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How the Geometries of Newton's Flat and Einstein's Curved Space-Time Explain the Laws of Motion

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This essay elucidates the way the geometries of space-time theories explain material bodies' motions. A conventional attempt to interpret the way that space-time geometry explains is to consider the geometrical structure of space-time as involving a causally efficient entity that directs material bodies to follow their trajectories corresponding to the laws of motion. Newtonian substantival space is interpreted as an entity that acts but is not acted on by the motions of material bodies. And Einstein's curved space-time is interpreted as an entity that causes the motions of bodies. This essay argues against this line of thought and provides an alternative understanding of the way space-time geometry explain the laws of motion. The workings of the way that Newton's flat and Einstein's curved space-time explains the law of motion is such that space-time geometry encodes the principle of inertia which specifies straight lines of moving bodies.

Keywords: Space-Time Geometry; Newton's Flat Space-Time; Einstein's Curved Space-Time; Laws of Motion; The Law of Inertia; The Principle of Equivalence.

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1 Introduction

This essay elucidates the way that space-time geometries of both Newtonian and Einsteinian physics explain material bodies' motions. For this purpose, it is necessary to investigate which specific contents of space-time theories explain the phenomena of motions and in which way their explanation of the motions of bodies takes place. A conventional attempt to comprehend the way space-time explains is to consider the geometrical structure of space-time as involving a causally efficient entity that directs material bodies to follow their trajectories corresponding to the laws of motion. Substantivalism admits the existence of a causally efficient entity,

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i.e., space, that explains the laws of motion. Newtonian substantival space is interpreted as an entity that acts but is not acted on by the motions of material bodies. And Einstein's curved space-time is interpreted as an entity that causes space-time to direct the motions of bodies.

This essay argues against this line of thought, and provides an alternative understanding of the way that space-time geometries of Newtonian and Einsteinian physics explain the behaviour of bodily motions. The key strategy of the argument of this essay is to employ the 'dynamical perspective of space-time' developed by DiSalle [4] and Brown [2]. Contrary to a conventional wisdom that the structure of space-time explains dynamical laws, the dynamical perspective views that dynamical laws encode the physical foundations of both flat and curved space-time. Both Newton's flat and Einstein's curved space-time geometry provide a set of coordinates encoding the principle of inertia which specifies inertial motions within Newtonian dynamics and the principle of equivalence within the general theory of relativity.

2 How Newton's Space-Time Geometry Explains Bodily Motions

The substantival interpretation of space (or space-time) can be found within Einstein's own writings which attempt to comprehend the geometrical structures of Newtonian space-time. Einstein considered Newtonian absolute space as an inertia-producing entity.

> The inertia-producing property of [Newtonian space] … is precisely not to be influenced, either by the configuration of matter, or by anything else. For this reason, one may call it 'absolute.' That something real has to be conceived as the cause for the preference of an inertial system over a non-inertial system [2].

Along these lines, Einstein interpreted Newtonian space as a substantival entity, i.e., an entity that involves causal effects toward the motion of material bodies satisfying Newtonian laws of motion. Earman characterizes Newton's view as substantivalism maintaining that

> [s]pace-time is a substance in that it forms a substratum that underlies physical events and processes, and spatiotemporal relations among such events and processes are parasitic on inhere in a substratum of space or space-time points [5].

Stein considers the geometrical structure of the affine connection within Newtonian space-time as being responsible for causing the behaviour of bodily motion. Nerlich also claims that

> without the affine structure there is nothing to determine how the particle trajectory should lie. ... It is because space-time has a shape that world lines lie as they do [11].

Along these lines, Newtonian substantivalism can be viewed as endorsing the causal efficacy of geometrical constitution of space-time which directs material bodies in accordance with the Newtonian laws of motion.

Newton, however, did not view his absolute space as an inertia generating entity which causes material bodies to follow the trajectories satisfying the laws of motion. Newton himself clearly stated that "[space is] not in the least mobile, nor capable of inducing change of motion in bodies" [13]. Accordingly, interpreting absolute space as a causally efficient entity is not consistent with Newton's own delineation. According to Brown and Pooley, how "all the free particles in the world behave in a mutually coordinated way" is a "prima facie mystery" in Newtonian mechanics [3]. It seems that the causation involving constitutions of space-time geometry fails to clarify the way space-time explains phenomena. Absolute space is not a causally efficient entity, given that it cannot engender 'ruts and grooves,' which cause material bodies to follow inertial or non-inertial trajectories.

