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Reconceptualizing Learning Goals and Teaching Practices: Implementation of Open-Ended Mathematical Tasks

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This study examines how open-ended tasks can be implemented with the support of redefined learning goals and teaching practices from a student-centered perspective. In order to apply open-ended tasks, learning goals should be adopted by individual student's cognitive levels in the classroom context rather than by designated goals from curriculum. Equitable opportunities to share children's mathematical ideas are also attainable through flexible management of lesson-time. Eventually, students can foster their meta-cognition in the process of abstraction of what they've learned through discussions facilitated by teachers. A pedagogical implication for professional development is that teachers need to improve additional teaching practices such as how to tailor tasks relevant to their classroom context and how to set norms for students to appreciate peer's mathematical ideas in the discussions.

Keywords: open-ended mathematical tasks, learning goals, teaching practices, student-centered
MESC Classification: C32, C72
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I. INTRODUCTION

Researchers have called for the use of open-ended tasks to implement good mathematics instruction (Sullivan, Warren, & White, 2000). Rather than listening to teachers' explanations or engaging passively in a provided activity, learning actually takes place by discussing mathematical ideas followed by the activity (Bransford, Brown, & Cocking, 1999; Chapin, O'Connor, & Anderson, 2013). When students construct their own ideas regardless of their levels of knowledge and participate in discussions built on

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the ideas from the given tasks, they would be engaged in the social interactions and the process of sense-making within the discussions might become more active (Stein & Stein, 2011). Therefore, it is crucial to provide appropriate tasks which allow students to approach their own mathematical thinking for them. In this study, this type of tasks is defined as *Open-Ended Mathematical Tasks* (OEMT). When teachers select a task, they should consider the followings: All students in the classroom are able to construct their knowledge; Students' cognition levels are varied. When selecting curricular materials, teachers have to consider that the materials are accessible to all students, even lower achievement levels. In other words, since there are diverse students in a classroom, we should give a single task for all students rather than multiple tasks adapted to individual student's level. This single task gives multiple entry points to solve the tasks and students use this as a common foundation in the following discussion about diverse mathematical ideas.

In this paper, we investigate how OEMTs can be a solution to solve this complex educational situation. The goal of this study is to discuss the new perspective for learning goals and practices in the use of OEMTs. We need such discussion since good mathematics instruction is leveraged by the consideration of appropriate learning goals and practices.

II. OPEN-ENDED MATHEMATICAL TASK

1. DEFINITION OF OPEN-ENDED MATHEMATICAL TASKS

In mathematics lessons, tasks trigger mathematical discussion and give a fundamental opportunity to learn (Stein & Lane, 1996). Instructors might differentiate how to implement these tasks in the consideration of students' context, learning goals, and mathematical content knowledge. Specifically, open-ended tasks have been shown to provide enhanced opportunity for students to investigate mathematics, to collaborate with peers, and to reason mathematically (Kosyvas, 2016). Furthermore, this type of tasks is powerful to practice students' metacognitive skills in decision making (Chan & Clarke, 2017).

In this study, OEMTs refer to the tasks which have different strategies to solve and/or derive various solutions (see Table 1, Pehkonen, 1997; Lowen, 1995). Pehkonen (1997) categorized task types in terms of goals and tasks: open goal, open task, closed goal, and closed task.

Table 1. The categorization of task types (Pehkonen, 1997)

	Closed goal	Open goal
Closed task	-	Investigations Problem fields Problem variations
Open task	Problem variation	Projects Problem Posing Open approach

When a goal of a task is not limited to a specific one and has diverse access points, this type of task pertains to the open-ended type of tasks such as projects and problem posing. As an instance of open-ended type, open approach refers to be solved in various ways. For example, 27+15 would be solved as follows:

20+10=30, 7+5=12, 30+12=42
27+3=30, 30+12=42
15+12=27, 15+15=30, 30+12=42
30+15-3=45-3=42

The definition of the open-ended task can be extended to the tasks which can be solved by composing the tasks from the given context. With a lack of required information, students might add information by themselves to pose newly modified OEMTs.

