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# Volatility and Z-Type Jumps of Euro Exchange Rates Using Outlying Weighted Quarticity Statistics in the 2010s*

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## Abstract

***Purpose*** – This paper examines the recently realized continuous volatility and discrete jumps of US Dollar/Euro returns using the frequency of five minute returns spanning the period from February 2010 through February 2018with periodicity filters.
***Design/Methodology*** - This paper adopts the nonparametric estimation. The realized volatility and Realized Outlying Weighted variations show non-Gaussian, fat-tailed, and leptokurtic distributions. Some significant volatility jumps in returns occurred from 2010 through 2018, and the very exceptionally large and irregular jumps occurred around 2010-2011, after the EU financial crisis, and 2015-2016. The outliers occurred somewhat frequently around the years of 2015 and 2016.
***Originality/value*** - When we include periodicity filters of volatility such as MAD, Short Half Scale, and WSD, the five minute returns of US Dollar/Euro exchange rates have smaller daily jump probabilities by 20-30% than when we do not include the periodicity filters of volatility. Thus, when we consider the periodicity filters of volatility such as MAD, Short Half Scale, and WSD, the five minute returns of US Dollar/Euro have considerably smaller jump probabilities.

**Keywords**: Euro, Exchange Rate, Periodicity Filter, Volatility, Z-Type Jump Statistic
**JEL Classifications**: F3, F4

## 1. Introduction

Volatility has been one of the most important fields in financial econometrics. It is almost impossible to forecast to the return rates of financial assets such as stocks and exchange rates, but it is possible to forecast some parts of volatility in asset returns. In particular, discontinuous jumps frequently occurred, especially right before and after, or during the world financial crisis in 2007 and 2008. This financial crisis reassures the importance of the estimation of irregular and discontinuous jumps.

Euro-Dollar exchange rates frequently were volatile around the world financial crisis of 2008. Since the uncertainty of Euro-Dollar exchange rates may seriously decrease world trade and influence Korea's trade as well as the world economy, it is important to estimate the volatility of Euro-Dollar exchange rates as key world currencies. The US Dollar exchange rate per Euro was 1.3412 on September 24, 2010, 1.0963 on March 5, 2015, and 1.1100 on September 5, 2015. This paper focuses on the volatility of the Euro-Dollar exchange rate and jumps, and examines the effectiveness of the estimation of the volatility of exchange rates. How do we estimate and predict the volatility and jumps in Euro-Dollar exchange rates in the 2010s? An important question, often left unaddressed, is whether one should incorporate jumps in the volatility process.

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While parametric approaches such as stochastic volatility and GARCH models contribute to the theoretical development of volatility models, they have several important drawbacks, since the parametric approaches rely on explicit functional forms which can be inherently mis-specified. However, the previous parametric approaches make it very difficult to estimate frequent jump diffusion in the volatile financial crisis era. Parametric models using low frequency data are, however, not informative enough to explain and forecast the volatility of exchange rates when discontinuous jumps occur frequently. Thus, this paper will adopt nonparametric approaches to forecast the volatility of Euro-Dollar exchange rates.

This paper is in line with the earlier research of Andersen, Bollerslev and Diebold (2007), Barndorff-Nielsen and Shephard (2005a/2005b/2006), Lee and Mykland (2008) and Yi (2014). To analyze jumps from continuous volatility, this paper uses a non-parametric realized volatility model to explain discrete jumps as well as continuous volatility. This paper analyzed US Dollar-Euro returns using five minute returns spanning the period of February 2010 through February 2018. This paper is, however, much different from the earlier models and contributes in the following aspects.

First, since it is almost impossible to analyze discontinuous jumps using low frequency dates, to analyze the recently realized discrete jump volatility of US Dollar/Euro exchange rates, this paper uses 5minute returns on US Dollar/Euro exchange rates. This paper characterizes the distributions of realized returns, and then uses several jump statistics to find whether there is a significant jump or not.

Second, more strikingly, this paper introduces more robust jump statistics using more efficient realized outlying weighted variances as developed by Boudt, Croux, and Laurent (2011a/2011b) than earlier jump statistics using the Bipower variance introduced by Andersen, Bollerslev and Diebold(2004/2007), Barndorff-Nielsen and Shephard (2004a/2004b/2005a/2005b/2006), Bibinger et al. (2014), Bollerslev and Todorov (2011), and Lahaye, Laurent and Neely (2011).

Third, this paper adopted periodicity window factors. Thus, this paper is different from Bibinger and Winkelmann (2015), Chatrath et al. (2014), Délèze and Hussain (2014) and Huang and Tauchen (2005) who did not consider the periodicity of volatility as this paper considers the periodicity of volatility in jump statistics.

Fourth, to obtain more robust jump estimators, this paper utilizes periodicity filters, and then compares the probability of jumps with those of no periodicity factors. In this respect, this paper is much different from previous studies, but extends the earlier contributions of Andersen, Bollerslev and Diebold (2007), Huang and Tauchen (2005) and Lee and Mykland (2008) who did not consider the periodicity window factors of volatility or volatility patterns.

Following Section 1, Section 2 reviews the previous studies. Section 3 introduces the models of realized volatility and jump statistics, which include the several periodicity filters of volatility. Section 4 explains the empirical results of several jump statistics associated with jump probabilities. Section 5 concludes empirical findings.

## 2. Literature Review

During past decades, Andersen and Bollerslev (1988b), Andersen et al. (2002), Chernov et al. (2003), Eraker (2004), Eraker, Jahannes and Polson (2003), Johannes (2004), and Pan (2002) analyzed jump volatility, mainly with parametric approach models such as stochastic volatility and GARCH models.

In recent decades, Andersen et al. (2001/2003), Andersen, Bollerslev, and Diebold (2002/2004), and Barndorff-Nielsen and Shephard (2004a/2004b/2005a/2005b/ 2006) introduced and developed nonparametric approaches which used high frequency daily and intraday asset return data. Bandi and Russell (2005a, 2005b) analyzed the microstructure of the volatility of financial assets. A nonparametric approach using realized volatility with high frequency intraday returns can capture discontinuous jump volatility as well as the continuous volatility of financial asset returns.

Thus, to examine continuous and discontinuous jump parts, Andersen and Bollerslev (1998a), Andersen et al. (2003b), Comte and Renault (1998), Fleming, Kirby and Ostdiek (2003), and Huang and Tauchen (2005) began to analyze the total variation, which can be separated into continuous variation parts and discontinuous jump parts.