The commentators on space-time theories have instead pointed out a different scheme interpreting the relationship between space-time geometry and motion. DiSalle argues that Euclidean geometric structure by no means causally explains its laws such as the Pythagorean theorem [4]. Likewise, Newton's space-time geometry does not causally explain the behaviors of material bodies that satisfy the laws of motion. The structure of space-time encodes the motions of bodies through geometric structures, which allow us to define the appropriate concepts of kinematics encapsulating the laws of motion. In other words, space-time structure, as a geometric entity, explains why the motions of bodies occur in a specific way through physically legitimate reference frames in which the laws of motion hold. Alexander in his preface of Leibniz-Clark correspondence states that

> one might interpret the Scholium as saying that space-time are ideal entities which it is helpful to consider in theory, ... identifying the set of frames of reference with respect to which the laws of dynamics would take the simplest forms [1].

Newtonian absolute space-time distinguishes the inertial frames in which the laws of motion satisfy.

The importance of kinematic concepts provided by space-time geometry in interpreting the substance-relation debate can be read within Newton's *De Gravitatione* [13]. Descartes' physics is criticized by Newton, because the analyses of Cartesian kinematic concept of space turn out to be inconsistent. Cartesian physics determines a given body's location relative to other immovable bodies without kinematical properties of the motions of a body with respect to immovable space, i.e. Newton's absolute space.

Newton at this point attempts to show that Descartes' relational space has not the appropriate kinematical structures to make sense of inertial trajectories of moving bodies. Newton instead endorsed an alternative conception of kinematics, i.e. true and absolute rotation, which is consistent with the dynamics:

> it is necessary that the definition of place, and hence of local motion, be referred to some motionless thing such as extension alone or space in so far as it is seen to be distinct from bodies [13].

What is significant as the kinematical conception of absolute space is that space maintains 'similar and immovable' over time [12]. Along these lines, Newton claimed that space is absolute in the sense that we can admit the trans-temporal identities of each and every location. [7]. Accordingly, the places of events can then be identified regardless of the passage of time. In this way, absolute space allows the definition of place that is truly at rest [12]. And true motion can then be defined as the translation of a body from one absolute place to another. This framework of kinematics allows us to distinguish inertial and non-inertial motion; the straight worldlines in Newtonian space-time represent inertial motions, while the curved ones are non-inertial ones. These kinematical properties of place and motion, which are described by geometry, endow Newtonian dynamics with appropriate foundations that support the laws of motion.

The conceptions of kinematics are constructed by means of geometry because places and motions are defined with points and lines. Furthermore, geometry plays a crucial role within Newtonian dynamics due to the fact that the trajectories of freely falling bodies are the same regardless of its internal constitution, i.e., whether the physical or chemical composition of the bodies is aluminium or gold [10]. Along these lines, kinematical properties endowed by the geometrical concepts of space-time are provided in order to exhibit the regularity relations between events, which are the laws of motion.

3 How Einstein's Space-Time Geometry Explains Bodily Motions

The causal property of Einstein's curved space-time is famously captured by Misner, Thorne, and Wheeler, who summarize this feature as "space acts on matter, telling how to move. In turn, matter reacts back on space, telling how to curve [10]." Within Einstein's general relativity, the gravitational interaction is interpreted as emerging from the causal property of curved space-time. This causal interpretation of curved space-time is manifest in capturing the workings of the gravitational field and the gravitational wave.

> The gravitational field is represented by a field on spacetime, $g_{\mu\nu}$, just like the electromagnetic field A_μ. They are both concrete entities: a strong electromagnetic wave can hit you and knock you down; and so can a strong gravitational wave [16].

