초록

본 연구는 국외 수학 교사교육 사례 보고의 일환으로, 미국의 예비교사들이 넓이 0.14m^2 를 모델링하고 설명하는 과정을 분석하고 논의하였다. 수학방법론을 수강한 총 94명의 예비교사들이 자신이 이해하는 바를 문장으로 서술하기, 교구나 그림 등을 통해 모델을 제시하기, 학생들의 수준을 고려하여 구두로 설명하기 등으로 이루어진 일련의 활동에 참여하였으며, 이 자료들이 분석에 이용되었다. 분석 결과, 개념들 간의 연계성, 양적 및 질적 추론, 적절한 용어의 사용, 개념적 이해 등에 있어 성공 및 오류 사례 간에 큰 차이가 있었다. 본 연구는 수학교사교육자들이 예비교사들에게 수학지식과 교수방법이 유기적으로 통합된 과제를 교사교육 초기부터, 그리고 지속적으로 제공할 것을 제안한다.

Abstract

This investigation engaged elementary and middle school pre-service teachers in a task of modeling and explaining the magnitude of 0.14m^2 and examined their responses. The study analyzed both successful and unsuccessful responses in order to reflect on the patterns of misconceptions relative to pre-service teachers' prior knowledge. The findings suggest a need to promote opportunities for pre-service teachers to make connections between different domains through meaningful tasks, to reason abstractly and quantitatively, to use proper language, and to refine conceptual understanding. While mathematics teacher educators (MTEs) could use such mathematical tasks to identify the mathematical content needs of pre-service teachers, MTEs generally use instructional time to connect content and pedagogy. More importantly, an early and consistent exposure to a combined experience of mathematics and pedagogy that connects and deepens key concepts in the program's curriculum is critical in defining the important content knowledge for K-8 mathematics teachers.

* 주요어 : 수학 예비교사, 측정, 넓이 측정, 측정 단위
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Ⅰ. Introduction

This study examines how a group of U.S. elementary and middle school pre-service teachers (PSTs) engage in a mathematical task in which they explain an area of measure through various representations. Researchers in the U.S. (e.g., Battista, 1982, 2003, 2004) have looked into the pedagogy and student misconceptions related to measurement, especially area and volume measurement. Some researchers in Korea have also reported on student conceptions about area and volume measurement and proposed new teaching methods (Kim & Kang, 2011; Lim & Park, 2011; Na, 2012; Park & Paik, 2010). However, few studies have investigated the understanding of prospective teachers in university-based teacher education programs about the topic as the baseline data to inform teacher education at the elementary or middle school level. Nor has research discussed how PSTs, through a curriculum of mathematics teacher education, could improve content and pedagogy together with the aim of supporting the beginning educators in revising or re-organizing their content knowledge and developing their teaching skills.

In light of the need to support pre-service teachers in reviewing important school mathematics topics, it is significant to engage pre-service teachers in meaningful mathematical tasks as part of their mathematics education courses. These tasks have the potential to connect content and pedagogy, while affording pre-service teachers the opportunity to reflect on how their future students can make connections between and among mathematical ideas in a task. In this study, we used a mathematical task of measurement, analyzed how the task helps to identify types of content needs for pre-service teachers, and discussed the way such a task could serve as a useful context for connecting content and pedagogy for future K-8 mathematics teachers.

The rationale for selecting measurement as a mathematical focus is as follows: though measurement is one of the core components of the elementary school mathematics curriculum, many students do not have thorough knowledge of relevant measurement concepts (Outhred., Mitchelmore, McPhail, & Gould, 2003). In particular, studies have reported that students have an inadequate understanding of area measurement, which involves the coordination of two dimensions. Studies have also shown that students struggle to transition from linear measurement to area measurement (Fernández, De Bock., Verschaffel, & van Dooren, 2014). The domain of measurement is an important foundation because "[it] offers an opportunity for learning and applying other mathematics, including number operations, geometric ideas, statistical concepts, and notions of functions" (National Council of Teachers of Mathematics [NCTM], 2000, p. 44). According to the suggested progression through grade levels, middle school teachers should be prepared to support students as they learn to make logical connections between different dimensions of measurement, and build upon what they have previously learned in earlier grades (Common Core State Standards Initiative [CCSSI], 2010).

As our study examines the variation of PSTs' responses to the mathematical task of area measurement, data analysis is guided by the following research questions: (1) What are the features noted in the successful cases of modeling and explaining the given magnitude? (2) What are the PSTs' struggles as they construct and explain the given magnitude, as noted in the unsuccessful cases?