In recent years, Bollerslev and Todorov (2011) estimated jump tails and premia which could not be explained by continuous volatility. Lahaye, Laurent and Neely (2011) examined jumps, cojumps, and macro announcements. Bibinger and Winkelmann (2015) estimated cojumps in high-frequency data with noise. Laakkonen and Lanne (2013) analyzed the impact of macroeconomic news on exchange rate volatility. Bibinger et al. (2014) estimated the spot covariation of assets. Dewachter et al. (2014) examined the intra-day impact of communication on euro-dollar volatility and jumps. Chatrath et al. (2014) analyzed currency jumps, cojumps, and the role of macro news, and Délèze and Hussain (2014) examined information arrival, jumps, and cojumps in European financial markets.

# 3. Measurement of Realized Volatility and Jumps

## 3.1. Volatility Model

Let's consider the following Brownian Semimartingale Process with jumps for the logarithmic price at time t, p(t) in the equation (1). Under this model, the diffusion component captures the smooth variation of the price process, while the jump component accounts for rare and large discontinuities in the observed prices, as shown in Andersen, Bollerslev and Dieboldv (2007) and Yi (2014).

$$dP(t) = \mu(t)dt + \sigma(t)dW(t) + \kappa(t)dq(t), \quad 0 \le t \le T \tag{1}$$

Where μ(t) is a continuous and locally bounded variation process, σ(t) id a strictly positive and stochastic volatility process,W(t) is standard Brownian Motion, q(t) is a counting process with time varying intensity κ(t) which is the size of the corresponding discrete jumps in the logarithmic price process, and dq(t) is a counting process with dq(t)=1 corresponding to a jump at time t, and dq(t) =0 otherwise.

In the equation, this paper disposes of T days of M equally spaced intraday returns. In equation (2), $r_{t,j}$ denotes the j-th intraday return of day t. M represents the observed intraday sampling frequency.

$$r_{t,j} = p(t-1+\frac{j}{M}) - p(t-1+\frac{j-1}{M}), \quad j=1, 2, ..., M. \tag{2}$$

The daily realized volatility is defined as the summation of realized intraday squared returns following the work of Barndorff-Nielsen and Shephard (2004a) in this paper. Thus, the daily realized volatility y($RV_t$) or variation of day t is represented in the summation of the varied frequently intraday realized squared variation in equation (3)

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2, \quad j=1, 2, ..., M. \tag{3}$$

The daily realized volatility converges at the increment of the quadratic variation process as the sampling frequency (M) of the underlying returns goes to infinity, or ((1/M)≡Δ) goes to zero as in Andersen and Bollerslev as well as Diebold(2007) pointed out. In reality, however, the jumps in stock prices occurred occasionally, and the occurrence of jumps is generally assumed to follow a Poisson, which is a continuous-time discrete process in which the realized volatility inherits the continuous sample path process and the discrete jump process. In the presence of jumps, the realized volatility is no longer a consistent estimator of integrated volatility. Thus, for $\Delta \rightarrow 0$, the daily realized volatility at day t converges in the probability of the sum of the continuous integrated variance and the daily summation of discrete N jumps a size of $\kappa_t$, as in equation (4).

$$\lim_{M \to \infty} RV_t = \int_{t-1}^{t} \sigma^2(s)ds + \sum_{j=1}^{N_t} \kappa_{t,j}^2 . \tag{4}$$

This paper adopts the standard realized Bipower variation ($BV_t$), which Barndorff-Nielsen and Shephard (2004a, 2004b/2006) developed. Huang and Tauchen (2005) showed that the realized Bipower variation ($BV_t$) converges in a probability of integrated variation, as $\Delta \rightarrow 0$ or M becomes sufficiently large.

$$BV_t = \mu_1^{-2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M}|r_{t,j-1}||r_{t,j}| = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M}|r_{t,j-1}||r_{t,j}|,$$

$$\lim_{M \to \infty} BV_t = \int_{t-1}^{t} \sigma^2(s)ds , \quad \mu_1 = \sqrt{\frac{2}{\pi}} = E(|Z|), \ Z \sim N(0,1). \tag{5}$$

Bipower variation ($BV_t$) is robust for jumps because it uses the product between two consecutive returns instead of the squared return. As M moves to infinity and there are no jumps in the volatility of financial assets, the joint distribution of the realized volatility and the realized Bipower variation has the property of normal distribution, as shown in Barndorff-Nielsen and Shephard (2006) and Huang and Tauchen (2005).

$$M^{\frac{1}{2}}\left(\int_{t-1}^{t}\sigma^4(s)ds\right)^{-\frac{1}{2}}\begin{pmatrix} RV_t - \int_{t-1}^{t}\sigma^2(s)ds \\ BV_t - \int_{t-1}^{t}\sigma^2(s)ds \end{pmatrix} \rightarrow N\left(0, \begin{pmatrix} v_{qq} & v_{qb} \\ v_{qb} & v_{bb} \end{pmatrix}\right) \tag{6}$$

where

$$\begin{pmatrix} v_{qq} & v_{qb} \\ v_{qb} & v_{bb} \end{pmatrix} = \begin{pmatrix} \mu_4 - \mu_2^2 & 2(\mu_3\mu^{-1} - \mu_2) \\ 2(\mu_3\mu^{-1} - \mu_2) & (\mu_1^{-4} - 1) + 2(\mu_1^{-2} - 1) \end{pmatrix},$$

$$\mu_1 = \sqrt{\frac{2}{\pi}}, \mu_2 = 1, \mu_3 = 2\sqrt{\frac{2}{\pi}}, \mu_4 = 3, v_{qq} = 2, v_{qb} = 2, v_{bb} = \left(\frac{2}{\pi}\right)^2 + \pi - 3,$$

## 3.2. Multi-Powered Volatility Jump Statistic

This paper estimates the realized jump as the difference of the realized volatility (RV) and the integrated variation (IV) for $\Delta \rightarrow 0$. As Barndorff-Nielsen and Shephard (2004a/2004b/2006) established, because Bipower variation ($BV$) becomes continuous integrated volatilty or a variation (IV) which cannot be influenced by jumps, ($RV_t$ - $BV_t$) can be a consistent estimate of jump contribution. Thus, the difference between realized volatility (RV) and any robust jump estimator of integrated variation ($\bar{IV}$) is an estimate of the realized jumps (RJ) for $\Delta \rightarrow 0$.

$$RJ_t(\triangle) \equiv RV_t(\triangle) - \widehat{IV}_t(\triangle) \rightarrow \sum_{t-1}^{t} \kappa^2(s). \tag{7}$$

