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ON THE STRUCTURAL CHANGE OF THE LEE-CARTER MODEL AND ITS ACTUARIAL APPLICATION

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Abstract. Over the past decades, the Lee-Carter model [1] has attracted much attention from various demography-related fields in order to project the future mortality rates. In the Lee-Carter model, the speed of mortality improvement is stochastically modeled by the so-called mortality index and is used to forecast the future mortality rates based on the time series analysis. However, the modeling is applied to long time series and thus an important structural change might exist, leading to potentially large long-term forecasting errors. Therefore, in this paper, we are interested in detecting the structural change of the Lee-Carter model and investigating the actuarial implications. For the purpose, we employ the tests proposed by Coelho and Nunes [2] and analyze the mortality data for six countries including Korea since 1970. Also, we calculate life expectancies and whole life insurance premiums by taking into account the structural change found in the Korean male mortality rates. Our empirical result shows that more caution needs to be paid to the Lee-Carter modeling and its actuarial applications.

1. Introduction

Over the past decades, mortality study has become an important area of research because most countries around the world have experienced a significant mortality improvement influencing their socioeconomic factors. In particular, the Lee-Carter model has attracted much attention from various demography-related researchers in order to forecast future mortality rates. Basically, the model assumes that the overall decline in mortality rates over time is governed by an index independent of age, called mortality index, which is stochastically modeled and projected through the time series analysis.

However, the modeling is applied to long time series, so there is a risk that important structural changes leading to large forecasting errors might have occurred during the period. In fact, the stability of the Lee-Carter model has been questioned in many studies ([3], [4], [5]). To address this issue, Coelho

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and Nunes [2] proposed a statistical methodology for testing structural changes in the mortality index. They considered both male and female mortality data for 18 developed countries and found the structural changes for all most every country, especially in male populations.

In this paper, following the method proposed in Coelho and Nunes [2], we intend to continue the search of structural changes for more countries and explore the actuarial implications in evaluating life insurance and annuity. For the purpose, we consider Korea, Taiwan, East Germany, UK, France and Japan. The first three are newly included in this research and the last three are for comparison with the result of Coelho and Nunes [2]. However, since the period of observation is different, the testing result could differ from that of Coelho and Nunes [2]. Also, we are interested in the impact of structural change on the actuarial valuation. That is, according to whether a structural change is taken into account or not, we would investigate the actuarial present values for life insurance and annuity and compare them to observe its effect.

The paper is organized as follows. In Chapter 2, we briefly review the Lee-Carter model and the testing methodology proposed in Coelho and Nunes [2]. In Chapter 3, we summarize the test results for structural change and, in Chapter 4, we examine the Korean male mortality data in more detail. In Chapter 5, we apply the testing result to the actuarial valuation. Finally, in Chapter 6, we conclude the paper with a short summary and discussion.

2. Preliminaries

2.1. The Lee-Carter Model

The Lee-Carter model [1] assumes that

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t}$$

where $m_{x,t}$ is the central death rate for age x and calendar year t, α_x the average value of the logarithm of the central death rate for age x, β_x the sensitivity of the log of mortality at age x to changes in the mortality index κ_t, and $\epsilon_{x,t}$ the error term that cannot be explained by the model. These errors are assumed to be white noise. The mortality index κ_t can be interpreted as how fast mortality improvement progresses over time. To ensure the identifiability of the model, two constraints are imposed upon the parameters:

$$\sum_{\forall x} \beta_x = 1 \; and \; \sum_{\forall t} \kappa_t = 0$$

Usually, κ_t is re-estimated using the estimates of α_x and β_x so that the implied number of deaths will equal the actual number of deaths, and the methods of time series analysis are applied to forecast κ_t and the future mortality rates. A lot of extensions to the Lee-Carter model and the estimation method are available. See, for instance, [6] and the papers cited therein. In this paper, we employ the original model and the estimation method based on the singular value decomposition.