Ⅱ. Theoretical Background

1. Place of measurement in school mathematics

Measurement has been recognized as an important foundational domain in both school mathematics and workplace practices (CCSSI, 2010; Kent, Bakker, Hoyles, & Noss, 2011; NCTM, 2000). Yet, despite this importance and usefulness, many studies have addressed the unsatisfactory results of students' learning of measurement in school (Clements & Bright, 2003; Sisman & Aksu, 2016; Zacharos, 2006). Out of the many factors causing students' difficulties in learning mathematics, this study pays particular attention to the emphasis on measurement concepts and their place in school mathematics.

Measurement deals with the coordination of continuous quantity and number (Smith, van den Heuvel-Panhuizen, & Teppo, 2011). As mentioned earlier, students are introduced to measurement during their early elementary years and are expected to make logical connections between different dimensions of measurement (CCSSI, 2010). Although this surface level progression seems reasonable, researchers urge us to take a closer look at the design and emphasis placed on the measurement concepts. Smith et al.(2011) point out that poor student conception is caused by, “less classroom attention to the measurement of continuous quantities than to developing students' understanding of base-10 number and arithmetic operations” (p. 618). This is reflected in relatively less instructional time, as well as less depth of exploration. Also, in many cases, the design of curriculum places and addresses spatial measurement (i.e., length, area, volume) in an isolated sequence rather than integrating different dimensions around central conceptual foundations (Smith et al., 2008 as cited in Barrett et al., 2011). As such, some researchers propose alternative approaches to promoting the integration of different mathematical domains. In a Russian experimental curriculum (Davydov, Gorbov, Mikulina, & Savel'eva, 1999), first grade students discuss various properties of quantities (e.g., length, volume, and weight) and understand a number as the expression of a relationship between a quantity to be measured and a measuring unit. The notion of the mathematical structure between quantity and unit is easily applicable across different dimensions of measurement without massive interventions (see Lee, 2006 for additional explanations). Similarly, Barrett et al.(2011) found that thoughtful design of spatial measurement activities across varying dimensions and tasks in the early grades could contribute to building a strong basis for other mathematics topics like proportional reasoning and rational number knowledge, since measurement units are closely related to these topics.

2. Students' misconceptions and errors in area measurement

The construction of unit and scale serve as the conceptual foundations of measurement (Davydov, 1990; Lehrer, Jaslow, & Curtis, 2003). Although these central concepts apply for all spatial measurements (i.e., length, area, volume), complexity increases as the dimensionality increases. For instance, when considering area measures, students will face additional challenges in thinking about central conceptions for measurement in two dimensions. As the task presented in this paper specifically utilizes the context of area measurement, this section briefly summarizes the many difficulties related to students' understanding of area measurement.

Area measurement relies on the idea of constructing an array of unit squares (Lehrer et al., 2003). Researchers report that students' often have difficulty understanding the unit structure associated with array and grid structure (Battista, Clements, Arnoff, Battista, & van Auken Borrow, 1998; Curry & Outhred, 2005). Students will use units of length measure for area, which often results in confusion between perimeter and area (Battista, 2003; Lehrer, 2003). Further, they attempt to use additive relationships rather than multiplicative relationships correctly for area measurement (Empson, Junk, Dominguez, & Turner, 2006). This is an indication of student confusion with dimensionality (i.e., one-dimensional length vs. two-dimensional area); because student difficulties are rooted in these weak conceptual understandings, it is common for many young students to apply area formulae without understanding what the products of length and width really mean (Battista et al., 1998; Lehrer et al., 2003).

3. Adults' understanding of area measurement

While there are some research studies on adults' understanding of measurement, including pre-service teachers and other college students, studies on these populations are relatively scarce compared to studies on grade school students. Baturo and Nason(1996, p. 261) reviewed elementary pre-service teachers' knowledge of area measurement and reported, "much of [the participants'] substantive knowledge was incorrect, and/or incomplete, and often unconnected" and the participants perceived area measurement, "as a set of units and formulae which was to be rote learnt and then applied." Menon(1998) suggests that teachers' shallow content knowledge and instructional practices are the causes for students' struggles in measurement concepts. In an examination of Malaysian pre-service teachers' knowledge of area formulae using clinical interviews, Yew, Zamri, and Lian(2010) reported a lack of conceptual knowledge related to the formula for the area of a rectangle, showing a strong indication of reliance on rote-learned formulae. Simon and Blume(1994) also suggest that many elementary pre-service teachers do not clearly understand that the relationship of the length and width of a rectangle to its area can be modeled by multiplication of length by width. Dorko and Speer(2015) investigated college students' understanding of area and volume units and reported that students who struggled with units seem to possess weak knowledge of array or dimensionality, while successful students tended to consider arrays and/or dimensionality. Overall, these studies indicate that the weak conceptual understanding of area measurement among adults (including teachers and pre-service teachers) has somewhat similar patterns to those reported in studies for grade school students.