Andersen, Bollerslev, and Diebold (2007), Barndorff-Nielsen and Shephard (2004a), and Huang and Tauchen (2005) proposed the relative standard normal Z-type jump statistic as a ratio statistic. This paper adopts their findings using Bipower variation,

$$Z_t = \frac{RV_t - BV_t}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M} IQ_t}} \tag{8}$$

to estimate $IQ_t \equiv \int_{t-1}^{t} \sigma^4(s)ds$. To estimate $IQ_t$, this paper uses Tri-Power Quarticity ($TP_t$) and Quad-Power Quarticity ($QP_t$) as proposed by Andersen, Bollerslev and Diebold (2007), and Barndorff-NielsenandShephard (2004a).

$$TP_t = M\mu_{4/3}^{-3}\left(\frac{M}{M-2}\right)\sum_{j=3}^{M} |r_{t,j-2}|^{4/3} |r_{t,j-1}|^{4/3} |r_{t,j}|^{4/3},$$
$$\mu_{4/3} \equiv 2^{2/3}\Gamma(7/6)\Gamma(1/2)^{-1} = E(|Z|^{4/3}). \tag{9}$$

$$QP_t = M\mu_1^{-4}\left(\frac{M}{M-3}\right)\sum_{j=4}^{M} |r_{t,j-3}||r_{t,j-2}||r_{t,j-1}||r_{t,j}|. \tag{10}$$

The Z-type jump statistics are written as

$$Z_{TP,t} = \frac{RV_t - BV_t}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M} TP_t}}, \quad Z_{QP,t} = \frac{RV_t - BV_t}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M} QP_t}} \tag{11}$$

When M moves to infinity and there are no jumps, $Z_{TP,t} \rightarrow N(0,1)$, $Z_{QP,t} \rightarrow N(0,1)$.
Andersen, Bollerslev and Diebold (2004), Barndorff-Nielsen and Shephard (2005a/2005b), and advocated the use of the log version of jump statistics because the jump statistics defined in equation (11) have a tendency to reject the null hypothesis of no jump a little bit more. According to their work, this paper uses the following jump statistics, which have better finite sample properties.

$$Z_{LTP,t} = \frac{\log(RV_t) - \log(BV_t)}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\frac{TP_t}{BV_t^2}}}, \quad Z_{LQP,t} = \frac{\log(RV_t) - \log(BV_t)}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\frac{QP_t}{BV_t^2}}} \tag{12}$$

Barndorff-Nielsen and Shephard (2005a/2006) also proposed maximum versions of log jump statistics, which more asymptotically converge to standard normal distribution. Thus, this paper uses the max versions of log jump statistics like Huang and Tauchen (2005).

$$Z_{MTP,t} = \frac{log(RV_t) - log(BV_t)}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M} max(1, \frac{TP_t}{BV_t^2})}}, \quad Z_{MQP,t} = \frac{log(RV_t) - log(BV_t)}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M} max(1, \frac{QP_t}{BV_t^2})}} \tag{13}$$

When M moves to infinity or $\Delta \rightarrow 0$ in high frequency data, all jump statistics in equations (11), (12), and (13) converge asymptotically to standard normal distribution under the null of no jump on day t.

Like Andersen, Bollerslev and Diebold (2007), to filter out daily realized jumps, whether they are significant or not, this paper adopts the following criterion of the significant jumps, of which jump statistics are larger than the critical values on day t at the significance level.

$$J_{t,\alpha}(\triangle) \equiv I[Z_t(\triangle) > \Phi_\alpha][RV_t(\triangle) - \widehat{IV}_t(\triangle)]. \tag{14}$$

Where $I[Z_t(\Delta) > \Phi_\alpha]$is the indicator function that is 1 if a jump has been detected on day t at the $\alpha$ significance level, and 0 otherwise.

## 3.3. Outlying Weighted Quarticity Jump Statistic

However, Tri-Power Quarticity($TP_t$) and Quad-Power Quarticity($QP_t$) are downward biased in the presence of zero returns like the Bipower variation($BV_t$), which is very sensitive to zero returns in the sample and furthermore, the bias is expected to be even larger in the finite sample because it is made of products of three or four consecutive absolute returns. Based on the work of Boudt, Croux and Laurent (2011a/2011b), for the impact of jumps on the Bipower variation ($BV_t$) to be negligible, the high-frequency returns need to be observed over extremely short time intervals ($\Delta \to 0$) to avoid the jump effects on two contiguous returns and not have the effect of jumps on the price process be spread out over these short intervals.

To overcome this drawback, this paper adopts the findings of Boudt, Croux and Laurent (2011b), who have proposed to replace Bipower variation (BV) with the Realized Outlying Weighted variation (ROWV) and $IQ$ with Realized Outlying Weighted Quarticity (ROWQ) in the jump statistics in equation (8),

$$ROWVar_t = c_w \frac{\sum_{i=1}^{M} w(d_{t,i}) r_{t,i}^2}{\frac{1}{M}\sum_{i=1}^{M} w(d_{t,i})}, \quad ROWQ_t = d_w \frac{\sum_{i=1}^{M} w(d_{t,i}) r_{t,i}^4}{\sum_{i=1}^{M} w(d_{t,i})}, \quad d_{t,i} = (\frac{r_{t,i}}{\sigma_{t,i}})^2. \tag{15}$$

where w(·) is the rejection of the weight function, $d_{t,i}$ measures the local outlyingness of intra-day i-th return($r_{t,i}$) of day t, and σ is instantaneous volatility. Boudt, Croux,and Laurent (2008a/2008b) adopted Median Absolute Deviation. $c_w$ and $d_w$ are correction factors. Now, if we substitute these into equations (11), (12) and (13), we obtain equations (11)', (12)' and (13)'.

$$Z_{R,t} = \frac{RV_t - ROWV_t}{\sqrt{(\nu_{bb} - \nu_{qq})\frac{1}{M} ROWQ_t}}, \tag{11'}$$

$$Z_{LR,t} = \frac{log(RV_t) - log(BV_t)}{\sqrt{(\nu_{bb} - \nu_{qq})\frac{1}{M}(\frac{ROWV_t}{ROWQ_t^2})}}, \tag{12'}$$

$$Z_{MLR,t} = \frac{log(RV_t) - log(BV_t)}{\sqrt{(\nu_{bb} - \nu_{qq})\frac{1}{M} max(1, \frac{ROWV_t}{ROWQ_t^2})}}. \tag{13'}$$

## 3.4. Daily Jump Statistic with Periodicity Filters

Andersen, Bollerslev, and Diebold (2007), Barndorff-Nielsen and Shephard (2004a) and

Huang and Tauchen (2005) did not consider this periodicity in their studies. This paper, however, adopts robust nonparametric estimators in the presence of jumps as Boudt, Croux, and Laurent (2011a) proposed.

