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Integral Formulas Involving Product of Srivastava's Polynomials and Galué type Struve Functions

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Abstract. The aim of this paper is to establish two general finite integral formulas involving the product of Galué type Struve functions and Srivastava's polynomials. The results are given in terms of generalized (Wright's) hypergeometric functions. These results are obtained with the help of finite integrals due to Oberhettinger and Lavoie-Trottier. Some interesting special cases of the main results are also considered. The results presented here are of general character and easily reducible to new and known integral formulae.

1. Introduction and Preliminaries

Nisar et al., in [10], defined Galué type Struve functions (GTSF) as a generalization of Struve functions as follows:

$$(1.1) \qquad {}_aw^{\upsilon,\delta}_{p,b,c,\xi}(z) = \sum_{k=0}^{\infty} \frac{(-c)^k}{\Gamma(\upsilon k+\delta)\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)} \left(\frac{z}{2}\right)^{2k+p+1}$$

$$(a \in \mathbb{N}, p, b, c \in \mathbb{C}),$$

where $\upsilon > 0, \xi > 0$ and δ is an arbitrary parameter. For the definition of the Struve function and other generalizations of it, the interested reader may refer to the papers (Bhow-mick [3, 4], Kanth [6], Singh [16, 17]).

If we set $\upsilon = 1 = a, \delta = 3/2$ and $\xi = 1$, equation (1.1) reduces the GTSF which is defined by Orhan and Yagmur [13, 14] to the following:

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$$H_{p,b,c}(z)=\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma\left(k+\frac{3}{2}\right)\Gamma\left(k+p+\frac{b+2}{2}\right)}\left(\frac{z}{2}\right)^{2k+p+1}\quad p,b,c\in\mathbb{C}. \tag{1.2}$$

Details related to the function $H_{p,b,c}(z)$ and particular cases of it can be found in Baricz [1, 2], Menaria et al. [8], Nisar et al. [9], and Purohit et al. [15].

Recall that the generalized Wright hypergeometric function ${}_p\psi_q(z)$ for $z, a_i, b_j \in \mathbb{C}$ and $\alpha_i, \beta_i \in \mathbb{R}$, with $\alpha_i, \beta_j \neq 0$ $(i = 1, 2, ..., p; j = 1, 2, ..., q)$ was defined as follows (see [20] for details):

$${}_p\psi_q(z)={}_p\psi_q\left[\begin{array}{l}(a_i,\alpha_i)_{1,p}\,;\\(b_j,\beta_j)_{1,q}\,;\end{array}\ z\right]=\sum_{k=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma(a_i+\alpha_i k)z^k}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j k)k!}. \tag{1.3}$$

The generalized Wright function was introduced by Wright [24] in the form of (1.3) under the condition:

$$\sum_{j=1}^{q}\beta_j-\sum_{i=1}^{p}\alpha_i>-1. \tag{1.4}$$

It is noted that the generalized (Wright) hypergeometric function ${}_p\psi_q$ in (1.3) whose asymptotic expansion was investigated by Fox [5] and Wright is an interesting further generalization of the generalized hypergeometric series:

$${}_p\psi_q\left[\begin{array}{l}(\alpha_1,1),...,(\alpha_p,1)\,;\\(\beta_1,1),...,(\beta_q,1)\,;\end{array}\ z\right]=\frac{\prod_{j=1}^{p}\Gamma(\alpha_j)}{\prod_{j=1}^{q}\Gamma(\beta_j)}\,{}_pF_q\left[\begin{array}{l}\alpha_1,\ \ldots,\alpha_p\,;\\\beta_1,\ \ldots,\beta_q\,;\end{array}\ z\right], \tag{1.5}$$

where ${}_pF_q$ is the generalized hypergeometric series defined by

$${}_pF_q\left[\begin{array}{l}\alpha_1,\ldots,\alpha_p;\\\beta_1,\ldots,\beta_q;\end{array}\ z\right]=\sum_{n=0}^{\infty}\frac{(\alpha_1)_n\cdots(\alpha_p)_n z^n}{(\beta_1)_n\cdots(\beta_q)_n n!}={}_pF_q(\alpha_1,\ldots,\alpha_p;\beta_1,\ldots,\beta_q;z), \tag{1.6}$$

where $(\lambda)_n$ is the Pochhammer symbol defined (for $\lambda \in \mathbb{C}$) by (see [19]).