4. Situating the study

The preceding three themes in this literature review suggest the recognition of several key points: the importance of measurement in school mathematics, concerns about students' persistent misconceptions and errors in area measurement, limited research on pre-service teachers' understanding, and the implications on future teaching practice. Despite the limited research on pre-service teacher knowledge of measurement in teacher education, Baturo and Nason(1996) were quite instrumental in that their study used eight tasks to examine teacher knowledge of area measurement and provided detailed work on PSTs' mathematical concept and process. Extending the work of Baturo and Nason(1996) and considering the gap in literature about pre-service teacher concepts and strategies regarding area measurement, this study seeks to investigate the knowledge of elementary and middle school PSTs. This investigation will focus on how PSTs reason and model in an attempt to explain the size of $0.14m^2$. It is expected that PSTs' approaches to the given task offer fertile ground for identifying pedagogical issues and opportunities related to young students' thinking. These approaches will also provide valuable insights for the kind of content knowledge that connects with classroom practice.

Ⅲ. Methods

1. Participants and context

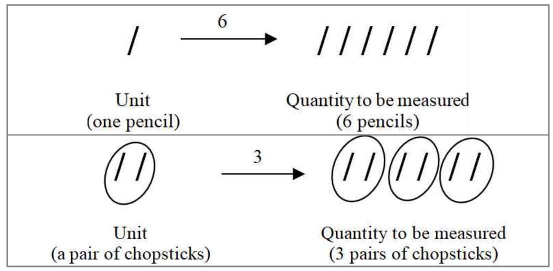
Data from pre-service elementary and middle school teachers was gathered over three semesters (n=94) across five sections of an elementary and middle school level mathematics methods course. The methods course was required for all participants in a teacher preparation program at a Midwestern university in the United States, one of the authors was the instructor of the course. All PSTs were working towards their initial teaching certifications for Kindergarten through eighth grade. As part of their major requirements, the PSTs took two mathematics content courses prior to this methods course; the studied course surveyed number theory, statistics, and geometry. The learning objectives of linear and area measurements in the mathematics content courses include understanding of measurable attributes of objects and the units, and systems as well as application of techniques, tools, and formulas to determine measurements.

The methods course was structured around several major concepts relevant to elementary and middle school mathematics, including the place value system, whole numbers and operations, fractional numbers and operations (as the primary foci), measurement, geometry, data analysis, and statistics (as the secondary foci). To provide contextual background, a description is provided below illustrating the aspects of measurement highlighted in the course.

For the purposes of the course, measurement is defined as the relationship between the measuring unit and the quantity to be measured, where the unit and quantity are the same kind of magnitudes (e.g., length, area, volume, etc.). The need for specification of the unit is also highlighted. For example, the following drawing (Figure 1) is shown along with a question: "How many are here?" (adapted from Davydov, 1990, p. 67 - 68).

[Fig. 1] Six chopsticks

A common initial reaction was 6. However, this answer is not sufficient when we do not know "how many of what." When the one line segment represents a pencil and the question was "how many pencils are here?" or one line segment represents a chopstick and the question was "how many pairs of chopsticks are here?" two different answers are possible depending on the unit of measure (see Figure 2).



[Fig. 2] Two methods of counting, using a different unit of measurement

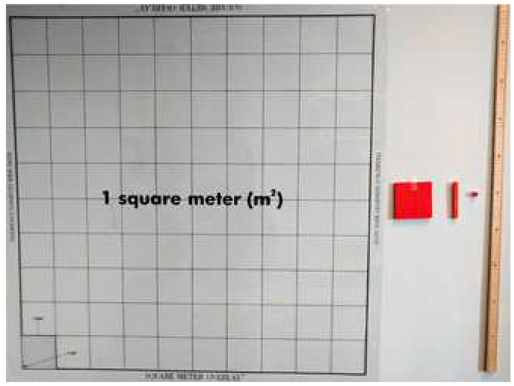
Likewise, when discussing different measuring unit conversions for different dimensions (e.g., length, area, volume, etc.), emphasis was placed on constructing the actual sizes of linear units (e.g., cm, dm, m), two-dimensional units (e.g., cm^2, dm^2, m^2), three-dimensional units (e.g., cm^3, dm^3, m^3), and on examining the relationship between magnitudes of units (e.g., How many cm^2 are in a dm^2?).

2. Task

The participants were asked to show the actual size of $0.14m^2$ and explain it in a way that a middle school student would likely be able to understand. This specific task was chosen to examine three issues relevant to understanding the meaning of measurement: (1) choosing an appropriate unit for measuring an attribute, (2) exploring the relationship between the size of a unit and the number of units required to measure, and (3) dealing with measurements that have whole units and parts of a unit (Grant & Kline, 2003). The magnitude 0.14 was chosen because the number 14 yields multiple factors of the area. The task also demanded drawing areas of the correct size to challenge participants to think with real-world referents before moving to abstract spaces of algebraic manipulation.