First of all, this paper is going to estimate daily jumps over given time intervals. To overcome the strong periodicity of the volatility of the financial asset, this paper also introduces estimators of return rates with the filters of periodicity $\widetilde{r_{t,i}} = \frac{r_{t,i}}{\widehat{f_{t,i}}}$, instead of just using ordinary return rates such as $r_{t,i}$, unlike Barndorff-Nielsen and Shephard (2005a/2005b/ 2006). These filters of periodicity are estimated by MAD, ShortH, and WSD estimation.

Therefore, this paper calculates jump statistics which consider the filters of periodicity. The estimators of return rates considering the filters of periodicity are denoted as $\widetilde{RV}$ in the following jump statistics. Also, we use ROWV instead of a Bipower variation (BV), and use Realized Outlying Weighted Quarticity ($ROWQ_t$) instead of $IQ$ in the following jump statistics.

### 3.4.1. *Jump Statistics without the Volatility Filter*

This paper calculates jump statistics which consider the filters of periodicity. The estimators of return rates considering the filters of periodicity are denoted as $\widetilde{RV}$ in the following jump statistics. Also, we use ROWVar instead of Bipower variation (BV), and use Realized Outlying Weighted Quarticity($ROWQ_t$) instead of $IQ$ in the following jump statistics,

$$PZ_{R,t} = \frac{\widetilde{RV}_t - ROWV_t}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}ROWQ_t}}, \tag{16}$$

Thus, Barndorff-Nielsen and Shephard (2004a), Andersen,Bollerslev, and Diebold (2007), and Huang and Tauchen (2005) did not consider this periodicity in their studies.

### 3.4.2. *Jump Statistics with the Volatility Filter*

This paper, however, adopts robust nonparametric estimators in the presence of jumps as proposed by Boudt, Croux and Laurent (2011a). To do this, this paper includes these periodicity filters. A variety of candidates were proposed by Boudt, Croux, and Laurent (2011a/011b).

First, this paper adopts median absolute deviation (MAD) and uses jump statistics ( $PZ_{R,t}^{MAD}$) using MAD periodicity factors as follows:

$$f_{t,i}^{MAD} = \frac{MAD_{t,i}}{\sqrt{\frac{1}{M}\sum_{j=1}^{M} MAD_{t,j}^2}}.$$

$$PZ_{R,t}^{MAD} = \frac{\frac{RV_t}{f^{MAD}} - ROWV_t}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}ROWQ_t}}. \tag{17}$$

Second, amongst the large number of robust scale estimators available[1], this paper adopts the Shortest Half scale estimator consistent in the presence of infinitesimal contaminations

[1] See Maronna, Martin, and Yohai (2006).

by jumps in the data[2]. Importantly, according to Boudt, Croux, and Laurent (2011a), it has the property of the smallest maximum bias possible that jumps can cause among a wide class of scale estimators. Then, the Shortest Half estimator (ShortH) for the periodicity factor and the jump statistics using ShortH( $PZ_{R,t}^{Short\ H}$) using the ShortH periodicity factors are as follows:

$$f_{t,i}^{Short\,H} = \frac{ShortH_{t,i}}{\sqrt{\frac{1}{M}\sum_{j=1}^{M} ShortH_{t,j}^{2}}} \,. \tag{18}$$

$$ShortH_{t,i} = 0.741\min\left\{\overline{r_{(h_{t,i});t,i}} - \overline{r_{(1);t,i}}, \ldots, \overline{r_{(n_{t,i});t,i}} - r_{(h_{t,i+1});t,i}\right\}.$$

$$PZ_{R,t}^{Short\ H} = \frac{\frac{RV_t}{f^{Short\ H}} - ROWV_t}{\sqrt{(\nu_{bb}-\nu_{qq})\frac{1}{M}ROWQ_t}}. \tag{19}$$

The shortest half dispersion is highly robust to jumps. Boudt, Croux, and Laurent (2011a) showed that a better trade-off between the efficiency of the standard deviation under normality and the high robustness to jumps of the shortest half dispersion is offered by standard deviation applied to the returns weighted in function of their outlyingness. Then, the weighted standard deviation (WSD) and the jump statistics using WSD ( $PZ_{R,t}^{WSD}$) using the WSD periodicity factors are as follows:

$$f_{t,i}^{WSD} = \frac{WSD_{t,i}}{\sqrt{\frac{1}{M}\sum_{j=1}^{M} WSD_{t,j}^{2}}} \,. \quad WSD_{t,j} = \sqrt{1.081\frac{\sum_{l=1}^{n_{t,j}} w[(\overline{r_{l:t,j}}/f_{t,j}^{SHortH})^2]\,\overline{r_{l:t,j}^2}}{\sum_{l=1}^{n_{t,j}} w[(\overline{r_{l:t,j}}/f_{t,j}^{SHortH})^2]}}$$

$$PZ_{R,t}^{WSD} = \frac{\frac{RV_t}{f^{WSD}} - ROWV_t}{\sqrt{(\nu_{bb}-\nu_{qq})\frac{1}{M}ROWQ_t}}, \tag{20}$$

# 4. Empirical Results

## 4.1. Data Description

The empirical analysis is based on data from Olsen and Associates in Zurich, Switzerland. The data set consists of five minute observations on US Dollar/Euro exchange rates from February 2010 through February 2018. All volatility measures are based on five minute returns as the first difference of the logarithm of US Dollar/Euro exchange rate, which results in a total of 265 high frequency return observations per day. This five minute interval is short enough for the underlying realized volatility measures to work long and well enough for market microstructure frictions not to overwhelm.

This paper removes holiday weekends and several fixed holidays. After removing holidays and other inactive trading holidays, we have a total of 2,529 days. The corresponding daily returns of the US Dollar/Euro for 2,529 days can be represented as

[2] See Rousseeuw and Leroy (1988).

$r_{t+1} \equiv r_{t+1,1} \equiv r_{t+\Delta,\Delta} + r_{t+2\Delta,\Delta} + \cdots + r_{t+1,\Delta}$, t= 1, 2,···, 2,529, where Δ is the 5 minute interval. Thus, this paper has a total of 670,185(=2,529 X 265) sample observations.