2. OPEN-DNDEDNESS BUT CLOSED

Open-endedness refers to be solved by more than one path, whereas closed implies only one acceptable pathway, response, approach, or line of reasoning (Sullivan, Warren, & White, 2000). Since the degree of open-endedness of tasks is depending on classroom situation, any mathematical task can be open-ended. However, overemphasis on standard arithmetic algorithms for arithmetic operations has made our students less motivated to find other paths and, eventually, the tasks themselves have been lost their open-endedness.

In school mathematics, students should be asked to solve the tasks with more than one strategy to foster the resilience of open-endedness. However, teachers tend to refuse to request multiple strategies in traditional mathematical classrooms. They do not want to have an unexpected situation with diverse ideas from different cognitive levels of students. Even though they have to learn the same mathematics in the same classroom, each student needs to pursue different learning goals. In addition, teachers require student-centered (or reform-based) practices rather than teacher-centered to recover open-endedness of tasks. Teachers can intentionally decide whether the tasks are closed or open. They might change closed tasks to open-ended and vice versa. The mathematics classrooms are dominated by the use of closed tasks traditionally, however, the educators have called to use OEMTs for standard based mathematical education (NCTM, 1995).

III. LEARNING GOALS IN THE LESSONS EMPLOYING OPEN-ENDED MATHEMATICAL TASKS

1. LEARNING GOALS IN STUDENT-CENTERED INSTRUCTION

In this paper, we focus particularly on students' new mathematical knowledge learning through the use of the OEMTs. This is different from the current approach to use mathematical textbooks, which is applying teacher-driven knowledge to OEMTs. Even though mathematical textbooks include such open-ended tasks, the way to employ the tasks are varied by the designated learning goals of the tasks. For example, when teachers implement a division task (8 stones are divided into 2 plates) in third grade, this task is supposed to use subtraction, direct modeling, and multiplication. However, some teachers might only focus on long division algorithm regardless of their students' understanding of division.

Most of students - over 90% by Mastery Learning theory (Carroll, 1963) - are traditionally believed to learn the mathematical knowledge presented by the textbooks and this knowledge is perceived as the same thing regardless of differentiated learners. However, it is evident that standardized and static goals suggested by designed curricula are difficult for students to attain. We believe all the students have the ability to understand mathematical ideas, which implies that students including the lower levels could do sense-making. In other words, the lower level of knowledge for struggling students might already be attained by the high-level achievement group of students. However, if the high-level of students are the reference group for mathematical content knowledge, other lower-level achievement groups of students could not have any opportunity to learn. How about grouping students by achievement levels? This also might give a privilege to only top students, not to most of the general students (Boaler, 2015). Therefore, traditional teacher-driven lessons have shown unsatisfied results when teachers adopted a specific knowledge for a specific level of students as the learning goal of each lesson and taught all students evenly.

What if all students in the classroom can construct their own knowledge and they use a different cognitive ability with the OEMTs, how can the learning goals be differentiated? First, the meaning of the learning goals would be shifted from teacher-centered to student-centered instruction, as the shift of scientific paradigms (Khun, 1970). In the classroom, teachers might decide the major learning goals for each lesson (teacher-centered). However, this practice would be changed with the use of OEMTs since teachers are able to anticipate potential students' mathematical ideas but this is the only possibility. Therefore, teachers should consider their own students' current cognitive and affective levels to make a decision of the scope of learning goals (student-centered). While referring to common mathematical content for students to learn in traditional instruction, the learning goals imply adaptive understanding depending on individual students' cognitive levels. Therefore, each student's learning accomplishment has the common areas as well as the different ones simultaneously. If the learning goals are set as a various understanding of individual students, these learning goals eventually refer to the development and the progress of each student's understanding. In other words, the achievement of students' learning goals is the continuous, dynamic, and integrated process of their sense-making. Further, the classrooms can develop their community-based understanding since this socio-cultural meaning making process is reciprocal to all members.

2. FROM CLASSROOMS: THE STUDENT-CENTERED LEARNING GOALS

In this section, a first-grade classroom situation is illustrated to show how the newly defined learning goals can be implemented by the OEMTs "Today's Number". This task is to generate various number expressions which have the same result as the given number by teachers (e.g., 20). The problem type can be changed by the variation of conditions as follows:

- Use addition once (the number of additions can be adjustable).
- Use addition once and subtraction once respectively (the different operations can be combined).
- Use three numbers (the numbers can be changed).
- Use fractions and whole-numbers (the type of numbers can be changed)
- The suggested conditions also can be combined (e.g., Find number expressions to make 20 with three numbers and with one addition and one subtraction).