To maintain the unfamiliarity of the task (so that participants could activate their concept of area measurement rather than memorized strategies), teaching strategies for decimal fractions or measures with decimal fractions were not specifically discussed when the task of modeling and explaining the size of $0.14m^2$ was presented. PSTs were only exposed to whole number operations, common fractions, construction of the actual size of various measurement units (e.g., building actual size of a square meter or a cubic meter), and their conversions (e.g., How many centimeters are there in 1 meter? How many square centimeters are there in one square meter?).

Participants were asked to work with their peers during the class time. A total of 94 PSTs jointly engaged in the task by forming pairs or groups of three (41 pairs and four groups of three PSTs). Although it was not a timed-task, it took between 20 – 40 minutes for most pairs/groups to complete the work of discussing and preparing their presentation of explanations. Each pair or group of PSTs summarized their ideas in three ways after reaching a consensus through discussion: (1) written statements describing their own understanding, (2) physical modeling using available materials, and (3) 5-minute verbal explanations presentable to middle school students. PSTs were informed that materials for modeling were available (e.g., ruler, base 10 block sets, square meter overlay) without specifying what needed to be used (see Figure 3).



[Fig. 3] Materials available for physical modeling

3. Data collection and analysis

The PSTs' written statements were collected, with observation notes taken by the researcher and a research assistant to document PSTs' use of physical materials and verbal explanations. An analysis of PSTs' written statements and observation notes was conducted via two levels: the first level of analysis provided descriptive information on frequencies of correctness of the size of $0.14m^2$ and types of chosen representations, while the second level of analysis followed some aspects of the open-ended coding and a double-coding procedure (Miles & Huberman, 1994; Strauss & Corbin, 1998) to examine the strategies PSTs used, as well as their reasoning process. Initially, the researcher independently reviewed the written data with a research assistant to identify recurring themes in strategies used for reasoning in both successful and unsuccessful cases of modeling and explanations. Later, the investigator and research assistant jointly revised and refined the independently identified themes through comparison and discussion, after which they jointly coded so that coding discrepancies could be resolved immediately.

Ⅳ. Results and Discussion

1. Correctness of presenting and reasoning the

magnitude of 0.14m^2

[Table 1] PSTs' performance of presenting and reasoning the magnitude of 0.14m^2

Presentation of Magnitude	Validity of Reasoning	Frequency (n=94)
Correct	Valid	56
	Unable to provide reasoning	2
Incorrect	Valid-Invalid[a]	6
	Invalid	20
	Unable to provide reasoning	2
Unable to present	Invalid	6
	Unable to provide reasoning	2

[a]These are the cases that proposed mixed reasoning.

Table 1 shows the overall snapshot of PSTs' performance in terms of correctness of the asked magnitude (0.14m^2) and reasoning behind their answers.

There was apparent alignment between the validity of reasoning and the correctness of magnitude for most cases (i.e., valid reasoning resulted in correct magnitude and invalid reasoning led to incorrect magnitude). However, several cases did not demonstrate this alignment. Six PSTs provided mixed reasoning. In all of these cases, PSTs were initially able to provide their reasoning abstractly (e.g., using conversion between units) but could not present a correct magnitude in other forms when asked to show the amount quantitatively. Also, there were three PST pairs/groups who could not provide any clear verbal or written explanations. These PSTs' presentations of magnitude were varied (i.e., correct, incorrect, and no presentations). There were no cases in which invalid reasoning led to the correct magnitude. More specific examples will be presented in the following sections when discussing the successful and unsuccessful cases.

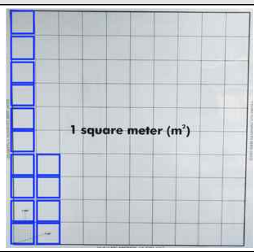
2. Features noted in successful cases of modeling and explaining

Here, successful cases refer to examples that presented correct magnitude with valid reasoning. A total of five strategies were identified. Having a decimal form of measure, the two most popular