$$(\lambda)_n=\begin{cases}1 & (n=0)\\ \lambda(\lambda+1)\ldots(\lambda+n-1) & (n\in N=\{1,2,3,\ldots\})\end{cases}$$

$$=\frac{\Gamma(\lambda+n)}{\Gamma(\lambda)}\quad(\lambda\in\mathbb{C}\backslash\mathbb{Z}_0). \tag{1.7}$$

$\mathbb{Z}_0$ denotes the set of nonpositive integers.

Now, we recall the following known functions. Srivastava's polynomials are defined is [18] as

$$S_n^m(x)=\sum_{k=0}^{\lfloor n/m\rfloor}\frac{(-n)_{mk}}{k!}A_{n,k}\,x^k\qquad(n=0,\ 1,\ 2,\ ...), \tag{1.8}$$

where m is an arbitrary positive integer and the coefficients $A_{n,k}\,(n,k \geq 0)$ are arbitrary constants, reals or complex. The family $\{S_n^m(x)\}_{n=0}^{\infty}$ gives a number of known polynomials as special cases on for suitably specialized coefficients $A_{n,k}$.

For our present investigation, we also need to recall the following Oberhettinger integral formula [12]:

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda} dx = 2\lambda a^{-\lambda}\left(\frac{a}{2}\right)^{\mu}\frac{\Gamma(2\mu)\Gamma(\lambda-\mu)}{\Gamma(1+\lambda+\mu)}, \tag{1.9}$$

provided $0 < \Re(\mu) < \Re(\lambda)$.

Also we recall the Lavoie-Trottier integral formula from [7]:

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}\left(1-\frac{x}{3}\right)^{2\alpha-1}\left(1-\frac{x}{4}\right)^{\beta-1} dx = \left(\frac{2}{3}\right)^{2\alpha}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \tag{1.10}$$

provided $\Re(\alpha) > 0, \Re(\beta) > 0$.

2. Main Results

The main purpose of this paper is to introduce four generalized integral formulas involving products of general class of polynomials and generalized Galué type Struve functions. The integral formulas are as follows:

Theorem 2.1. *Let $a \in \mathbb{N}$, $\lambda, \mu, p, b, c \in \mathbb{C}$ such that $0 < \Re(\mu) < \Re(\lambda + p + 1)$. Let $\upsilon > 0$, $x > 0, n, k \geq 0$ and δ be an arbitrary parameter. The following integral holds:*

$$\int_0^\infty x^{\mu-1}(X)^{-\lambda} S_n^m(y/X)\,{}_a w_{p,b,c,\xi}^{\upsilon,\delta}(y/X)\,dx$$

$$= \sum_{l=0}^{\lfloor n/m\rfloor}\frac{(-n)_{ml}}{l!}A_{n,l}y^{l+p+1}2^{-\mu-p}a^{\mu-\lambda-l-p-1}\Gamma(2\mu)$$

$$\times {}_3\psi_4\left[\begin{matrix}(\lambda+l+p+2,2),(\lambda+l-\mu+p+1,2),(1,1);\\ (\delta,\upsilon),\left(\frac{p}{\xi}+\frac{b}{2}+1,a\right),(\lambda+l+p+1,2),(\lambda+l+\mu+p+2,2);\end{matrix}\ -cy^2/4a^2\right], \tag{2.1}$$

where $X = x + a + \sqrt{x^2+2ax}$.