## 4.2. Basic Statistics and Density Function Distribution

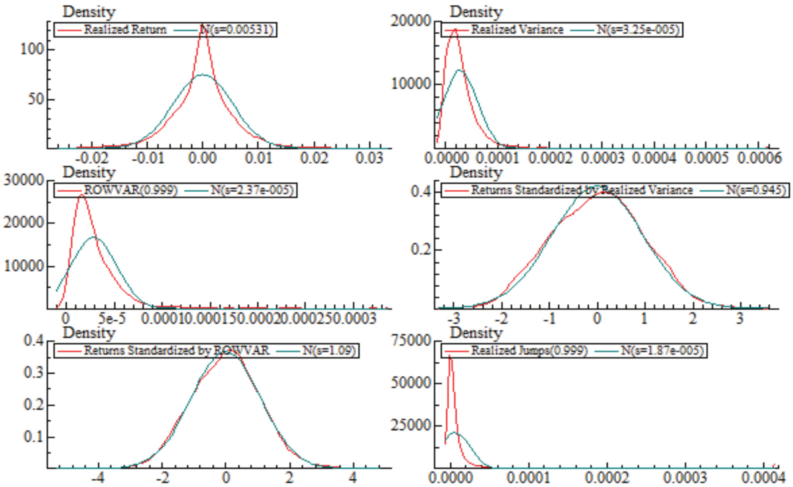
Table 1 shows the basic statistics of the Realized Returns (RR), the Realized Variances (RV), the Realized BipowerVariation (BV),the Realized Jumps (RJ_BV) using Realized Bipower Variation (BV), the realized jump statistics (RJ_Stat) using tripower variation, the Realized Outlying Weighted variation (ROWVAR), the realized jumps (RJ_ROWVAR) using Realized Outlying Weighted variation (ROWVAR), and Max Outlyingness, respectively. In Table 1 min represents minimum values, max represents maximum values, and std.dev means standard deviation.

**Table 1.** Basic Statistics

| Variable | Min | Mean | Max | Std.dev |
|---|---|---|---|---|
| RR | -0.023923 | 3.3216e-005 | 0.030437 | 0.0053114 |
| RV | 0 | 2.8906e-005 | 0.00060597 | 3.2452e-005 |
| BV(0.999) | 0 | 2.8328e-005 | 0.00060597 | 3.1397e-005 |
| RJ_BV(0.999) | 0 | 1.4589e-006 | 0.00012327 | 6.1075e-006 |
| RJ-Stat | 2.6346 | 6.457 | 50.629 | 3.5556 |
| ROWVAR(0.999) | 6.9577e-007 | 2.8694e-005 | 0.00032191 | 2.3743e-005 |
| RJ_ROWVAR(0.999) | 0 | 6.0531e-006 | 0.00040362 | 1.8659e-005 |
| Max Outlyingness | 6.941 | 54.335 | 2563.3 | 104.08 |

**Note**: Min represents minimum values, mean represents the average, max represents maximum values, and std.dev means the standard deviation. The number in parenthesis stands for the significance level.

**Fig. 1.** Density Functions




**Note**: This figure compares the density function of the realized return (RR), the realized variance (RV), Realized Outlying Weighted variation(ROWVAR),the returns standardized by RV, the returns standardized by ROWVAR, and the realized jumps with the normal density function.

Fig. 1 compares the density functions of the Realized Outlying Weighted variation (ROWVAR), the realized returns standardized by the realized variance, the realized returns standardized by Realized Outlying Weighted variation(ROWVAR) and the realized jump statistics (RJ_Stat) of the US Dollar/Euro(RR) and the realized jumps with the normal density function, respectively. In Fig. 1, the solid lines represent the realized density functions, and the dashed lines represent the normal density functions (N) with the standard deviations(s).

The density function of the Realized Outlying Weighted variation (ROWVAR at 0.999 critical level) and the realized jump statistics (RJ_Stat) of the US Dollar/Euro show non-Gaussian, fat-tailed, and leptokurtic distributions when compared to the normal distribution, as expected. However, both the returns standardized by RV and the returns standardized by ROWVAR show asymptotically normal distributions.

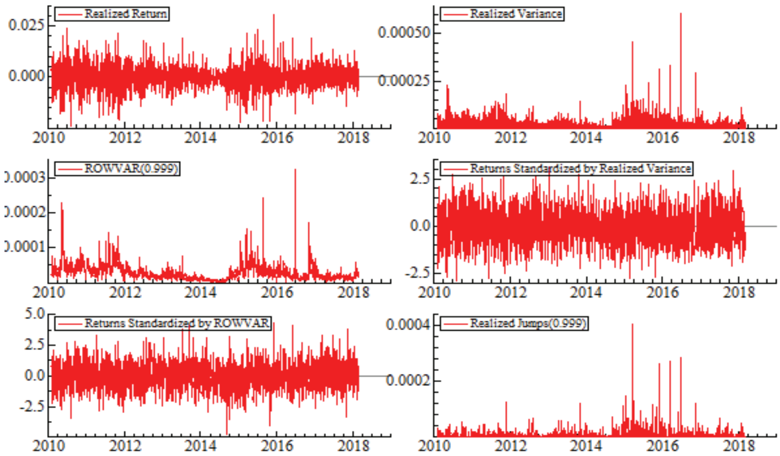
## 4.3. Volatility and Jump Distribution

Fig. 2 refers to the graph of the volatility of realized return (RR), the realized variance (RV), realized Outlying Weighted variation (ROWVAR), returns standardized by RV, returns standardized by ROWVAR, and the realized jumps.

In the left top panel, the realized returns (RR) of the US Dollar/Euro appeared to very volatile around zero, and some large jumps occurred during 2010-2011 of the Euro crisis, and 2015-2016. After 2012, the returns became less volatile. Thus, the size of jumps for the exchange rate of the US Dollar/Euro increased extraordinarily around 2010 and 2016. After the 2nd half of 2016, the size of jumps for the returns of the US Dollar/Euro became smoother, or relatively smaller, due to the counter actions of most countries. Fig. 2 also shows the stylized volatility clustering effects and dynamic dependence of the returns of the US Dollar/Euro over the period.

The returns standardized by RV and the returns standardized by ROWVAR for the returns of the US Dollar/Euro also show volatility clustering effects like the returns of the US Dollar/Euro, and also show that some jumps occurred in the second half of 2010, and extremely large jumps occurred especially more during 2015-2016, as shown in Fig. 2.

**Fig. 2.** Volatility and Jump Distribution




**Note**: This figure shows the volatility of realized return (RR), the realized variance (RV), realized Outlying Weighted variation (ROWVAR), returns standardized by RV, returns standardized by ROWVAR, and realized jumps.