Gravitational waves are basically small fluctuations of the space-time metric $g_{\mu\nu}$, which are described as 'ripples' of space-time. When the gravitational field in empty space is very weak, the space-time metric $g_{\mu\nu}$ is split approximately into the Minkowski metric $\eta_{\mu\nu}$ and the metric perturbation $h_{\mu\nu}$, i.e., $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ [10]. By neglecting terms in $h_{\mu\nu}$-squared and higher powers, 'linearized field equations' can be derived from Einstein's field equations. The result is a set of linear second-order partial differential equations for $h_{\mu\nu}$. And with the choice of Lorentz gauge, which corresponds to the choice of coordinates, these equations can be transformed so as to be similar in form to a set of Maxwell's equations, which predict electromagnetic waves. This linearized theory of gravitation, just like its electromagnetic counterparts, has a solution that predicts gravitational waves travelling with the speed of light. Accordingly, gravitational waves seem to pass through space just like electromagnetic waves. In this way, commentators of Einstein's curved space-time maintain the interpretation admitting the causal role of geometry along the line with the causal role of Maxwell's electromagnetic field [16].

However, this analogy between the causal properties of gravitational and non-gravitational waves breaks down, when one considers the difference between the gravitational and the non-gravitational energy-momentum [9, 14]. The latter quantity is a tensor, while the former is not. Although physically meaningful quantities should be tensors, the gravitational energy-momentum does not pass this test. It is instead a pseudo-tensor, which is a variable (and may even vanish) dependent on the choice of coordinate system:

> its non-tensorial nature means that there is no well-defined, intrinsic 'amount of stuff' present at any given point. In particular, unlike a genuine tensor, [a pseudo-tensor] can be made to vanish at any given moment by a suitable coordinate transformation. [This] therefore cannot really be telling us about the local interchange of energy-momentum between gravity and matter [9].

And Norton expresses the same view:

> In so far as I can understand this response, it really just tells us that a pseudo-tensor should be given no physical interpretation. It should merely be used as a mathematical intermediary in computing the gravitational energy and momentum of extended systems [14].

Can this problem be avoided by considering the gravitational energy-momentum of some extended system? Norton's answer is "no" [14]. The total energy and momentum of some extended system cannot be meaningfully defined since the total energy and momentum involves the contributions of both gravitational and non-gravitational interaction. In fact, in order to recover a total energy from the summation of gravitational and non-gravitational interaction, there should be a rest frame where space-time geometry is time-independent. This time-independent space-time geometry represents the astronomical system completely motionless and isolated from any external interaction. These total quantities can be defined only within special yet unrealistic cases [14]. The motion, radiation and mass loss of stars along with gravitational waves influence space-time geometry and prevent the recovery a total energy. Accordingly, Salmon's notion of "causal interaction" and "causal process" [17], which are based on the exchange of conserved quantities, do not fit within the general theory of relativity.

Instead, the physical significance of gravitational waves is captured in terms of the inertial motions of Einstein's general theory of relativity:

> When the rock is hit by a strong burst of electromagnetic radiation, the natural motions of the parts of the rock do not (significantly) change. Rather the parts of the rock are differently accelerated by forces that overcome the contracting forces between the parts of the rock. When the rock is hit by gravitational radiation, however, no additional accelerative forces are applied. Rather the natural motions are no longer towards the rock's centre but are radically divergent. So divergent, in fact, that the electromagnetic binding forces of the rock are no longer sufficient to accelerate the parts of the rock away from their natural trajectories" [15].

The distortion by a gravitational wave is in fact captured by considering its effect on the motions of bodies that the wave passes. Here the equation of geodesic deviation, which describes the relative motions of two or more bodies, describes that a set of particles, which follow their own geodesics, is deformed by the tidal acceleration generated by the gravitational wave. In other words, this equation describes the oscillations in the separation between neighbouring inertially moving particles, which are described as the oscillating curvature tensor of a gravitational wave.

We can see that the essential role of the specification of the inertial motions of bodies in the above account properly fits within the framework of Einstein's general theory of relativity, if we look at how current standard textbooks motivate Einstein's field equations (EFE) [10]. Here, the physics of the curvature tensor **R** on the left hand side of EFE is motivated from a Newtonian equation that expresses the relative acceleration of neighbouring test particles – so called "the tidal acceleration of two nearby particles." And its corresponding general relativistic expression, i.e. the equation of geodesic deviation, then provides the expression of the curvature tensor **R**. Accordingly, the geometrical structure of space-time, i.e. the curvature tensor in fact encodes the geometric relationships between events, which are specified by the equations of geodesic motion and geodesic deviation. Geroch's analogy also makes this point well:

> What is the sum total of all the information (of a geometrical character) that these ants could accumulate about the surface of the earth? … The ants have access only to points on the earth and distances between nearby points; we have access only to events in space-time and intervals between nearby points. The ants, from the information they have, can detect curvature … This analogy suggests, then, that, using only geometrical constructions involving the events in space-time and the intervals, we may introduce a quantity which may be thought of as the "curvature of space-time geometry" itself [8].