Providing enrichment tasks to some students who solve the given tasks easily, students can use an open number task to generate expressions and be asked to compare their strategies with each other.

Let's look at an instructional case implemented by a mathematics specialist (Kim, 2018). After one month into the first semester, the "Today's Number" task was introduced to the first graders: " ______= 20". The students were individually engaged in the task for five minutes and found various number expressions with their own strategies. The teacher gave the opportunity to present their number expressions and strategies to anyone who desired.

Many students used addition between two numbers as the following examples:

11+9=20, 12+8=20, 13+7=20, 14+6=20, 15+5=20, 16+4=20, 17+3=20, 18+2=20, 19+1=20, 9+11=20, 8+12=20, 7+13=20, 6+14=20, 5+15=20, 4+16=20, 3+17=20, 2+18=20, 1+19=20

Some students generated addition expressions which might include the foundational idea of multiplication (i.e., iteration):

10+10=20, 5+5+5+5=20, 4+4+4+4+4=20, 2+2+2+2+2+2+2+2+2+2=20, 1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1=20

Other students also found diverse expressions with mathematical quantity (e.g., identity element, using a pattern, estimating):

20-0=20, 20+0=20

21-1=20, 22-2=20, 23-3=20, …

5+6+6+4-1=20

5×4=20, 40÷2=20

One student wrote the expressions in the folded notebook.

20-0=20 30-10=20 40-20=20 50-30=20 60-40=20 70-50=20 80-60=20 90-70=20 100-80=20 110-90=20 120-100=20 130-110=20 140-120=20	150-130=20 160-140=20 170-150=20 180-160=20 190-170=20 200-180=20 210-190=20 220-200=20 230-210=20 240-220=20 250-230=20 260-240=20 270-250=20	280-260=20 290-270=20 300-280=20 310-290=20 320-300=20 330-310=20 320-300=20 330-310=20 340-320=20 350-330=20 360-340=20 370-350=20 380-360=20	390-370=20 400-380=20 410-390=20 420-400=20 430-410=20 420-400=20 430-410=20 440-420=20 450-430=20 460-440=20 470-450=20 480-460=20 490-470=20 500-480=20

During the lesson which intended to understand new mathematical knowledge (e.g., various number expressions and their connections) through the OEMT, one of the student's crucial responses was 5+6+6+4-1=20. This student did not use number facts making 20 with two numbers but use guess-and-check. At first, he added any two numbers (5 and 6), then added 6 since the sum of the first two numbers (5 and 6) is smaller than 20. Then, he added 4 to 17 as a partial sum. But the new sum 21 is greater than 20 by 1, so he took away 1 to make 20 eventually. The student did solve the problem by using his knowledge already established rather than by adding and/or subtracting numbers on purpose.

What mathematics content might first graders learn on the basis of constructed ideas through the OEMT? What learning goals can be set in this context? Before describing the answers to the questions, we can infer what is already known for students' current ideas and knowledge based on their documented works as follows:

- Two-digit and one-digit addition with regrouping
- Two-digit and one-digit subtraction without regrouping
- Adding the same number
- Adding multiple numbers
- Subtracting between three numbers
- Commutative property of addition
- Multiplication
- Identity property of addition
- Mixed addition and subtraction

It is evident that the listed knowledge was constructed by the students in the classroom. Notably, if this task was implemented in another classroom, those students would construct similar but different mathematical knowledge. More generally speaking, students in different classrooms will construct their own responses. That is, students could learn different mathematical content knowledge in different classrooms even though they use the same tasks.