As seen in the right panel at the bottom of Fig. 2, very significant volatility jumps of returns (RJ) occurred during 2015 through 2016. Thus, the volatility of returns of the US Dollar/Euro should include not only the continuous volatility and significant discrete jumps volatility process as Andersen, Bollerslev and Diebold (2007) argued.

## 4.4. Jump Statistic and Probability with Non-Filtered Returns

To overcome the drawback of the downward bias of Quad-power quarticity and Tri-power quarticity in the presence of zero returns and have a more robust and efficient estimator, the integrated variance (IV) is replaced by ROWV, and integrated quarticity (IQ) is replaced by Realized Outlying Weighted Quarticity (ROWQ) as in equations (11)', (12)' and (13)'. Then, we also consider a no periodicity case and aperiodicity case of volatility, and compare the two cases. Also, we examine jump statistics in intraday observations (n=MT).

First, we use ROWQ and intra-day periods, but we do not use any periodicity filter in jump statistics. In Z statistic equation (21), we use non-filtered realized returns (RV) instead of the filtered returns realized returns($\tilde{RV}$). Table 2 shows that at the significance level $\alpha$=0.999($\Phi_{\alpha}$ =3.09023), 2,049 significant jumps occurred among days during 2010-2018, and the proportion of days with significant jumps appeared to be 81.02%, which corresponds to about one jump per every 1.23 days.

**Table 2.** Jump Probability Without a Periodicity Filter

| a.($\alpha$=0.999) | $Z_{R,t}$ | $Z_{LR,t}$ | $Z_{MLR,t}$ |
|---|---|---|---|
| Critical Value | 3.09023 | 3.09023 | 3.09023 |
| Expected Jumps under Ho =No Jump | 2.529 | 2.529 | 2.529 |
| Detected Number of Jumps | 2,049 | 2,046 | 2.028 |
| Probability of Jumps | 0.8102 | 0.8090 | 0.8019 |
| b.($\alpha$=0.995) | $Z_{R,t}$ | $Z_{LR,t}$ | $Z_{MLR,t}$ |
| Critical Value | 2.57583 | 2.57583 | 2.57583 |
| Expected Jumps under Ho =No Jump | 12.645 | 12.645 | 12.645 |
| Detected Number of Jumps | 2,060 | 2,059 | 2,051 |
| Probability of Jumps | 0.8142 | 0.8142 | 0.8110 |
| c.($\alpha$=0.990) | $Z_{R,t}$ | $Z_{LR,t}$ | $Z_{MLR,t}$ |
| Critical Value | 2.32635 | 2.32635 | 2.32635 |
| Expected Jumps under Ho =No Jump | 25.29 | 25.29 | 25.29 |
| Detected Number of Jumps | 2,070 | 2,067 | 2,061 |
| Probability of Jumps | 0.8185 | 0.8173 | 0.8149 |

**Note**: This table shows the Z jump frequencies and jump probabilities of the realized returns of the US Dollar versus Euro without considering volatility periodicity filters. This table shows that the critical levels ($\alpha$), critical values, the numbers of expected jumps, the detected numbers of jumps using ROWV (Realized Outlyingness Weighted Variance), and Realized Outlying Weighted Quarticity (ROWQ). $Z_{R,t}$, $Z_{LR,t}$, and $Z_{MLR,t}$ represent the jump statistics of simple Z jump, log Z jump, and max Z log jumps, respectively.

Secondly, turning to the log version of jump statistics in equation (22), in Table 2 at significance level $\alpha$=0.999($\Phi_{\alpha}$=3.09023), 2,046 significant jumps occurred among 2,529 days during 2010-2018, and the proportion of days with significant jumps appeared to be 80.90%. Thirdly, according to the max log version of jump statistics in equation (23), as shown in Table 2, 2,260 significant jumps occurred among 2,529 days during 2010-2018, and the proportion of days with significant jumps appeared to be 80.19% at the significance level $\alpha$ =0.999($\Phi_\alpha$=3.09023) which corresponds to about one jump per every 1.25 days.

At the significance level $\alpha$ =0.995( $\Phi_\alpha$ =2.57583), according to the Z statistic, 2,060 significant jumps occurred at a proportion of 81.42%. According to the log Z statistic and max log Z statistic, 2,059 and 2,051 significant jumps occurred with high proportions of 81.42% and 81.10%, respectively. At the significance level $\alpha$=0.990($\Phi_\alpha$=2.32635), according to the Z statistic, 2,070 significant jumps occurred at a proportion of 81.85%. According to the log Z statistic and max log Z statistic, 2,067 and 2,061 significant jumps occurred with high proportions of 81.73% and 81.49%, respectively, as shown in Table 2.

## 4.5. Jump Statistic and Probability with Filtered Returns and Periodicity Filters

### *4.5.1. Median Absolute Deviation(MAD) Filter*

Table 3 refers to the jump detection probability with filtered returns with the MAD periodicity window using intra-day periods for the outlyingness jump statistics. According to the Z jump statistic, at the significance level $\alpha$=0.999($\Phi_\alpha$=3.09023), 1,589 significant jumps occurred among 2,529 days during 2010-2018, and the proportion of days with significant jumps appeared to be 62.83%. According to the log version of jump statistics, 1,551 significant jumps occurred with a proportion of jumps at61.33%. According to the max log version of jump statistics, 1,549 significant jumps occurred with the proportion of jumps at 61.25% among 2,529 days during 2010-2018.

**Table 3.** Standard Z Daily Jump Probability Using a MAD Filter

| **a.($\alpha$=0.999)** | $PZ_{R;t}^{MAD}$ | $Log\,PZ_{R;t}^{MAD}$ | $\max Log\,PZ_{R;t}^{MAD}$ |
|---|---|---|---|
| Critical Value | 3.09023 | 3.09023 | 3.09023 |
| Expected Jumps under Ho =No Jump | 2.529 | 2.529 | 2.529 |
| Detected Number of Jumps | 1,589 | 1,551 | 1,549 |
| Probability of Jumps | 0.6283 | 0.6133 | 0.6125 |
| **b.($\alpha$=0.995)** | $PZ_{R;t}^{MAD}$ | $Log\,PZ_{R;t}^{MAD}$ | $\max Log\,PZ_{R;t}^{MAD}$ |
| Critical Value | 2.57583 | 2.57583 | 2.57583 |
| Expected Jumps under Ho =No Jump | 12.65 | 12.65 | 12.65 |
| Detected Number of Jumps | 1,679 | 1,657 | 1,657 |
| Probability of Jumps | 0.6639 | 0.6552 | 0.6552 |
| **c.($\alpha$=0.990)** | $PZ_{R;t}^{MAD}$ | $Log\,PZ_{R;t}^{MAD}$ | $\max Log\,PZ_{R;t}^{MAD}$ |
| Critical Value | 2.32635 | 2.32635 | 2.32635 |
| Expected Jumps under Ho =No Jump | 25.29 | 25.29 | 25.29 |
| Detected Number of Jumps | 1,716 | 1,693 | 1,692 |
| Probability of Jumps | 0.6785 | 0.6694 | 0.6690 |

**Note**: This table shows the Z jump frequencies and jump probabilities of the realized returns of the US Dollar versus Euro, considering MAD filters. This table shows the critical levels($\alpha$), critical values, the number of expected jumps, the detected numbers of jumps using ROWV (Realized Outlyingness Weighted Variance) and Realized Outlying Weighted Quarticity (ROWQ). $PZ_{R;t}$, $Log\,PZ_{LR;t}$, and $\max Log\,Z_{MLR;t}$ represent the jump statistics of the simple Z jump, log Z jump, and max Z log jumps with MAD jump periodicity, respectively.