2.2. Singular Value Decomposition

The ordinary regression methods cannot be used in estimating the parameters of the Lee-Carter model because there are no given regressors in (1). To overcome this difficulty, Lee and Carter [1] applied the method of singular value decomposition (SVD) to estimate the parameters. First, by the identifiability condition, the estimate of α_x is given as

$$\hat{\alpha}_x = \frac{1}{T}\sum_{t=1}^{T} \ln(m_{x,t})$$

Then we create a matrix $Z_{x,t} = \ln(m_{x,t}) - \hat{\alpha}_x$ to estimate the parameters β_x and κ_t using the SVD method as follow.

$$SVD(Z_{x,t}) = UdV^{'} = \sum_{i=1}^{r} d_i U_{x,i} V_{i,t}$$

where d_i, represents the ordered singular values of $Z_{x,t}, U_{x,t}$ and $V_{i,t}$ denote the left and right singular vectors, respectively. The first rank of SVD produces $\hat{\beta}^{(1)} = U^{'}_{x,1}$ and $\hat{\kappa}^{(1)} = d_1 V_{1,t}$. Finally, by applying the constraints, the coefficient can be obtained as $\hat{\beta}^{(1)} = U^{'}_{x,1}/\sum_{\forall x} U_{x,1}$ and $\hat{\kappa}^{(1)} = d_1 V_{1,t}\sum_{\forall x} U_{x,1}$.

2.3. Testing for a Structural Change

To identify the existence of a structural change in the mortality index, we use the test proposed in Harvey et al. [7] which is valid regardless of whether it is difference stationary or trend stationary. The model that is described here corresponds to Model A in [7]. This test is relatively easy to implement because it only considers two linear regression models and the parameters are estimated using the ordinary least squares method. If κ_x is assumed to be trend stationary, the regression equation is given as

$$\kappa_x = \alpha + \beta t + \gamma DT_t(\tau) + u_t, t = 1, 2, \dots, T,$$

where $DT_t(\tau)$ is a dummy variable defined as 0 if $t \leq T_B$ and $t - T_B$ if $t > T_B$, with $T_B = \lfloor \tau T \rfloor$ denoting the possible change of date in trend, τ a fractional date with $\tau \in (0, 1)$, and u_t stationary shocks. In contrast, if κ_x is assumed to be difference stationary, the appropriate regression model equation would be

$$\kappa_x = \beta + \gamma DU_t(\tau) + u_t, t = 2, 3, \dots, T,$$

where the dummy variable $DU_t(\tau) = 0$ if $t \leq T_B$ and $DU_t(\tau) = t - T_B$ if $t > T_B$, $\Delta\kappa_t = \kappa_t - \kappa_(t-1)$, and u_t also stationary shocks. In both equations assuming trend stationary and difference stationary, the hypothesis of no change in the trend slopes correspond to $H_0 : \tau = 0$ and the alternative hypothesis is $H_1 : \tau \neq 0$. If κ_t is assumed to be trend stationary, the test statistic is given by

$$t_0^* = \sup_{\tau \in \Lambda} |t_0(\tau)|$$

where the supremum is taken over a set $\Lambda = [\tau_L, \tau_U]$, with $0 < \tau_L < \tau_U < 1$. On the other hand, if κ_t is assumed to be difference stationary, the test statistic is given by

$$t_1^* = \sup_{\tau \in \Lambda} |t_1(\tau)|$$

The change fractions in t_0^* and t_1^* will be denoted as τ_0^* and τ_1^*, respectively. To avoid erroneous conclusions on the existence of a structural change, we do not consider the possible change fractions that are too close to the beginning or the end of the sample. The trimming parameters τ_L and τ_U equal to the first and the last 10% of the series, respectively as suggested in [7]. Finally, we compute a weighted average of t_0^* and t_1^* as

$$t_\lambda^* = \lambda t_0^* + m(1-\lambda)t_1^*$$

where the weight λ is given by

$$\lambda = \exp\{-(500 S_0^* S_1^*)^2\}$$

with S_0^* and S_1^* denoting the stationary test statistics of KwiatkowskiPhillips SchmidtShin (KPSS) [8] obtained from the residual series of ordinary least square regression evaluated at τ_0^* and τ_1^*, respectively. The value of λ converges asymptotically to 1 if the series is trend stationary or converges to 0 if the series is difference stationary, so that t_0^* or t_1^* is selected. The critical value of t_λ^* is provided in [7] and it equals 2.563 with the constant m = 0.853 for 5% confidence level.