[Table 2] Reasoning/strategies used: successful cases

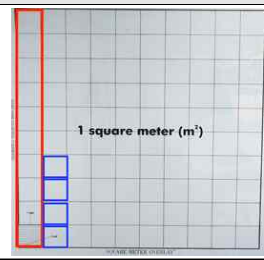
Connections to other mathematics topics	Description	Frequency (n = 56)
Fractional numbers (see Example 1)	Explanations address the referent unit and the relationship between the referent unit and the proposed magnitude (Lee, Brown, & Orrill, 2011).	26
Base-10 Place value system (see Example 2)	Explanations address one or more of the following properties that characterize the base-10 place value system (Ross, 2002): 1. Additive property. The quantity represented by the whole numeral is the sum of the values represented by the individual digits. 2. Positional property. The quantities represented by the individual digits are determined by the positions that they hold in the whole numeral. 3. Base-ten property. The values of the positions increase in powers of ten from right to left. 4. Multiplicative property. The value of an individual digit is found by multiplying the face value of the digit by the value assigned to its position.	18
Area formula (Multiplication) (see Example 3)	Explanations highlight that area is an attribute of two-dimensional regions and provide two factors for length and width that can produce the given area measure.	10
Unit conversions as known facts (see Example 4)	Explanations are based on the rules associated with conversion units of area by moving the decimal point left or right or treating the conversions as known facts.	2

interpretations were based on PSTs' understanding of the base-10 place value system and decimal fractions, though there was some overlap in PSTs' explanations because decimal fractions have a base-10 positional system. The distinction was made based on the presence of key features underlying place value and fractions. Table 2 shows the descriptions of utilized strategies and their usage frequencies. This is followed by specific examples of PSTs' work.

Representations Used for Modeling

Explanation
Example 1(a) We need to focus on the whole first. What is the whole? The whole is 1 square meter [*picked up the entire 1 square meter overlay*]. 0.14m^2 is the same as 14/100m^2. This means that the whole is cut into 100 equal-sized pieces and we are discussing 14 of them. Because 1/100 of a square meter is this [*pointed out one of the blue sections*], we need 14 of those for 0.14m^2 [*pointed out all blue sections*]. Example 1(b) [*Before starting explanation, wrote '0.14' on the board.*] How do we read this decimal correctly? Is it "zero point one four"? Can we read this more precisely and in a mathematically correct way? We read this as "zero and fourteen-hundredths." So, what we are looking for is fourteen-hundredths of this [*picked up the entire 1 square meter overlay*]. One hundredth of this is that [*pointed out one of the blue sections*]. So, we need 14 of those [*pointed out all blue sections*].

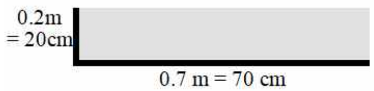
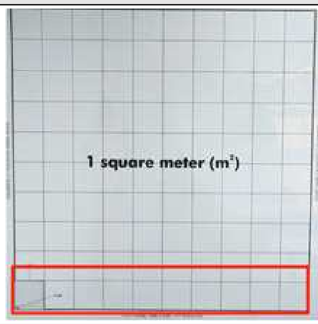
[Fig. 4] Successful case connecting with the meaning of fractional numbers

Example 1. These PSTs expanded their understanding of fractional numbers (common fractions or decimal fractions) to interpret this two-dimensional quantity (see Figure 4). They defined the referent unit (the whole) first and explained how a part can be named based on the size of the whole.

Representations Used for Modeling

Explanation
0.14 has three places: ones place, tenths place, and hundredths place. In this case, the size for ones place unit is this [*picked up the entire 1 square meter overlay*]. It is one square meter. The size of tenths place unit should be 10 times smaller than one square meter. So, the one long strip of the whole square meter is one-tenth of a square meter [*pointed out the red colored column*]. The size of hundredths place unit should be 10 times smaller than the previous place unit, which is one-tenth of a square meter. So, this one square [*pointed out one of the blue colored sections*] is one-hundredth of a square meter. In 0.14m^2, there is nothing in the ones place, 1 in the tenths place, and 4 in the hundredths place. This means that we need one full column [*pointed out the red section*] and four of these [*pointed out all blue sections*]."

[Fig. 5] Successful case connecting with the place value system

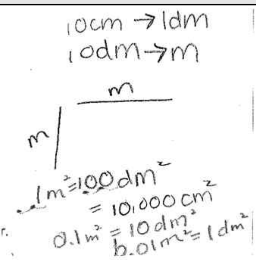
Example 2. The PSTs who presented this case focused on the place value concept. They tried to model and explain by highlighting the size of the unit in each place. These PSTs considered the given quantity as $0.14m^2=0\cdot m^2+(1\times 0.1m^2)+(4\times 0.01m^2)$. This explanation relies on an understanding of the place value system and its application in base-10. Figure 5 illustrates what materials they used to model the magnitude and how they explained the given quantity using those materials.

Representations Used for Modeling

Explanation
Example 3 (a) Area is length times width [wrote A=L×W]. The area is $0.14m^2$. One possible case is 0.2 meter by 0.7 meter. [When asked to show the actual magnitude, a meter stick was used to show the length and width, stating that $0.2m$ is the same as $20cm$ and $0.7m$ is the same as $70cm$.]
Representations Used for Modeling

Explanation
Example 3 (b) 0.14 is 1 by 0.14. So, the area of a rectangle that is 1 meter long and 0.14 meters wide is 0.14 square meters. [When asked to show the actual magnitude, the square meter overlay was used to show the length and width. Pointing out the red colored section.]