At the significance level $\alpha$=0.995($\Phi_\alpha$=2.57583), according to the Z statistic, 1,679 significant jumps occurred with a proportion of 66.39%. According to the log Z statistic and max log Z statistic, 1,657 and 1,657 significant jumps occurred with high proportions of 65.52% and 65.52%, as shown in Table 3. At the significance level $\alpha$=0.990($\Phi_\alpha$=2.32635), according to the Z statistic, 1,716 significant jumps occurred at a proportion of 67.85%. According to the log Z statistic and max log Z statistic, 1,693 and 1,692 significant jumps occurred with high proportions of 66.94% and 66.90%, respectively, as shown in Table 3.

### *4.5.2. Shortest Half Scale(ShortH) Periodicity Filter*

Table 4 presents the jump detection probability with filtered returns with the Shortest Half Scale periodicity filter using intra-day periods for the outlyingness jump statistics. According to the Z jump statistic, at the significance level $\alpha$=0.999($\Phi_\alpha$=3.09023), 1,587 significant jumps occurred among 2,529 days during 2010-2018, and the proportion of days with significant jumps appeared to be 62.75%. According to the log version of jump statistics, 1,546 significant jumps occurred, with the proportion of jumps at 61.13%. According to the max log version of jump statistics, 1,544 significant jumps occurred with the proportion of jumps at 61.05% among 2,529 days during 2010-2018.

**Table 4.** Standard Z Daily Jump Probability Using the Shortest Half Scale Filter

| **a.($\alpha$=0.999)** | $PZ_{R:t}^{ShortH}$ | $LogPZ_{R:t}^{ShortH}$ | $\max LogPZ_{R:t}^{ShortH}$ |
|---|---|---|---|
| Critical Value | 3.09023 | 3.09023 | 3.09023 |
| Expected Jumps under Ho =No Jump | 2.529 | 2.529 | 2.529 |
| Detected Number of Jumps | 1,587 | 1,546 | 1,544 |
| Probability of Jumps | 0.6275 | 0.6113 | 0.6105 |
| **b.($\alpha$=0.995)** | $PZ_{R:t}^{ShortH}$ | $LogPZ_{R:t}^{ShortH}$ | $\max LogPZ_{R:t}^{ShortH}$ |
| Critical Value | 2.57583 | 2.57583 | 2.57583 |
| Expected Jumps under Ho =No Jump | 12.645 | 12.645 | 12.645 |
| Detected Number of Jumps | 1,677 | 1,651 | 1,650 |
| Probability of Jumps | 0.6631 | 0.6524 | 0.6524 |
| **c.($\alpha$=0.990)** | $PZ_{R:t}^{MAD}$ | $LogPZ_{R:t}^{MAD}$ | $\max LogPZ_{R:t}^{MAD}$ |
| Critical Value | 2.3263 | 2.3263 | 2.3263 |
| Expected Jumps under Ho =No Jump | 25.29 | 25.29 | 25.29 |
| Detected Number of Jumps | 1,724 | 1,699 | 1,696 |
| Probability of Jumps | 0.6817 | 0.6718 | 0.6706 |

**Note**: This table shows the Z jump frequencies and jump probabilities of the realized returns of the US Dollar versus Euro, considering Short Half Scale (Short H) periodicity filters. This table also shows the critical levels($\alpha$), critical values, the number of expected jumps, the detected number of jumps using ROWV (Realized Outlyingness Weighted Variance) and Realized Outlying Weighted Quarticity (ROWQ). $PZ_{R:t}$, $LogPZ_{LR:t}$, and $\max LogZ_{MLR:t}$ represent the jump statistics of the simple Z jump, log Z jump, and max Z log jumps with the Short Half Scale (Short H) jump periodicity, respectively.

At the significance level $\alpha$=0.995($\Phi_\alpha$=2.57583), according to the Z statistic, 1,677 significant jumps occurred with a proportion of 66.31%. According to the log Z statistic and max log Z statistic, 1,651 and 1,650 significant jumps occurred with high proportions of 65.24% and 65.24%, respectively, as shown in Table 4. Thus, the proportions of jumps are a little bit lower or higher than those of the MAD filter case, but not appreciably different.

At the significance level $\alpha$=0.990( $\Phi_\alpha$ =2.32635), according to the Z statistic, 1,724 significant jumps occurred with a proportion of 68.17%. According to the log Z statistic and max log Z statistic, 1,699 and 1,696 significant jumps occurred with high proportions of 67.18% and 67.06%, respectively, as shown in Table 4. Thus, the proportions of jumps are a little bit lower or higher than those of the MAD filter case, but not appreciably different.

### *4.5.3. Weighted Standard Deviation(WSD) Filter*

Table 5 presents jump detection probability with filtered returns with the WSD periodicity filter using intra-day periods for the outlyingness jump statistics. According to the Z jump statistic, at the significance level $\alpha$=0.999($\Phi_\alpha$=3.09023), 1,285 significant jumps occurred among 2,529 days during 2010-2018, and the proportion of days with significant jumps appeared to be 50.81%. According to the log version of jump statistics, 1,259 significant jumps occurred with the proportion of jumps at 49.78%. According to the max log version of jump statistics, 1,256 significant jumps occurred with the proportion of jumps at 49.66% among 2,529 days during 2010-2018.