Proof. Using (1.1) and (1.8) in the integrand of (2.1) and then interchanging the order of the integral sign and the summation, which is verified by uniform convergence of the involved series under the given conditions, we get

$$I_1 = \int_0^\infty x^{\mu-1}(X)^{-\lambda} S_n^m(y/X)\,{}_a w_{p,b,c,\xi}^{\upsilon,\delta}(y/X)\,dx$$

$$= \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{nl} y^l \sum_{k=0}^{\infty} \frac{(-c)^k}{\Gamma(\upsilon k+\delta)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)} \left(\frac{y}{2}\right)^{2k+p+1}$$

$$\times \int_0^{\infty} x^{\mu-1} (X)^{-(\lambda+l+2k+p+1)}\, dx. \tag{2.2}$$

We can apply the integral formula (1.9) to the integral in (2.2) and obtain the following expression under the valid conditions:

$$I_1 = \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{n,l} y^l \sum_{k=0}^{\infty} \frac{(-c)^k}{\Gamma(\upsilon k+\delta)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)} \left(\frac{y}{2}\right)^{2k+p+1}$$

$$\times 2(\lambda+l+p+1+2k)\, a^{-(\lambda+l+p+1+2k)} \left(\frac{a}{2}\right)^{\mu} \frac{\Gamma(2\mu)\,\Gamma(\lambda+l-\mu+p+1+2k)}{\Gamma(\lambda+l+\mu+p+2+2k)}$$

$$= \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{n,l} y^l 2^{-\mu-p} a^{\mu-\lambda-l-p-1} y^{p+1} \Gamma(2\mu) \sum_{k=0}^{\infty} \frac{(-c)^k}{\Gamma(\upsilon k+\delta)\,\Gamma\left(ak+\frac{p}{\xi}+\frac{b+2}{2}\right)}$$

$$\times \frac{\Gamma(\lambda+l+p+2+2k)\,\Gamma(\lambda+l-\mu+p+1+2k)}{\Gamma(\lambda+l+p+1+2k)\,\Gamma(\lambda+l+\mu+p+2+2k)} \left(\frac{y}{2a}\right)^{2k}.$$

In accordance with the definition of (1.3), we obtain the result (2.1). This completes the proof of the theorem. □

Theorem 2.2. *Let $a \in \mathbb{N}$, $\lambda, \mu, p, b, c \in \mathbb{C}$ such that $0 < \Re(\mu) < \Re(\lambda+p+1)$. Let $\upsilon > 0$, $x > 0, n, k \geq 0$ and δ be an arbitrary parameter. The following integral holds:*

$$\int_0^{\infty} x^{\mu-1} (X)^{-\lambda} S_n^m (xy/X)\, {}_a w_{p,b,c,\xi}^{\upsilon,\delta} (xy/X)\, dx$$

$$= \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{n,l} 2^{-\mu-l-2p} a^{\mu-\lambda-1} y^{l+p+1} \Gamma(1+\lambda-\mu)$$

$$\times {}_3\psi_4 \left[\begin{matrix} (\lambda+l+p+2,2),(2\mu+2l+2p,4),(1,1); \\ (\delta,\upsilon),\left(\frac{p}{\xi}+\frac{b}{2}+1,a\right),(\lambda+l+p+1,2),(\lambda+\mu+2l+2p+2,4); \end{matrix} \; -cy^2/4 \right]. \tag{2.3}$$

Proof. Proceeding as in the proof of Theorem 2.1, we get the integral formula (2.3). □

Theorem 2.3. *Let* $a \in \mathbb{N}$, $\alpha, \beta, p, b, c \in \mathbb{C}$ *such that* $\Re(\alpha) > 0, \Re(\beta + p + 1 + 2k) > 0$. *Let* $\upsilon > 0$, $x > 0, n, k \geq 0$ *and* δ *be an arbitrary parameter. The following integral holds:*

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(\mathrm{A})^{2\alpha-1}(\mathrm{B})^{\beta-1} S_n^m\left(y\mathrm{B}(1-x)^2\right) {}_a w_{p,b,c,\xi}^{\upsilon,\delta}\left(y\mathrm{B}(1-x)^2\right) dx$$

$$= \left(\frac{2}{3}\right)^{2\alpha} \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{n,l} y^l \left(\frac{y}{2}\right)^{p+1} \Gamma(\alpha)$$

$$\times {}_2\psi_3 \left[\begin{matrix} (\beta + l + p + 1, 2), (1, 1); \\ (\delta, \upsilon), \left(\frac{p}{\xi} + \frac{b}{2} + 1, a\right), (\alpha + \beta + l + p + 1, 2); \end{matrix} \; -cy^2/4 \right], \tag{2.4}$$

where $\mathrm{A} = \left(1 - \frac{x}{3}\right)$ *and* $\mathrm{B} = \left(1 - \frac{x}{4}\right)$.

