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Orthogonal NOMA Strong Channel User Capacity: Zero Power Non-Zero Capacity Transmission

Kyuhyuk Chung*★

Abstract

Recently, orthogonal non-orthogonal multiple access (O NOMA) with polar on-off keying (POOK) has been proposed to mitigate the severe effect of the superposition. However, it is observed that the performance of the O NOMA strong channel user is better than that of the perfect successive interference cancellation (SIC), i.e., the performance of a single user transmission with binary phase shift keying (BPSK). Can the performance of the BPSK modulation be better that that of itself? It is not normal. It should be clearly understood theoretically, with the ultimate bound, i.e., the channel capacity. This paper proves that the channel capacity of the O NOMA strong channel user is non-zero with zero power allocation. Thus, it is shown that the interference is transformed effectively into the meaningful signal.

Key words : Non-orthogonal multiple access, successive interference cancellation, power allocation, channel capacity, binary phase shift keying

Ⅰ. Introduction

The fifth generation (5G) and beyond mobile networks require the more users to be served. Therefore, the standard bodies have been considering non-orthogonal multiple access (NOMA) [1-4]. In NOMA, the superposition principle in the power-domain is exploited to increase the system capacity. At the same time, such principle degrades the NOMA performance. Specifically, in [1], [2], the existing modulation techniques do not achieve the perfect successive interference cancellation (SIC) performance, for the power allocation factor greater than 20%. Such performance degradation is recovered with the symmetric superposition coding, for the power allocation factor up to 50% [3], [4]. However, the perfect SIC performance is not achieved for the entire power allocation factor, i.e., from 0% to 100%. Therefore, in order to mitigate the severe effect of the superposition, recently the orthogonal NOMA (O NOMA) with polar on-off keying (POOK) is proposed [5]. This paper proves that the channel capacity of the O NOMA strong channel user is non-zero with zero power allocation.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \le \alpha \le 1$, and the channel gains are h_1 and h_2 with $|h_1| > |h_2|$. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2

* Dept. of Software Science, Dankook University
★ Corresponding author
E-mail : khchung@dankook.ac.kr, Tel : +82-32-8005-3237
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signal s_2, with $E\left[|s_1|^2\right]=E\left[|s_2|^2\right]=1$. The superimposed signal is expressed by

$$x=\sqrt{aP}s_1+\sqrt{(1-a)P}s_2 \tag{1}$$

Before the SIC is performed on the user-1 with the better channel condition, the received signals of the user-1 and the user-2 are represented as

$$\begin{aligned} r_1 &= |h_1|\sqrt{aP}s_1+(|h_1|\sqrt{(1-a)P}s_2+n_1) \\ r_2 &= |h_2|\sqrt{(1-a)P}s_2+(|h_2|\sqrt{aP}s_1+n_2) \end{aligned} \tag{2}$$

where n_1 and $n_2 \sim N(0,N_0/2)$ are additive white Gaussian noise (AWGN). The notation $N(\mu,\Sigma)$ denotes the normal distribution with mean μ and variance Σ. In the standard NOMA, the SIC is performed only on the user-1. Then the received signal is given by, if the perfect SIC is assumed,

$$y_1=r_1-|h_1|\sqrt{(1-\alpha)P}s_2=|h_1|\sqrt{\alpha P}s_1+n_1 \tag{3}$$

The channel model is depicted in Fig. 1.

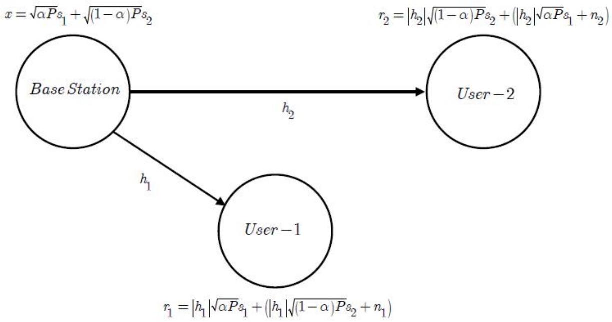

Fig. 1. System and Channel Model.