2.4. Unit Root Test

The unit root test is used to decide whether a correct break date are selected from the suggested break date corresponding to t_0^* or t_1^*. The test starts by considering t_λ^* as a pretest. If t_λ^* is rejected in a certain confidence level, then an optimal test for a unit root in the presence of a structural change should be used. In contrast, if t_λ^* cannot be rejected, then an optimal test for a unit root without allowing for a structural change should be used. If the pretest of a structural change shows that a structural change does exist, the corresponding generalized least squares (GLS) estimation for the model in equation (2) can be rewritten as

$$\kappa_t = X_t(\tau_1^*)\theta_0 + u_t, \quad t = 1, 2, \ldots, T$$

where $X_t(\tau_1^*) = (1, t, DT_t(\tau_1^*))$ and $\theta_0 = (\alpha, \beta, \gamma)'$. The GLS estimation is implemented by estimating the following quasi-difference transformation of equation using the ordinary least squares,

$$\kappa_{c,t} = X_{c,t}(\tau_1^*)\theta_0 + u_{c,t}, \quad t = 1, 2, \ldots, T$$

where $\kappa_{c,t} = \kappa_t - \rho\kappa_(t-1)$ and $X_{c,t}(\tau_1^*) = X_t(\tau_1^*) - \rho X_{t-1}(\tau_1^*)$ for $t = 2, 3, \ldots, T, \kappa_{c,1} = \kappa_1, X_{c,1}(\tau_1^*) = X_1(\tau_1^*)$, and $\rho = 1 - c/T$. The value of c is the quasi-difference parameter and can be chosen according to a local power criterion as explained in Harris et al. [9]. Let $\hat{\theta}_{OLS}$ be the ordinary least squares estimator of θ_0 in equation (3) and $\hat{u}_t$ the residuals of the regression given as $\hat{u}_t = \kappa_t - X_t(\tau_1^*)\hat{\theta}_{OLS}$. The residual $\hat{u}_t$ series is then used to examine the existence of the unit root using the augmented Dickey Fuller (ADF) [10] regression as follows.

$$\hat{u}_t = \phi\hat{u}_{t-1} + \sum_{j=1}^{p} \delta_j \hat{u}_{t-j} + e_{p,t}, t = p + 2, \ldots, T$$

The number of lag p can be chosen by a modified Akaike Information Criterion as explained in [11]. The values of c and their critical values can be found in [9]. On the other hand, when no structural change is found by the test of [7], a unit root test without allowing for a structural change should be used such as the ADF-GLS optimal test. The test has the following two modifications:
1. The dummy variable for a structural change is excluded from the $X_t(\tau_1^*)$ vector.
2. The optimal value of c equals 13.5.

3. The Test Result for Structural Changes

The data used in this research are obtained from Human Mortality Database (HMD). Since the data periods are not equal for each country, we use the period since 1970. However, in the case of Korea, the HMD provides a shorter period of data, so we obtain the data available from the KOSTAT website. The data, consisting of gender and age specific central death rates from age 0 up to 99, cover the following periods: South Korea (1970-2016), Japan (1970-2014), Taiwan (1970-2014), United Kingdom (1970-2014), France (1970-2015), and East Germany (1970-2013).

The identification of a structural change begins by considering two scenarios. The result of the first scenario is obtained by assuming that κ_t series is known to be trend stationary, while the result of the second scenario is obtained by assuming that κ_t series is known to be difference stationary. The values of t_0^* and t_1^* then refer to the maximum assuming trend stationary and difference stationary, respectively. In the following Figures, we depict the κ_t series for both male and female. The vertical dash line represents the break year suggested by t_0^* or t_1^* depending on the result of unit root test. In the figure, vertical long dash line indicates that t_λ^* is statistically significant. It means that a structural