So, within the general theory of relativity, the geometric trajectories of inertial motions play the key role.

My account of the way Newtonian flat space-time and Einsteinian curved space-time explain is not about the causal relationship between bodily motion and space-time geometry, but about the appropriate relationships among a set of coordinates encapsulating the law of inertial motions. While kinematic concepts encapsulating inertial motions within Newtonian physics define flat space-time geometry, the relationships among a set of coordinates encapsulating inertial motions within Einsteinian physics, i.e. free falling motions, define curved space-time. And this idea is clearly captured by Misner, Thorne, and Wheeler:

> [I]t was the whole point of Einstein that physics looks simple only when analyzed locally. To look at local physics, however, means to compare one geodesic of one test particle with geodesics of other test particles travelling (1) nearby with (2) nearly the same directions and (3) nearly the same speeds. Then one can "look at the separations between these nearby test particles and from the second time-rate of change of these separations and the 'equation of geodesic deviation' ... read out the curvature of spacetime [10].

Einstein's equivalence principle forbids locally inertially moving observers from determining a global inertial frame. The disorientation between a set of locally inertial frames is defined as the curvature of space-time geometry.

Accordingly, the way that Einsteinian curved space-time explains the motion of body is that the trajectories of two neighbouring free-falling bodies encode the geometry of curved space-time. This is the same as the way that Newtonian flat space-time explains. Just as flat space-time signifies is the fact that the kinematic concepts capturing inertial motions are based on Euclidean geometry, what curved space-time signifies is the fact that the relationship among a set of coordinates capturing inertial motion is based on semi-Riemannian geometry. Ellis and Williams states this very clearly.

> As in the case of particle world-lines, the relative separation of neighbouring light rays can be used to detect space-time curvature, ... In the space-time context, Euclid's axiom that parallel straight lines never meet is replaced by an equation (the equation of geodesic deviation) determining how the distance between neighbouring geodesics varies as a result of space-time curvature [6].

Accordingly, the structure of space-time within Einstein's general relativity is determined by the behaviours of bodies just like its predecessors. Just as the geometry of Newtonian space-time encodes information about the law of inertia that inertially moving particles move straight lines with constant velocity, the curvature of space-time of Einstein's general relativity encodes the information that neighboring inertially moving particles exhibit a relative acceleration.

4 Conclusion

Note that this essay does not argue that Einstein's theory does not revolutionize the Newtonian theory of gravitation. The former has many unique features not involved in the latter. Einstein's general relativity is essentially a field theory, while its predecessor is not. Einstein's general relativity is based on the equivalence principle and the light postulate, while its predecessor is not. The initial value problem of the Newtonian theory is well defined, but the one of its successor is not. Einstein's general relativity has symmetries unthinkable within its predecessor. It seems that all these make Einstein's theory unique in the development of physics. Hence, this essay is not about the comparison of the overall theoretical structure between Newtonian physics and Einstein's general relativity. The focus of this essay is rather the commonness of the relationship between space-time geometry and motion.

Just as the law of Newtonian inertia requires the kinematic properties that founds absolute rest, Einstein's equivalence principle requires a set of coordinate systems that founds the curvature of space-time. Accordingly, space-time geometry in Einstein's general relativity is determined by the behaviors of bodies just like its predecessors. Just as the geometries of Newtonian space-time encode information about the law of inertia that inertially moving particles move straight lines with constant velocity, the curvature of space-time of Einstein's general relativity encodes information that neighboring inertially-moving particles exhibit a relative acceleration. Newton's law of inertia and Einstein's equivalence principle play the role of the axiomatic principles, i.e., foundational principles of Newtonian and Einsteinian physics. These two principles lay foundations of the two physics upon the common unifying concepts of inertial motions.

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