As stated above, students are expected to learn knowledge by assimilating and accommodating from their prior knowledge through the OEMTs which allow the construction of diverse knowledge. Since students might have different levels of assimilation and accommodation skills, it is not an easy problem to set the same learning goal for all students in the classroom to understand specific mathematical content knowledge. Even though the students were first graders, some of them could use the second abstraction based on their ideas constructed by the first abstraction through the given task. For example, some students might solve the tasks with various ways and classify them based on similarity (first abstraction). Then, they might make sense that the sum of the two numbers does not change with different orders like the first example (11+9=9+11=20) as the second abstraction. Undoubtedly, not every student in the classroom might construct this knowledge. Therefore, this commutative property of addition can be introduced after a relatively larger group of students are ready for formalizing this knowledge. From the Piagetian perspective, this naming process is not learning objects to be constructed by students but transferring objects from teachers to students as social knowledge (See Chapin, O'Connor, & Anderson, 2013; Kamii, 1994; Skemp, 1987). The mathematical lessons are not one-day situations but consecutive day-by-day. Teachers should bridge mathematical ideas from today's lesson to the following lessons and orchestrate the lessons for students to connect their mathematical knowledge and ideas. The development of the cognitive ability to construct their knowledge by themselves based on the connections between mathematical knowledge is the essence of teaching and learning, and this is the fundamental reason why students learn mathematics. However, only when students are the main subject to build this connection, the kind of knowledge can be developed.

VI. TEACHING PRACTICES

1. GIVING OPPORTUNITIES TO ALL FOR SHARING THEIR OWN CONSTRUCTED IDEAS

The students who are engaged in the OEMTs such as the "Today's Number" task can construct their own mathematical ideas regardless of the level of learning ability. In this context, it is important that teachers should give opportunities for students' sharing ideas. Some researchers (Chapin et al., 2013; Smith & Stein, 2011) recommended teachers give the opportunity for sharing selected ideas in the individual or group activities, which are worthwhile to discuss with the whole group members. In particular, if all students in the classroom have the same learning content, this teaching practice would be more valuable to implement.

Every student should have equitable opportunity to share their ideas (Chapin et al., 2013; Kim, 2018; Ronfeldt, 2003). For this, teachers might ask, "Is there any other ideas?", "Anyone to say something?" Furthermore, some students who are shy or prefer to listening than talking also can be purposefully designated by calling their name, "Can you share your idea?" Teachers also encourage other students who do not share their ideas from the worksheets (e.g., "Tom did something new which is not shared with us. Tom, can you tell (or share with) us what you did?). Like this, teachers should understand that some students would not share their ideas due to their emotional characteristic, not due to a deficiency of their knowledge. Teachers must keep in mind that one case of negative reaction to a student's experience from teachers or peers can make them never share again in school.

2. COORDINATION OF LESSON TIME

Chapin et al. (2013) suggested five talk moves for effective teaching practice in the mathematics lessons. One of the talk moves is waiting time which does not include talking. This practice provides a time for students to organize and rehearse their thoughts. In the same vein, to coordinate lesson time is easy to overlook since it does not need actual performance by teachers. However, this coordination of lesson time is one of the important teaching practices when using OEMTs in lessons. This practice might not work well in traditional classrooms with limited 40-minute lessons. During this one period, teachers and students strive to achieve a specific learning goal from the given content. However, if teachers desire to design the lesson built on students' constructed ideas on the spot, 40-minute is never enough to implement meaningfully (Ronfeldt, 2003). If teachers use this teaching practice with the OEMTs in their classroom, they need a different length of periods and much longer lesson time than the general classrooms (Kim, 2018). The core of mathematical instructions is for students to construct relationships between mathematical ideas built on curricula materials. Therefore, students should be involved in this constructing process, even though they might not make any meaningful relationships. If students were engaged in closed tasks, it might be difficult for them to build such relationships and present their ability when solving the tasks. However, this situation will not happen in the case of the implementation of OEMTs since all students can make their own construction (Wickett, Ohanian, & Burns, 2002). Especially, the use of OEMTs can facilitate to build this knowledge construction. Thus, the prerequisite for the effective lesson is for students to have enough time for contemplating by themselves (Charalambous & Pitta-Pantazi, 2015).

From the teacher-centered approach, teachers might criticize the fidelity of curricular standards. If they build the lessons on students' ideas, then it might be impossible to deal with all the suggested standards for each grade level. However, such concerns are stemmed from the lessons which are dealing with only one specific knowledge per period of a lesson. On the other hand, the lessons with the OEMTs (e.g., Today's Number) spent at least 3 to 4 periods lessons and the significant amount of knowledge can be integrated as we see in the example of "_______=20". Thus, students can have enough time to learn at one's level. As a result, the lessons that employed OEMTs can handle even higher levels of knowledge than suggested curricular standards and develop and promote further students' understanding at their levels.