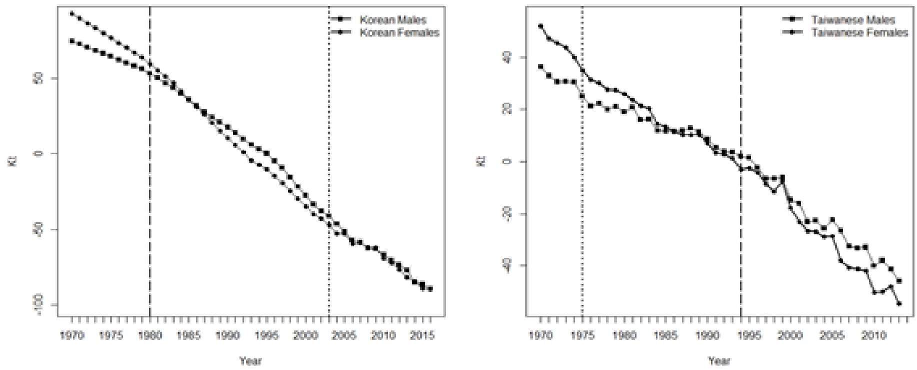

FIGURE 1. κ_t of Korean males and females (Left) and κ_t of Taiwanese males and females (Right)

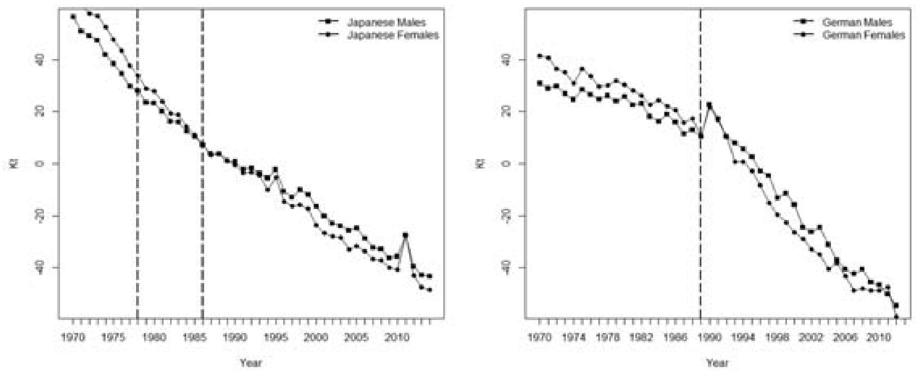

FIGURE 2. κ_t of Japanese males and females (Left) and κ_t of East German males and females (Right)

change does exist. In contrast, short dash line means that t_λ^* is statistically insignificant. In other words, there is no structural change.

In the case of Korean males, the null hypothesis H_0 is rejected at a significance level of 5%, meaning that a structural change exists. Meanwhile, H_0 cannot be rejected at a significance level of 5% for Korean females. Since there is a unit root in the κ_t series of Korean males, the break year suggested by t_1^* should be selected. According to the test result assuming difference stationary, a structural change happened in 1980. Taiwan shows a similar result to Korea. We found a structural change only for Taiwanese male in 1994, but no structural change for Taiwanese female. For Japan, we found a structural change for both male (1978) and female (1986). For UK and France, the test result shows that there is no structural change. The result for East German looks interesting. As

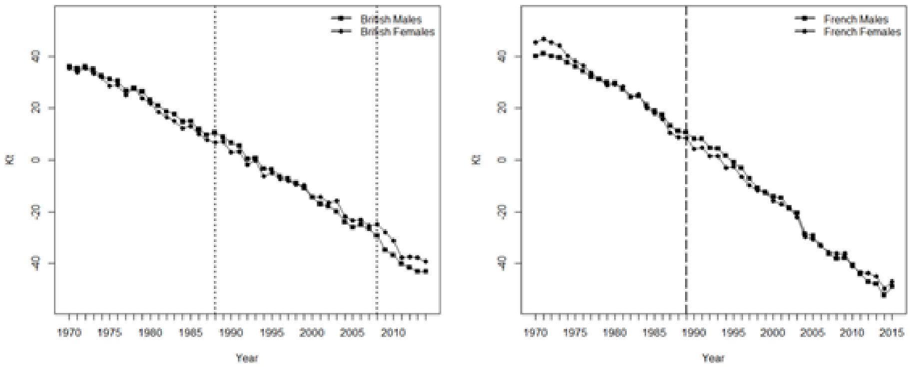

FIGURE 3. κ_t of British males and females (Left) and κ_t of French males and females (Right)

shown in Figure 2, a structural change happened for East German male and female in 1989. The result is related to a historical event where the Berlin Wall was opened in November 9^{th} 1989. Germans were officially reunited in October 3rd 1990. As can be seen in Figure 2 (right), the mortality improvement of East Germany was accelerated around the reunification. The test results are recapitulated in Table 1.