Proof. Using (1.1) and (1.8) in the integrand of (2.4) and then interchanging the order of the integral sign and the summation, which is verified by uniform convergence of the involved series under the given conditions, we get

$$I_2 = \int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(\mathrm{A})^{2\alpha-1}(\mathrm{B})^{\beta-1} S_n^m\left(y\mathrm{B}\,(1-x)^2\right) {}_a w_{p,b,c,\xi}^{\upsilon,\delta}\left(y\mathrm{B}(1-x)^2\right) dx$$

$$= \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{n,l} y^l \sum_{k=0}^{\infty} \frac{(-c)^k}{\Gamma(\upsilon k + \delta)\Gamma\left(ak + \frac{p}{\xi} + \frac{b+2}{2}\right)} \left(\frac{y}{2}\right)^{2k+p+1}$$

$$\times \int_0^1 x^{\alpha-1}(1-x)^{2(\beta+l+p+1+2k)-1}(\mathrm{A})^{2\alpha-1}(\mathrm{B})^{\beta+l+p+1+2k-1} dx. \tag{2.5}$$

We can apply the integral formula (1.10) to the integral in (2.5) and obtain the following expression:

$$I_2 = \left(\frac{2}{3}\right)^{2\alpha} \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{n,l} y^l \sum_{k=0}^{\infty} \frac{(-c)^k}{\Gamma(\upsilon k + \delta)\Gamma\left(ak + \frac{p}{\xi} + \frac{b+2}{2}\right)} \left(\frac{y}{2}\right)^{2k+p+1}$$

$$\times \frac{\Gamma(\alpha)\Gamma(\beta + l + p + 1 + 2k)}{\Gamma(\alpha + \beta + l + p + 1 + 2k)}$$

$$= \left(\frac{2}{3}\right)^{2\alpha} \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{n,l} y^l \left(\frac{y}{2}\right)^{p+1} \Gamma(\alpha)$$

$$\times \sum_{k=0}^{\infty} \frac{(-c)^k \Gamma(\beta + l + p + 1 + 2k)}{\Gamma(\upsilon k + \delta)\Gamma\left(ak + \frac{p}{\xi} + \frac{b+2}{2}\right)\Gamma(\alpha + \beta + l + p + 1 + 2k)} \left(\frac{y}{2}\right)^{2k}. \tag{2.6}$$

In accordance with the definition of (1.3), we obtain the result (2.4). This completes the proof of the theorem. □

Theorem 2.4. *Let* $a \in \mathbb{N}$, $\alpha, \beta, p, b, c \in \mathbb{C}$ *such that* $\Re(\alpha) > 0, \Re(\beta + p + 1 + 2k) > 0$. *Let* $\upsilon > 0$, $x > 0, n, k \geq 0$ *and* δ *be an arbitrary parameter. The following integral holds:*

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(\mathrm{A})^{2\alpha-1}(\mathrm{B})^{\beta-1} S_n^m\left(yx(\mathrm{A})^2\right) {}_a w_{p,b,c,\xi}^{\upsilon,\delta}\left(yx(\mathrm{A})^2\right) dx$$

$$= \left(\frac{2}{3}\right)^{2(\alpha+p+1)} \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{n,l} y^l \left(\frac{y}{2}\right)^{p+1} \Gamma(\beta)$$

$$\times {}_2\psi_3 \left[\begin{matrix} (2\alpha + 2p + 2l + 2, 4), (1,1); \\ (\delta, \upsilon), \left(\frac{p}{\xi} + \frac{b}{2} + 1, a\right), (2\alpha + \beta + 2p + 2l + 2, 4); \end{matrix} -4cy^2/81 \right]. \tag{2.7}$$

Proof. Proceeding as in the proof of Theorem 2.3, we get the integral formula (2.7). □

3. Special Cases

In this section, we derive in Corollaries 3.1 - 3.4 some new integral formulae by using known generalized Struve function. We also derive as example a result involving the Hermite polynomials.