III. Brief Review of POOK

We, now, briefly review POOK [5]. On-off keying (OOK) is the simplest modulation technique. The carrier is sent or not. Assume the binary phase shift keying (BPSK) modulation for the user-1, with $s_1 \in \{+1,-1\}$. Then POOK, with $s_2 \in \{+\sqrt{2},0,-\sqrt{2}\}$, is the inter user interference s_1 dependent OOK. The power is normalized as

$$E\left[|s_2|^2\right]=\frac{1}{4}(+\sqrt{2})^2+\frac{1}{2}(0)^2+\frac{1}{4}(-\sqrt{2})^2=1 \tag{4}$$

Compare the standard OOK, $s_{OOK} \in \{+\sqrt{2},0\}$ with

$$E\left[|s_{OOK}|^2\right]=\frac{1}{2}(\sqrt{2})^2+\frac{1}{2}(0)^2=1 \tag{5}$$

POOK is given by, with the information input bits for the user-1 and the user-2 being $b_1, b_2 \in \{0,1\}$, as

$$\begin{cases} s_1(b_1=0)=+1 \\ s_1(b_1=0)=-1 \end{cases} \tag{6}$$
$$\begin{cases} s_2(b_2=0\,|\,b_1=0)=+\sqrt{2} \\ s_2(b_2=1\,|\,b_2=0) \end{cases} \quad \begin{cases} s_2(b_2=0\,|\,b_1=1)=-\sqrt{2} \\ s_2(b_2=1\,|\,b_2=1) \end{cases}$$

POOK and BPSK are compared in Fig. 2.

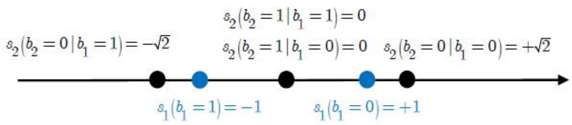

Fig. 2. Polar on-off keying and BPSK.

IV. O NOMA Strong Channel User Capacity

In this section, we prove that the O NOMA strong channel user capacity is non-zero with zero power allocation. In O NOMA, it is interesting that even if no power is allocated to the user-1 $(\alpha=0)$, the user-1 can receive information from the base station, at a meaningful probability of errors of 1/4, less than 1/2, which is a meaningless probability of errors [5].

Theorem 1: The channel capacity of the strong channel user in O NOMA is non-zero with zero power allocation, if there exists the interference.

Proof) If there exists the interference, then the received signal for the strong channel user is given by, (which is the same as in the equation (2) before the perfect SIC is performed)

$$r_1 = |h_1|\sqrt{\alpha P}s_1 + (|h_2|\sqrt{(1-\alpha)P}s_2 + n_2) \tag{7}$$

In this case, we consider the capacity [6] in bit/s/Hz,

$$c_1^{(b;0\,NOMA)} = \max_{p_{B_1}(b_1)} H(r_1) - H(r_1 \mid b_1) \tag{8}$$

$$= -\int_{-\infty}^{\infty} p_{R_1}(r_1)\log_2(r_1)dr_1$$

$$+\int_{-\infty}^{\infty} p_{R_1|B_1}(r_1|b_1=0)p_{B_1}(b_1=0)\log_2 p_{R_1|B_1}(r_1|b_1=0)dr_1$$

$$+\int_{-\infty}^{\infty} p_{R_1|B_1}(r_1|b_1=1)p_{B_1}(b_1=1)\log_2 p_{R_1|B_1}(r_1|b_1=0)dr_1$$

where the entropy $H(x) = -E[\log_2 p_X(x)]$, $p_X(x)$, is the probability density function (PDF),

$$p_{R_1|B_1}(r_1|b_1=0) = \frac{1}{2}\frac{1}{\sqrt{2pN_o/2}}e^{-\frac{(r_1|h_1|\sqrt{aP}+|h_1|\sqrt{2(1-a)P})^2}{2N_o/2}} \tag{9}$$

$$+\frac{1}{2}\frac{1}{\sqrt{2pN_o/2}}e^{-\frac{(r_1+|h_1|\sqrt{aP})^2}{2N_0/2}}$$

and

$$p_{R_1|B_1}(r_1|b_1=1) = \frac{1}{2}\frac{1}{\sqrt{2pN_o/2}}e^{-\frac{(r_1+|h_1|\sqrt{aP})^2}{2N_0/2}}$$

$$+\frac{1}{2}\frac{1}{\sqrt{2pN_o/2}}e^{-\frac{(r_1|h_1|\sqrt{aP}+|h_1|\sqrt{2(1-a)P})^2}{2N_o/2}} \tag{10}$$