4. Modeling and Forecasting κ_t for Korean Male

4.1. Modeling κ_t for Korean Male

According to the test result in fifth column of Table 1, there exists a unit root for almost every country and both genders, except for British female and French female. This statistical evidence can be used to determine the most appropriate model for the κ_t series. If a unit root exists, an appropriate ARIMA(p, 1, q) model would be used. Otherwise, we use a trend stationary model as

$$\kappa_t = d_0 + d_1 t + u_t$$

where d_0 is an intercept, d_1 the slope of the linear trend and u_t a stationary ARMA process. To illustrate the effect of structural change in mortality forecast and its actuarial implication, we further explore the Korean male mortality index. Since a unit root exists for Korean male, ARIMA(p, 1, q) would be appropriate. First, we model the κ_t series by considering both with and without a structural change. The fitted results of modeling the κ_t series are summarized in Table 2 and Table 3.

Table 2 shows some candidate models for κ_t with the complete series. Based on the model evaluations considering the significance of the parameters, minimum AIC and maximum Log-Likelihood, ARIMA(1,1,1) should be chosen. The

TABLE 1. Structural change and unit root test results

	Results of a struc. change tests			ADF-GLS test statistic	Results of Unit Root Tests with		Conclusion	
	t_0^*	t_1^*	t_λ^*		ADF-GLS	ADF-GLS break	$I(0)$ or $I(1)$	Break Year
Male								
S. Korea	12.99	3.85	3.29	-2.49	-2.56	-2.49	I(1)	1980
Taiwan	11.28	1.50	4.74	-2.79	-1.20	-2.79	I(1)	1994
Japan	10.06	1.54	8.68	-2.81	-2.72	-2.81	I(1)	1978
UK	8.61	1.63	1.70	-3.06	-3.06	-3.42	I(1)	-
France	11.23	1.94	1.68	-2.65	-2.65	-3.76	I(1)	-
E. Germany	17.82	2.92	3.92	-2.34	-1.38	-2.34	I(1)	1989
Female								
S. Korea	6.63	2.33	1.99	-1.17	-1.17	-2.95	I(1)	-
Taiwan	6.31	0.89	0.76	-2.18	-2.18	-2.67	I(1)	-
Japan	11.46	1.55	4.77	-1.56	-2.12	-1.56	I(1)	1986
UK	2.96	1.05	0.90	-4.32	-4.32	-4.92	I(0)	-
France	3.38	1.07	0.91	-3.63	-3.63	-2.88	I(0)	-
E. Germany	10.30	2.35	2.94	-3.07	-1.85	-3.07	I(1)	1989

Rejection of the null hypothesis of no structural change at $\alpha = 0.05$.

Rejection of the null hypothesis of a unit root at $\alpha = 0.05$. The critical value is -3.19 for ADF-GLS without break and [-3.45, -3.09] for ADF-GLS with break.

TABLE 2. Korean male mortality index without allowing for a structural change

Variables	ARIMA before break			
	ARIMA(0,1,0)	ARIMA(0,1,1)	ARIMA(1,1,0)	ARIMA(1,1,1)
Constant	-3.565***	-3.559***	-3.543***	-3.424***
MA(1)	-	0.191*	-	0.860***
AR(1)	-	-	0.312**	-0.614***
Log Likelihood	-83.890	-82.510	-81.550	-79.320
AIC	171.790	171.020	169.110	166.640
Q(4) p-value	0.000	0.000	0.001	0.877

* Significant at $\alpha = 10\%$

** Significant at $\alpha = 5\%$

*** Significant at $\alpha = 1\%$

results allowing for the break in 1980 can be seen in Table 3. By using the same procedure to select the best model, ARIMA(0,1,0) or random walk model with a