If we employ the same method as in proofs of Theorems 2.1 - 2.4, we obtain the following four corollaries with the help of (1.2) which is well known generalized Struve function due to Orhan and Yagmur [13, 14]. For the conditions $\upsilon = a = 1, \delta = 3/2$ and $\xi = 1$, the above Theorems reduce to:

Corollary 3.1. *Let the conditions of Theorem 2.1 be satisfied, then the following integral holds:*

$$\int_0^\infty x^{\mu-1}(X)^{-\lambda} S_n^m(y/X)\, H_{p,b,c}(y/X)\, dx$$

$$= \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{n,l} y^{l+p+1} 2^{-\mu-p} a^{\mu-l-p-1} \Gamma(2\mu)$$

$$\times {}_3\psi_4 \left[\begin{matrix} (\lambda + l + p + 2, 2), (\lambda + l - \mu + p + 1, 2), (1,1); \\ \left(p + \frac{b+2}{2}, 1\right), (\lambda + l + p + 1, 2), (\lambda + l + \mu + p + 2, 2), (3/2, 1); \end{matrix} -cy^2/4a^2 \right]. \tag{3.1}$$

Corollary 3.2. *Let the conditions of Theorem 2.2 be satisfied, then we have*

$$\int_0^\infty x^{\mu-1}(X)^{-\lambda} S_n^m(xy/X) H_{p,b,c}(xy/X)\,dx$$

$$= \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{n,l} 2^{-\mu-l-2p} a^{\mu-\lambda-1} y^{p+l+1} \Gamma(\lambda-\mu+1)$$

(3.2)
$$\times {}_3\psi_4 \left[\begin{matrix} (\lambda+l+p+2,2),(2\mu+2l+2p,4),(1,1); \\ \left(p+\frac{b+2}{2},1\right),(\lambda+l+p+1,2),(\lambda+\mu+2l+2p+2,4),(3/2,1); \end{matrix} -cy^2/4 \right].$$

Corollary 3.3. *Under the conditions of Theorem 2.3, we have*

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(\mathrm{A})^{2\alpha-1}(\mathrm{B})^{\beta-1} S_n^m\left(y\mathrm{B}(1-x)^2\right) H_{p,b,c}\left(y\mathrm{B}(1-x)^2\right) dx$$

$$= \left(\frac{2}{3}\right)^{2\alpha} \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{n,l} y^l \left(\frac{y}{2}\right)^{p+1} \Gamma(\alpha)$$

$$\times {}_2\psi_3 \left[\begin{matrix} (\beta+l+p+1,2),(1,1); \\ \left(p+\frac{b+2}{2},1\right),(\alpha+\beta+l+p+1,2),(3/2,1); \end{matrix} -cy^2/4 \right]. \tag{3.3}$$

Corollary 3.4. *Let the condition of Theorem 2.4 be satisfied, we have*

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(\mathrm{A})^{2\alpha-1}(\mathrm{B})^{\beta-1} S_n^m\left(yx(\mathrm{A})^2\right) H_{p,b,c}\left(yx(\mathrm{A})^2\right) dx$$

$$= \left(\frac{2}{3}\right)^{2(\alpha+p+1)} \sum_{l=0}^{\lfloor n/m \rfloor} \frac{(-n)_{ml}}{l!} A_{n,l} y^l \left(\frac{y}{2}\right)^{p+1} \Gamma(\beta)$$

$$\times {}_2\psi_3 \left[\begin{matrix} (2\alpha+2p+2l+2,4),(1,1); \\ \left(p+\frac{b+2}{2},1\right),(2\alpha+\beta+2p+2l+2,4),(3/2,1); \end{matrix} -4cy^2/81 \right]. \tag{3.4}$$

Further, If we set $m=2$ and $A_{n,l}=(-1)^l$ then the general class of polynomials stated in equation (1.8) can be reduced into the form:

$$S_n^2(x) \to x^{\frac{n}{2}} H_n\left(\frac{1}{2\sqrt{x}}\right), \tag{3.5}$$

where $H_n(x)$ denotes the well known Hermite polynomials, which are defined as

$$H_n(x) = \sum_{l=0}^{\lfloor n/m \rfloor} (-1)^l \frac{n!}{l!(n-2l)!} (2x)^{n-2l} . \tag{3.6}$$

In this case for example, Theorems 2.1 and 2.3 yield the following results involving the Hermite polynomials and the generalized Struve functions.