Note that based on the translation property of the entropy, the equation (8) is simplified as

$$c_1^{(b;0\,NOMA)} = -\int_{-\infty}^{\infty} p_{R_1}(r_1)\log_2(r_1)dr_1 \tag{11}$$

$$+\int_{-\infty}^{\infty} p_{R_1|B_1}(r_1|b_1=0)\log_2 p_{R_1|B_1}(r_1|b_1=0)dr_1$$

$$+\int_{-\infty}^{\infty} p_{R_1|B_1}(r_1|b_1=1)p_{B_1}(b_1=1)\log_2 p_{R_1|B_1}(r_1|b_1=0)dr_1$$

where

$$p_{R_1}(r_1) = \frac{1}{4}\frac{1}{\sqrt{2\pi N_o/2}}e^{-\frac{(r_1|h_1|\sqrt{aP}+|h_1|\sqrt{2(1-a)P})^2}{2N_o/2}} \tag{12}$$

$$+\frac{1}{4}\frac{1}{\sqrt{2\pi N_o/2}}e^{-\frac{(r_1+|h_1|\sqrt{aP})^2}{2N_0/2}}$$

$$+\frac{1}{4}\frac{1}{\sqrt{2\pi N_o/2}}e^{-\frac{(r_1+|h_1|\sqrt{aP})^2}{2N_0/2}}$$

$$+\frac{1}{4}\frac{1}{\sqrt{2\pi N_o/2}}e^{-\frac{(r_1|h_1|\sqrt{aP}+|h_1|\sqrt{2(1-a)P})^2}{2N_o/2}}$$

If we plug $\alpha = 0$ into the equation (11),

$$c_1^{(b;0\,NOMA)} \simeq \frac{1}{2} \tag{13}$$

where the approximate sign happens due to the approximate entropy calculation. The calculation error is small and tolerable, resorting to the $68-95-99.7$ rule, for $N(0,1^2)$ and $Q(x) = \int_x^{\infty} e^{-\frac{z^2}{2}}/\sqrt{2\pi}\,dz$,

$$Q(3) \simeq 0.0015,\ Q(2) \simeq 0.025\ Q(1) \simeq 0.16 \tag{14}$$

Q.E.D.

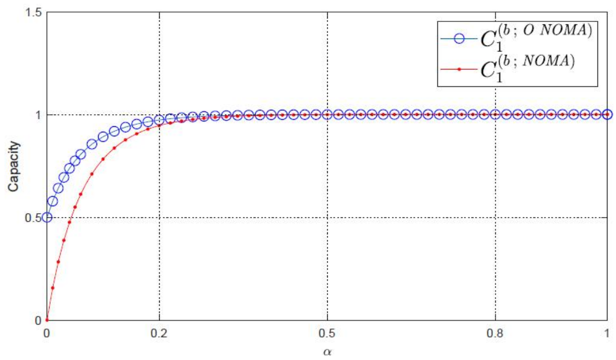

Fig. 3. Capacities of perfect SIC NOMA and O NOMA for the user-1.

Ⅴ. Results and Discussions

Assume that the channel gain is $|h_1| = 1$.

The total transmit signal power to one-sided power spectral density ratio is $P/N_o = 10$, $(10\,dB = 10\log_{10}(10))$. The channel capacities of the perfect SIC NOMA [6] with the binary constellation and O NOMA in the equation for the user-1 are shown in Fig. 1, with different power allocations, $0 \le \alpha \le 1$. As shown in Fig. 1, the channel capacity of O NOMA is better than that of the perfect SIC NOMA, for the user-1, for the entire range of the power allocation factor. A striking feature for O NOMA is that even though no power is allocated to the user-1, i.e., at $\alpha = 0$, the user-1 can transmit the input information at the maximum rate of 0.5 bit/s/Hz. Specifically, the capacity of O NOMA is better than that of the perfect SIC NOMA, less than the power allocation factor $\alpha \simeq 0.2$. The maximum improvement of 0.5 bit/s/Hz is observed at $\alpha = 0$.

Ⅵ. Conclusion

In this paper, the perfect SIC performance achieving O NOMA was proved. Furthermore, it was shown that the O NOMA user-1 capacity is even better than the perfect SIC NOMA user-1 capacity. Consequently, O NOMA could be considered in NOMA.

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