At the significance level $\alpha$ =0.995( $\Phi_\alpha$ =2.57583), according to the Z statistic, 1,364 significant jumps occurred with a proportion of 53.93%. According to the log Z statistic and max log Z statistic, 1,347 and 1,343 significant jumps occurred with high proportions of 53.26% and 53.10%, respectively, as shown in Table 5. At the significance level $\alpha$=0.990($\Phi_\alpha$ =2.32635), according to the Z statistic, 1,397 significant jumps occurred with a proportion of 55.24%. According to the log Z statistic and max log Z statistic, 1,381 and 1,379 significant jumps occurred with high proportions of 54.61% and 54.53%, respectively, as shown in Table 5. Thus, the proportions of jumps appeared almost similar to those of the MAD and Shortest Half Scale filter.

**Table 5.** Standard Z Daily Jump Probability Using a WSD Filter

| **a.($\alpha$=0.999)** | $PZ_{R:t}^{WSD}$ | $Log PZ_{R:t}^{WSD}$ | $\max Log PZ_{R:t}^{WSD}$ |
|---|---|---|---|
| Critical Value | 3.09023 | 3.09023 | 3.09023 |
| Expected Jumps under Ho =No Jump | 2.529 | 2.529 | 2.529 |
| Detected Number of Jumps | 1,285 | 1,259 | 1,256 |
| Probability of Jumps | 0.5081 | 0.4978 | 0.4966 |
| **b.($\alpha$=0.995)** | $PZ_{R:t}^{WSD}$ | $Log PZ_{R:t}^{WSD}$ | $\max Log PZ_{R:t}^{WSD}$ |
| Critical Value | 2.57583 | 2.57583 | 2.57583 |
| Expected Jumps under Ho =No Jump | 12.645 | 12.645 | 12.645 |
| Detected Number of Jumps | 1,364 | 1,347 | 1,343 |
| Probability of Jumps | 0.5393 | 0.5326 | 0.5310 |
| **c.($\alpha$=0.990)** | $PZ_{R:t}^{MAD}$ | $Log PZ_{R:t}^{MAD}$ | $\max Log PZ_{R:t}^{MAD}$ |
| Critical Value | 2.32635 | 2.32635 | 2.32635 |
| Expected Jumps under Ho =No Jump | 25.29 | 25.29 | 25.29 |
| Detected Number of Jumps | 1,397 | 1,381 | 1,379 |
| Probability of Jumps | 0.5524 | 0.5461 | 0.5453 |

**Note**: This table shows the Z jump frequencies and jump probabilities of the realized returns of the US Dollar versus Euro, considering WSD filters. This table shows the critical levels($\alpha$), critical values, the number of expected jumps, the detected number of jumps using ROWV (Realized Outlyingness Weighted Variance) and Realized Outlying Weighted Quarticity (ROWQ). $PZ_{R:t}$, $Log PZ_{LR:t}$, and $\max Log Z_{MLR:t}$ represent the jump statistics of the simple Z jump, log Z jump, and max Z log jumps with WSD jump periodicity, respectively.

Thus, when we consider the periodicity filters of volatility such as MAD, Short Half Scale, and WSD, the five minute returns of the US Dollar/Euro have considerably smaller daily jump probabilities in Table 3, Table 4, and Table 5. In this respect, this paper is significantly different from previous studies by Andersen, Bollerslev and Diebold (2007), Huang and Tauchen (2005) and Lee and Mykland (2008), who did not consider the periodicity of volatility.

If we disregard this volatility periodicity or periodicity pattern, the jump probability may have a remarkable bias on the accuracy of the jump estimation and jump probability. Even though the periodicity of volatility occurs frequently in the five minute returns of US Dollar/Euro exchange rates, if we don't consider the periodicity of the volatility of US Dollar/Euro exchange rates, the estimators of the jump detection and jump probability will be overestimated.

## 5. Conclusion

This paper analyzed the recently realized continuous volatility and discrete jump volatility of US Dollar-Euro returns using ultra-high frequency five minute returns spanning a period from February 2010 through February 2018. In addition to the realized Bipower variance, this paper also used the Realized Outlying Weighted Variation (ROWV), Realized Outlying Weighted Quarticity (ROWQ) and several periodicity filters such as MAD, Short Half Scale, and WSD to obtain Z-type jump estimators.

Main findings and contributions of this paper are summarized as below. Firstly, the density functions of Realized Return, Realized Variance, Realized Outlying Weighted variation, and the realized jump statistics of the US Dollar/Euro show non-Gaussian, fat-tailed, and leptokurtic distributions.

Secondly, the jumps for the realized returns, the realized volatility, and the Realized Outlying Weighted variation of the US Dollar-Euro showed stylized volatility clustering effects and dynamic dependence of the returns of the US Dollar/Euro over the period, and the jumps for the realized returns were extraordinarily large around 2010-2012 and 2015-2016. The realized volatility and Realized Outlying Weighted variation also showed that some jumps occurred during 2015 and 2016, and extremely large jumps occurred around2010, 2015, and 2016.

Thirdly, some significant volatility jumps of returns occurred from2001 through 2009, and the very exceptionally large and irregular jumps occurred a little bit more frequently around 2015-2016. The jump statistics of the US Dollar-Euro appeared large, and the outlyingness occurred somewhat frequently around 2015 and 2016.

Fourthly, this paper examines the realized jumps, and then use several jump statistics to identify whether the observed jumps are significant or not. To calculate the jump statistics and overcome the drawback of the downward bias of Quad-power quarticity and Tri-power quarticity in the presence of zero returns and get a more robust and efficient estimator, the integrated variance (IV) is replaced by ROWV, and the integrated quarticity (IQ) is replaced by a Realized Outlying Weighted Quarticity (ROWQ).

Fifthly, we compared the results of daily jump detection using a non-filtered return test and filtered return tests. When we included the periodicity filters of volatility such as MAD, Short Half Scale, and WSD using Realized Outlying Weighted Variation, Realized Outlying Weighted Quarticity (ROWQ), the five minute returns of US Dollar/Euro exchange rates have much smaller daily Z-type jump probabilities by 20-30% than when we did not include periodicity filters of volatility.

Therefore, if we do not consider periodicity filters of volatility, we may overestimate the jump probabilities of US Dollar/Euro exchange rates. This may lead to a serious bias in the estimation of volatility and jump probabilities of US Dollar/Euro exchange rates. Thus, we have to consider periodicity filters of volatility to find more robust and efficient estimates of jumps and jump probabilities for US Dollar-Euro exchange rates. If we can include more exchange rates, not only US Dollar-Euro exchange rates, but also Korean Won-US Dollar, Korean Won-Japanese Yen, and Korean Won - Chinese Yuan exchange rates with 1 or 2 minute frequencies as well as 5 minute frequencies over a longer period, it may bring about more informative and robust jump statistics. It, however, will be very expensive to obtain data. Thus, this will be left for future work.

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