Table 3. Korean male mortality index allowing for a structural change

Variables	ARIMA after break			
	ARIMA(0,1,0)	ARIMA(0,1,1)	ARIMA(1,1,0)	ARIMA(1,1,1)
Constant	-3.983***	-3.981***	-3.980***	-3.962***
MA(1)	-	0.055***	-	-0.514***
AR(1)	-	-	0.073***	0.622***
Log Likelihood	-63.250	-63.180	-63.150	-62.920
AIC	130.500	132.360	132.310	133.850
Q(4) p-value	0.755	0.000	0.000	0.927

*** Significant at $\alpha = 1\%$

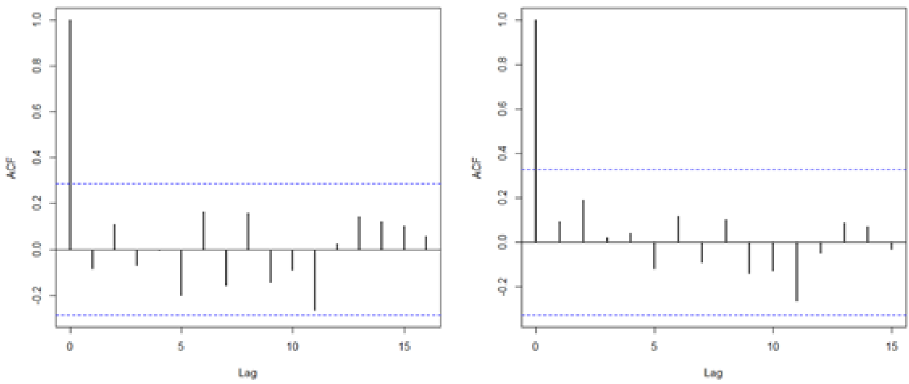

Figure 4. ACF of ARIMA(1,1,1) before break and ARIMA(0,1,0) after break

drift could be selected. We also consider residual diagnostic analysis to confirm the selected model. For the purpose, we consider the autocorrelation function and the Ljung-Box test statistic. According to Figure 4, there seems to be no correlation among residuals in both ARIMA(1,1,1) and ARIMA(0,1,0) models. In line with ACF analysis, the p-values of Ljung-Box Q-Statistics shows that the randomness assumption for the residuals is satisfied for the two models.

4.2. Forecasting κ_t for Korean Male

In this subsection, we forecast the future mortality index. The models selected in the previous section are used. In this study, we forecast the future mortality for 50 years. A comparison of ARIMA(1,1,1) before break and ARIMA(0,1,0) after break can be shown as in Figure 5. The forecasted values of κ_t obtained by allowing for a structural change show more rapid mortality improvement compared to the forecasted values of κ_t obtained without allowing

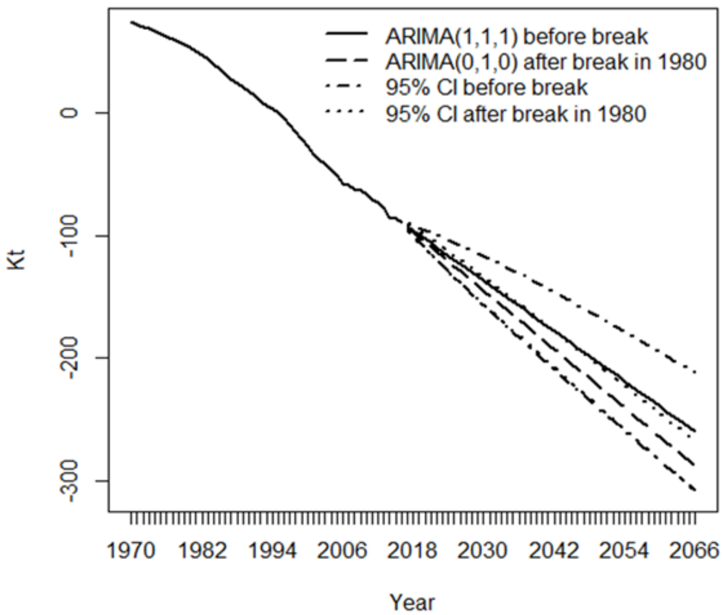

FIGURE 5. The future mortality rates with and without structural change

for a structural change. The selected model assuming a structural change also shows narrower confidence interval compared to the selected model without allowing for a structural change.