3. ACCEPTING DIFFERENT STRATEGIES

Mathematical tasks have diverse strategies to be solved but students can also get the same strategy with peers when they solve OEMTs with their own mathematical expressions. After a student statement of expressions and strategies, teachers might ask to the whole classroom, “Is there anyone who has the same expression?” Then, the teachers might also ask, “Any other strategies?” These questions will help students recognize various strategies to solve the tasks and try other strategies.

Some teachers who have implemented traditional mathematical instruction express their struggling with making rich class discussions due to the students’ disengagement even if the teachers are eager to facilitate discussions. However, if teachers utilize OEMTs and promote the construction of students’ ideas to share, this story might be changed. Rather, it is noticeable the students keep raising their hands, hoping to share their ideas. The students are saying, “I would like to share!”, “Me too!”, and “I would like to hear A’s opinions” (Kim, 2018; Kim & Lee, 2008).

4. META-COGNITION OR REFLECTIVE ABSTRACTION

Elementary students in the concrete operational stage, which is the third stage out of four in the cognitive development (Piaget, 1964), begin to abstract concepts from their empirical facts. If applying this approach to mathematical lessons, the subject of constructing knowledge is students, not teachers. However, wrap-up activities which teachers are generally doing at the end of lessons would get rid of the opportunity for reflective abstraction (Ronfeldt, 2003; Wickett, Ohanian, & Burns, 2002). Although teachers might be skeptical that their students have enough meta-cognitive ability to make connections about what they have learned in the lesson, the students should have more opportunities to use their meta-cognition in the mathematics classrooms (Kamii, 1994; Kim, 2018, in press; Wickett, Ohanian, & Burns, 2002). This is why the work of teaching is facilitating and supporting students’ sense-making rather than delivering teachers own knowledge to students (Kim, 2018; Kamii, 1992; Empson, & Levi, 2011). From this perspective, a teacher’s debriefing statements about the whole lesson might be actually the same products which should be drawn by students’ meta-cognition.

How do these teaching practices look like in the classroom with the exemplary task, “Today’s Number”? After the whole group discussion, teachers should ask students to find similarity and difference between expressions and strategies to compare each other. It might difficult for students to abstract these concepts rigorously. However, this does not mean teachers should instead abstract their ideas. If then, students cannot promote their meta-cognition throughout mathematics classes. Students have to know meta-cognition is the major mental activity in lessons and teachers also help them get used to this activity gradually.

Students who find the similarity (or difference) will share their ideas and other students need to explain what they are told in their own language (e.g., Is there someone who can explain this with your words?). Following the restatements, teachers should give the authority to evaluate the value of the statement to students such as “Is that the same meaning with previous explanation?” rather than “Good job!”, “That’s right!”, or “I don’t agree with that”. Such questioning makes students concentrate on peer’s comments. It might be also difficult to respond to the questions without careful listening.

V. CONCLUSION

In this study, we investigated the possibility to build good mathematics instruction through the use of the OEMTs which influence what students should learn and how teachers can support substantially. Students’ mathematical knowledge is extended and transformed by the engagement of mathematical tasks. During the lesson, students might be expected to achieve the same goal together. However, if the tasks are approachable by varied strategies and ideas, students could develop their understanding of mathematical knowledge through the process of comparing peer’s ideas. Since each student builds new mathematical knowledge on their own prior knowledge, what they might learn should be differentiated. As a result, teachers should recognize all students have different learning goals through the use of the OEMTs. In the meanwhile, teachers need to support their students to engage in rich discussions. To orchestrate mathematical discussion, teachers can give equitable opportunities to share students’ ideas and encourage them to share their thinking with supporting talk moves (Chapin et al., 2013). This can be strengthened by reorganizing a series of lessons innovatively and pressing students to use their meta-cognition.

Student-centered approach can be challenging to some teachers even though they have enough curricular resources. A more feasible way to access the approach can begin with the change of the tasks. When teachers implement OEMTs, they will see diverse mathematical thinking explicitly. At this time, the professional development to foster their teaching practice such as how to use OEMTs and how to facilitate discussions will be necessary. Eventually, this change will also affect their teaching belief and value in their teaching to think about mathematics instruction fundamentally.

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