[Fig. 6] Successful case with applying area formula

Example 3. Some PSTs suggested finding the length and width of a rectangle with $0.14m^2$ as the area. Two different sets of factors were specifically presented (see Figure 6). Although these PSTs abstractly found the length and width, they were successful in presenting the actual size of the given area measure with valid explanations.

Example 4. As shown in Figure 7, some PSTs explained the size of a given quantity by first using completely computational and symbolic representations, then finding the actual magnitude of the given measure. They noted the conversions between two-dimensional units abstractly (e.g., moving around the decimal point).

Representations Used for Modeling

Explanation
1 square meter is the area created by 1 meter length by 1 meter width. 10 centimeters is the same as 1 decimeter, and 10 decimeters is the same as 1 meter. So, 1 square meter is the same as 10 decimeters by 10 decimeters, which is 100 square decimeters. It is also the same as 100 centimeters by 100 centimeters that is 10,000 square centimeters. 0.1 [*read as zero point one*] square meter is 10 square decimeters [*showed how the decimal point moved*] and also 0.01 [*read as zero point zero one*] square meter is 1 square decimeter [*showed how the decimal point moved*]. So, 0.14 [*read as zero point one four*] square meters is 14 like this [*picked up a flat piece from the base 10 block set*] because it is a square decimeter, which is one decimeter by one decimeter."

[Fig. 7] Successful case with symbolic representations

3. Features noted in unsuccessful cases of modeling and explaining

Here, unsuccessful cases refer to all of the cases that could not present both correct magnitude and valid reasoning. A total of 38 PSTs proposed invalid

[Table 3] Reasoning/strategies used: unsuccessful cases

Attempts to connect mathematics topics	Description	Frequency (n = 38)
Area formula (Multiplication) (see Example 5)[a]	Explanations highlight that area is an attribute of two-dimensional regions and attempt to find two equal factors for length and width that can produce the given area measure (i.e., length = width).	14
Confusion with dimensionality (see Example 6)	Explanations were based on the relationships between first dimensional units (length) rather than second dimensional units (area).	7
Additive relationship vs. Multiplicative relationship[a] (see Example 7)	Explanations show confusion between additive relationship and multiplicative relationship.	5
Confusion with place value whole numbers and decimals (see Example 8)	Explanations include incorrect application of place value concept.	6
None	Unable to provide both reasoning and the actual size (4) Able to present the actual size, but unable to provide reasoning (2).	6

[a]These categories include six PSTs who proposed mixed reasoning. PSTs initially proposed valid reasoning at the abstract level, but used invalid reasoning when asked to show the actual size of the given area.

reasoning or were unable to provide reasoning at all. Six PSTs initially provided valid reasoning at the abstract level (manipulation of symbols only), but they changed it to invalid explanations when asked to show the actual size of the given area. Six PSTs were unable to provide proper reasoning at all. Among them, two PSTs showed the correct size of the asked area, but failed to explain why it showed $0.14m^2$.

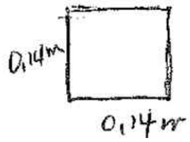
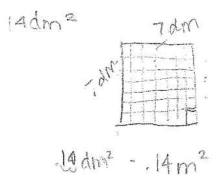
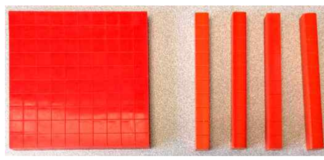
Similar strategies used for the successful cases were used but incorrect or incomplete answers and/or reasoning were presented. Table 3 shows the descriptions of strategies utilized and the frequencies of usage, which was then followed by specific examples of PSTs' work. Since most of PSTs in these categories provided incomplete explanations, or altered their own due to confusion, the examples below (see Figure 8) highlight attempts rather than full explanations.

Ⅴ. Conclusion and Implication

1. Conclusion

The study investigated PSTs' approaches to modeling and explaining an area measurement represented in decimal notation. The literature on pre-service teacher knowledge of area measurement tends to draw an alarming picture of low and disconnected mathematical knowledge. Our findings offer a more concrete diagnosis of their knowledge. Our findings suggest that PSTs engaged in flexible strategies to represent an area, while tapping into their prior knowledge about the area. Our findings, just like other researchers (e.g., Baturo & Nason, 1996; Dorko & Speer, 2015; Yew et al., 2010), also reveal that PSTs' struggled to connect and apply prior knowledge in an unfamiliar context.