5. Actuarial Valuations for Korean Males

5.1. Forecasting Korean Males Life Expectancy

As explained in the previous section, the projected series decrease more rapidly as a result of structural change. Thus the life expectancy would be increased as illustrated in Figure 6. The life expectancies in the graph are computed by fixing a life table at a certain time point. That is, for a life aged x, its life expectancy is calculated as

$$e_{x,t} = \sum_{k=1_k}^{\infty} p_{x,t} = p_{x,t} +_2 p_{x,t} +_3 p_{x,t} + \cdots .$$

using the life table at time t. However, assuming the Lee-Carter model requires us to reflect the mortality improvement expected for the forthcoming years in the probability calculation. So the life expectancy would be obtained as

$$e_{x,t} = p_{x,t} + p_{x,t} * p_{x+1,t+1} + p_{x,t} * p_{x+1,t+1} * p_{x+2,t+2} + \cdots .$$

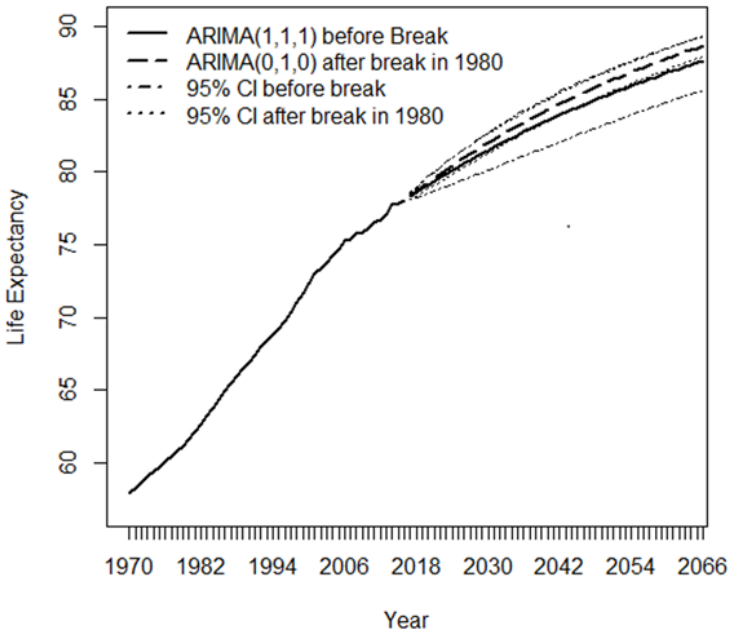

FIGURE 6. The life expectancies with and without structural change assumption

Moreover, while calculating the life expectancy in (4), we need an additional assumption on how long the future mortality improvement based on the Lee-Carter model would continue. For the purpose, we choose 20, 30, 40 and 50 years. The life expectancies for a new born, a life aged 40 and 65 are provided in Table 4, Table 5 and Table 6, respectively. In the square bracket are the confidence intervals at 95% confidence level. For instance, Table 4 shows the life expectancies for a new born. The life expectancy increases by 0.649 years or 0.789%, and keeps increasing as the assumed period of mortality improvement increases. The difference reaches 1.034 years or 1.185% assuming 50 years of mortality improvement. Similarly, Tables 5 and 6 show the results for a life aged 40 and 65 in year 2018.

TABLE 4. Korean male life expectancy of a new born in year 2018

Mortality impr.	With SC		Without SC		Difference	% change
20	83.3	[82.5, 84.0]	82.6	[81.0, 84.1]	0.649	0.768
30	85.2	[84.5, 86.0]	84.4	[82.5, 86.1]	0.831	0.985
40	86.9	[87.6, 86.1]	86.0	[83.9, 87.7]	0.953	1.109
50	88.3	[87.5, 89.0]	87.3	[85.2, 89.0]	1.034	1.185

Table 5. Korean male life expectancy of a life aged 40 in year 2018

Mortality impr.	With SC		Without SC		Difference	% change
20	44.4	[43.7, 45.1]	43.9	[42.4, 45.2]	0.574	1.309
30	45.9	[45.1, 46.5]	45.1	[43.5, 46.6]	0.734	1.627
40	46.8	[46.0, 47.6]	46.0	[44.1, 47.6]	0.849	1.848
50	47.2	[46.5, 48.0]	46.4	[44.4, 48.1]	0.920	1.984