Corollary 3.5. *Let* $a \in \mathbb{N}$, $\lambda, \mu, p, b, c \in \mathbb{C}$ *such that* $0 < \Re(\mu) < \Re(\lambda + p + 1)$. *The following integral holds:*

$$\begin{aligned} &\int_0^\infty x^{\mu-1} (X)^{-\lambda-\frac{n}{2}} y^{\frac{n}{2}} H_n \left(\frac{1}{2\sqrt{y/X}} \right) {}_a w_{p,b,c,\xi}^{\upsilon,\delta} (y/X)\, dx \\ &= \sum_{l=0}^{\lfloor n/2 \rfloor} \frac{(-n)_{2l}}{l!} (-1)^l y^{l+p+1} 2^{-\mu-p} a^{\mu-\lambda-l-p-1} \Gamma(2\mu) \\ &\times {}_3\psi_4 \left[\begin{array}{c} (\lambda+l+p+2,2), (\lambda+l-\mu+p+1,2), (1,1); \\ (\delta,\upsilon), \left(\frac{p}{\xi}+\frac{b}{2}+1, a\right), (\lambda+l+p+1,2), (\lambda+l+\mu+p+2,2); \end{array} -cy^2/4a^2 \right]. \end{aligned} \tag{3.7}$$

Corollary 3.6. *Let* $a \in \mathbb{N}$, $\alpha, \beta, p, b, c \in \mathbb{C}$ *such that* $\Re(\alpha)$, $\Re(\beta)$, $R(p) > -1$. *The following integral holds:*

$$\begin{aligned} &\int_0^1 x^{\alpha-1} (1-x)^{2\beta+n-1} (\mathrm{A})^{2\alpha-1} (\mathrm{B})^{\beta+(n/2)-1} y^{\frac{n}{2}} \\ &\times H_n \left(\frac{1}{2\sqrt{y\mathrm{B}(1-x)^2}} \right) {}_a w_{p,b,c,\xi}^{\upsilon,\delta} \left(y\mathrm{B}(1-x)^2 \right) dx, \\ &= \left(\frac{2}{3}\right)^{2\alpha} \sum_{l=0}^{\lfloor n/2 \rfloor} \frac{(-1)^l (-n)_{2l}}{l!} y^l \left(\frac{y}{2}\right)^{p+1} \Gamma(\alpha) \\ &\times {}_2\psi_3 \left[\begin{array}{c} (\beta+l+p+1,2), (1,1); \\ (\delta,\upsilon), \left(\frac{p}{\xi}+\frac{b}{2}+1, a\right), (\alpha+\beta+l+p+1,2); \end{array} -cy^2/4 \right]. \end{aligned} \tag{3.8}$$

Remark 3.7. If we set $n = 0$, then we observe that the general class of polynomials $S_n^m(x)$ reduces to unity, i.e. $S_0^m(x) \to 1$, and we get known results due to Nisar et al. [11].

4. Conclusion

In the present paper, we used generalized (Wright) hypergeometric functions to investigated new integrals involving the generalized Struve functions and Srivastava polynomials. Certain special cases of involving Struve functions have been investigated in the literature by a number of authors [21, 22, 23] using different arguments. The results presented in this paper can easily be altered to deal with similar new interesting integrals by making suitable parameter substitutions. Further, for given suitable special values for the coefficient $A_{n,l}$, the general class of polynomials give many known classical orthogonal polynomials as particular cases. These include the Hermite, the Laguerre, the Jacobi, the Konhauser polynomials. In this sequel, one can obtain integral representation of more generalized special functions, which have extensive applications in physics and the engineering sciences.

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