Considering the various cases of successful and unsuccessful modeling and the explanations presented here, it is clear that modeling and explaining the magnitude of $0.14m^2$ does involve knowledge that goes beyond recalling and applying a formula. We believe

Example 5	Example 6
 In this example, PSTs interpreted 0.14m^2 as the area of a square whose length and width are both 0.14m. When asked to show how 0.14m×0.14m can be 0.14m^2, they used a calculator and found that the answer is 0.0196m^2. With this confusion, their explanation ended without conclusion.	 In this example, PSTs selected one long and four unit base 10 blocks. In their explanation, they only referred to the linear measuring units, stating "this piece [the long piece] is one decimeter and one decimeter is one-tenth of a meter. So, we need one of those. Also, this piece [the unit piece] is one centimeter. It is one hundredth of a meter. So, we need four of them." When asked which part of the materials were used to show 0.14m^2, they traced one face of each pieces to show that they were talking about the area.
Example 7	**Example 8**
 In this example, PSTs immediately noted that 0.14$m^2$is the same as 14dm^2. When asked how big 14dm^2 is, they drew a 7dm by 7dm square, concluding that 7dm×7dm=14dm^2=0.14m^2. When asked to explain why 7dm×7dm=14dm^2, they noticed their mistake, but could not come up with physical or pictorial representation for 0.14m^2.	 This pair of PSTs immediately chose the above base 10 blocks [one flat piece and four long pieces]. When asked for explanation, they said, "This flat piece is one hundred. The one long piece is a tenth of the flat. So, we need one flat and four longs." This pair did not refer to the name of measuring units at all.

[Fig. 8] Examples of unsuccessful cases

this is an important criterion for defining a rich and meaningful mathematical task in mathematics education course work. Our task requires the knowledge of what it means to measure, the ability to differentiate various dimensions of magnitudes (e.g., length vs. area), the ability to broaden the prior understanding of the domain of numbers, and the ability to flexibly use physical or pictorial models to support reasoning. The successful cases from the PSTs' work demonstrated these abilities. Based on the findings regarding the strategies PSTs used and the conceptions they held, this section presents several implications teacher educators can consider when designing and creating meaningful mathematical learning experiences for PSTs in mathematics education courses.

First, while it is critical for PSTs to engage in meaningful mathematical tasks, it is equally important to dwell on exactly what we mean by meaningful and rich tasks as part of our curriculum in mathematics education. Our findings indicate that an important criterion of meaningful mathematics tasks for PSTs is whether the task can promote connections between different mathematics domains. As mentioned earlier, one of the criticisms for grade school curriculum is teaching mathematical concepts in an isolated, unintegrated way. This might be true in PSTs' past learning experiences as well. Considering that PSTs are in the transitional period from students to teachers, it is important to review content through a math task that connects various concepts, and more importantly one that uses the opportunity to connect content to the pedagogy of engaging students in similar tasks. In this study, most successful cases show the efforts of extending and elaborating on the concept of area based on a number of related concepts (e.g., numbers and operations in base 10, place value understanding, rational numbers)—while some unsuccessful cases indicate difficulties in extending and elaborating on the concept of area beyond the area formula.

Second, meaningful math tasks for PSTs should provide the opportunity to develop reasoning skills both abstractly and quantitatively. This kind of reasoning is one of the Standards for Mathematical Practice that we expect PSTs to foster in students at all levels (CCSSI, 2010). Related to our task in this study, an important reasoning skill is “considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects” (CCSSI, 2010, p. 6). It was interesting that some PSTs clearly demonstrated mathematical precision in specifying units of measure as they used the actual manipulatives available to visualize the magnitude of units (e.g., actual size of a square meter overlay). This was interesting because in several unsuccessful cases, PSTs could manipulate algebraic symbols abstractly and demonstrated elaborate conversions, but failed to present the actual size of the asked area measurement. This implies that the procedural skills involved in applying rules or formulae do not necessarily represent students' understanding of the meaning of said quantities.

Third, a rich math task in a mathematics education course provides PSTs the opportunity to reflect on appropriate and effective language use as the classroom teacher. It is difficult to imagine our PSTs dwelling on language issues in the mathematics classroom when they take mathematics content courses, in most part because those courses may not be designed for future teachers. Regarding the use of appropriate language of measurement with precision, the PSTs who presented unsuccessful reasoning or magnitude of $0.14m^2$ used vague terms when referring to measuring units (e.g., saying “14 like this” rather than “14 square decimeters” or saying “One hundredth of this is that). Although it is hard to generalize with the limited work samples provided in this paper, PSTs' ability to “reason both abstractly and quantitatively” and “attend to precision,” both of which are standards for mathematical practice (CCSSI, 2010), seem to go hand in hand. For example, PSTs referred to $0.14m^2$ in a variety of ways such as “point one-four meters, squared,″ ″point one-four meters-squared″ or ″point one-four square meters,″ and some PSTs stated, “$0.14m^2=0.14m\times0.14m$.” This finding adds more credence to the view that our PSTs need precision. PSTs need precision not only in their mathematical work, but also in using appropriate language to communicate their mathematical thinking and reasoning.