Table 6. Korean male life expectancy of a life aged 65 in year 2018

Mortality impr.	With SC		Without SC		Difference	% change
20	20.3	[19.8, 20.7]	19.9	[19.1, 20.7]	0.327	1.640
30	20.4	[20.0, 20.9]	20.1	[19.2, 21.0]	0.359	1.787
40	20.5	[20.0, 20.9]	20.1	[19.2, 21.0]	0.361	1.797
50	20.5	[20.0, 20.9]	20.1	[19.2, 21.0]	0.361	1.797

5.2. Actuarial Applications

This subsection examines the effect of structural change on actuarial valuations. The first we consider is the actuarial present value (APV) of whole life insurance which will be denoted by $A_{x,t}$ and calculated as

$$A_{x,t} = \nu q_{x,t} + \nu^2 p_{x,t} q_{x+1,t+1} + \nu^3 p_{x,t} p_{x+1,t+1} q_{x+2,t+2} + \cdots .$$

Table 7 shows the actuarial present value of whole life insurance on a life aged 40 in year 2018 evaluated at 6% effective rate of interest per annum. The same effective interest rate is used in the following calculations. The impact of structural change looks quite significant. The actuarial present value drops by 3.578% if we assume 20 years of mortality improvement, and by 5.543% if 50 years.

Table 7. APV of whole life insurance for a Korean male aged 40 in year 2018

Mortality impr.	With SC	Without SC	Difference	% change
20	0.09328	0.09675	-0.00346	-3.578
30	0.08681	0.09090	-0.00409	-4.505
40	0.08314	0.08766	-0.00452	-5.154
50	0.08154	0.08632	-0.00478	-5.543

Next we consider the actuarial present value of whole life annuity. Similarly as before, our calculation is based on

$$\ddot{a}_{x,t} = 1 + \nu p_{x,t} + \nu^2 p_{x,t} p_{x+1,t+1} + \nu^3 p_{x,t} p_{x+1,t+1} p_{x+2,t+2} + \cdots .$$

The calculation result is presented in Table 8 where we consider a life aged 65 reaching a common retirement age. As shown in the table, the APV increases marginally as we assume a longer period of mortality improvement.

TABLE 8. APV of whole life annuity for a Korean male aged 65 in year 2018

Mortality impr.	With SC	Without SC	Difference	% change
20	11.787	11.689	0.097	0.834
30	11.821	11.718	0.103	0.880
40	11.822	11.719	0.103	0.882
50	11.822	11.719	0.103	0.882

The last we consider is the net annual premium of whole life insurance calculated as

$$P_{x,t} = \frac{A_{x,t}}{\ddot{a}_{x,t}}$$

We tabulate our calculation result for a life age 40 in Table 9. As shown in the table, the net premium drops by nearly 6% if we assume 50 years of mortality improvement.

TABLE 9. Annual premium of whole life insurance for a Korean male aged 40 in year 2018

Mortality impr.	With SC	Without SC	Difference	% change
20	0.00583	0.00607	-0.00024	-3.937
30	0.00539	0.00567	-0.00028	-4.916
40	0.00514	0.00545	-0.00030	-5.595
50	0.00504	0.00536	-0.00032	-5.999

6. Conclusion

In this paper, following the statistical test proposed in Coelho and Nunes [2], we found a significant evidence for structural change in the Korean male mortality index but not in the Korean female. As with Korea, Taiwan exhibits a structural change only for male. For Japan, both males and females had experienced a structural change in 1978 and 1986, respectively. No structural change was found for United Kingdom and French for both male and female. And, a noticeable mortality improvement could be observed for both male and female in East German population. As mentioned earlier, the three countries, Japan, UK and French, were already investigated in Coelho and Nunes [2]. However, they do not coincide probably due to the different observation periods. Therefore, it might be worthwhile to study the robustness of the test in the future research. Using the empirical evidence on the structural change of the Korean male mortality index, we constructed an appropriate model to project the future mortality rates, and applied it to some actuarial calculations. Depending on the product type and the assumed period of future mortality improvement, the actuarial present values could be affected significantly. Hence, more caution needs to be paid to the Lee-Carter modeling and its applications.

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