Fourth, we suspect that children and adults are not so different in their misconceptions regarding some mathematical concepts. The misconceptions about the measurement of area do not necessarily disappear with time and maturity when students experience little about using the concept of area in later courses. In fact, the logic and patterns of errors shown in the PSTs' unsuccessful cases resemble students' misconceptions as reported in other studies. From difficulties with numerical procedures caused by insufficient understanding to misapplication of linear relationship in non-linear situations (Fernández, Llinares, van Dooren, De Bock, & Verschaffel, 2012), there is still a great need to improve PSTs' conceptual understanding of measurement and their ability to make connections to multiple domains of mathematics in order to build a strong conceptual foundation. In the same vein, there is a need to highlight meaningful use of various representations. Some cases in this study show PSTs employing inflexible or rote ways of using manipulatives. For example, PSTs who employed the approach in Example 8 somewhat demonstrated their understanding of the base-10 system as evidenced in their interpretation of a long piece as one-tenth of the flat piece. However, they seemed to believe that the flat piece in the base-10 block sets always represents 100, as it was frequently used in whole number representations.

Perhaps, past learning experiences for these PSTs focused greatly on the technical aspects of dealing with the domain of measurement. However, despite their past learning experiences, teachers are expected to provide appropriate support to allow students the opportunity to articulate mathematical ideas and connect related concepts. To do so, PSTs should be afforded the opportunities to deepen their understanding of measurement concepts and practice modeling along with logical and coherent explanations in teacher preparation programs.

2. Implications

The field of mathematics teacher education has developed an increasingly diverse body of teacher knowledge and skills in the mathematics classroom. In addition to learning to write lesson plans, most mathematics education courses (including the mathematics methods course in the university-based teacher education programs) integrate content and pedagogy, even addressing affective issues in the mathematics classroom. Although our findings from this study largely point to the lack of PST skills in integrating key concepts to make sense of unfamiliar concepts and contexts, we do not mean to dwell on the deficit. Rather, the implication of these findings indicates the need for learning opportunities designed for mathematics teachers in math content courses—where the focus is more on integrating important concepts in school mathematics, as opposed to the mastery of advanced mathematics—especially for K-8 mathematics teachers.

Pre-service teachers take mathematics courses mostly from the mathematics department, and instructors may teach content knowledge using the same methods they would employ for science/engineering majors. In the midst of mathematics education reform, pre-service teachers ought to learn mathematics in the manner in which they will be teaching it, which is unlike the lecture format found in traditional classrooms. Mathematics courses need to have a renewed focus on providing powerful learning experiences for pre-service teachers. Regarding the nature of powerful learning experiences, the implication of our findings points to the rich task and contexts in which pre-service teachers apply key math concepts and use appropriate and precise language to model and explain their thinking and reasoning. Our field has come to recognize that teachers who perform highly in advanced mathematics are not necessarily well versed in explaining the fundamental mathematics concepts such as area and volume to children. In this study, the participants were asked to explain mathematics for middle graders, and we wonder how their responses might have changed if they had to explain for their peers or much younger children.

Although this study investigated elementary and middle school PSTs' knowledge of area measurement as the baseline data, it has yet to complete a comprehensive line of research regarding implementation of a curriculum for mathematics teacher education that is designed to improve content and pedagogy of PSTs. In the U.S. context of mathematics teacher education, teacher′s content knowledge has been recognized as the panacea for all the problems confronting students in the U.S. classroom. In this vein, one could well argue that the low content knowledge of the participants explains the unsuccessful responses away. Having said that, Cooney(1994)′s statement still rings true: ″there is little evidence about the relationship of elementary teachers′ knowledge of mathematics to the way mathematics is taught,″ (p.

107). More importantly, there is little work in teacher education in Korea or the U.S. on providing pre-service teachers with the kind of learning experiences in teacher education where pre-service teachers revisit school mathematics, reflect on their own misconceptions and struggles in the curriculum (i.e., the baseline data in our study), and improve curriculum and instruction. Therefore, future research looking into a curriculum that reforms mathematics teacher education and instruction, through collaboration between mathematics educators (at each developmental period from Pre-K through 12 education) and mathematics education researchers, is warranted. This research could include (1) an extended version of this study categorizing the nuances of pre-service teachers′ relational knowledge and resourcefulness in school mathematics and proposing programs to improve them and implement them in the classroom and (2) any longitudinal studies of PSTs and their initial knowledge of mathematics and pedagogy respectively, improved knowledge and attitude through learning (or the absence of it) in teacher education, and the ways in which teaching is enacted in actual classrooms during the first year of instruction.

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