J. Appl. Math. & Informatics Vol. **37***(2019), No. 1 - 2, pp. 37 - 52*
https://doi.org/10.14317/jami.2019.037

RIESZ TRIPLE ALMOST LACUNARY χ^3 SEQUENCE SPACES DEFINED BY A ORLICZ FUNCTION-I

N. SUBRAMANIAN, A. ESI* AND M. AIYUB

Abstract. In this paper we introduce a new concept for Riesz almost lacunary χ^3 sequence spaces strong $P-$ convergent to zero with respect to an Orlicz function and examine some properties of the resulting sequence spaces. We introduce and study statistical convergence of Riesz almost lacunary χ^3 sequence spaces and some inclusion theorems are discussed.

AMS Mathematics Subject Classification : 40A05,40C05,40D05.
Key words and phrases : analytic sequence, modulus function, double sequences, chi sequence, Riesz space.

1. Introduction

Throughout w, χ and Λ denote the classes of all, gai and analytic scalar valued single sequences, respectively. We write w^3 for the set of all complex triple sequences (x_{mnk}), where $m, n, k \in \mathbb{N}$, the set of positive integers. Then, w^3 is a linear space under the coordinate wise addition and scalar multiplication.

We can represent triple sequences by matrix. In case of double sequences we write in the form of a square. In the case of a triple sequence it will be in the form of a box in three dimensional case.

Some initial work on double series is found in *Apostol [1]* and double sequence spaces is found in *Hardy [7]*, *Deepmala et al [9], Subramanian et al. [10-15]* and many others. Later on some initial work on triple sequence spaces are found in *Esi [2]-[5] Esi and Catalbaş [3], Esi and Savas [4], Savas and Esi [6], Sahiner et al. [8]* and many others.

Let (x_{mnk}) be a triple sequence of real or complex numbers. Then the series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is called a triple series. The triple series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is convergent if and only if the triple sequence (S_{mnk}) is convergent, where

$$S_{mnk} = \sum_{i,j,q=1}^{m,n,k} x_{ijq}(m, n, k = 1, 2, 3, ...) .$$

Received June 21, 2018. Revised October 22, 2018. Accepted October 29, 2018.
*Corresponding author.

A triple sequence $x = (x_{mnk})$ is said to be analytic if

$$sup_{m,n,k} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

The vector space of all triple analytic sequences are usually denoted by Λ^3. A sequence $x = (x_{mnk})$ is called triple entire sequence if

$$|x_{mnk}|^{\frac{1}{m+n+k}} \to 0 \text{ as } m, n, k \to \infty.$$

The vector space of all triple entire sequences are usually denoted by Γ^3. Let the set of sequences with this property be denoted by Λ^3 and Γ^3 is a metric space with the metric

$$d(x, y) = sup_{m,n,k} \left\{ |x_{mnk} - y_{mnk}|^{\frac{1}{m+n+k}} : m, n, k : 1, 2, 3, ... \right\}, \quad (1)$$

for all $x = \{x_{mnk}\}$ and $y = \{y_{mnk}\}$ in Γ^3. Let $\phi = \{finite\ sequences\}$.

Consider a triple sequence $x = (x_{mnk})$. The $(m, n, k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by $x^{[m,n,k]} = \sum_{i,j,q=0}^{m,n,k} x_{ijq}\delta_{ijq}$ for all $m, n, k \in \mathbb{N}$,

$$\delta_{mnk} = \begin{bmatrix} 0 & 0 & ...0 & 0 & ... \\ 0 & 0 & ...0 & 0 & ... \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & ...1 & 0 & ... \\ 0 & 0 & ...0 & 0 & ... \\ & & & & \end{bmatrix}$$

with 1 in the $(m, n, k)^{th}$ position and zero otherwise.

A sequence $x = (x_{mnk})$ is called triple gai sequence if $((m + n + k)! |x_{mnk}|)^{\frac{1}{m+n+k}} \to 0$ as $m, n, k \to \infty$. The triple gai sequences will be denoted by χ^3.

2. Definitions and Preliminaries

A triple sequence $x = (x_{mnk})$ has limit 0 (denoted by $P - limx = 0$) (i.e) $((m + n + k)! |x_{mnk}|)^{1/m+n+k} \to 0$ as $m, n, k \to \infty$. We shall write more briefly as $P - convergent\ \ to\ \ 0$.

Definition 2.1. An Orlicz function is a function $M : [0, \infty) \to [0, \infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function M is replaced by $M(x + y) \leq M(x) + M(y)$, then this function is called modulus function. Different classes of sequence defined by Orlicz function have been introduced and investigated by *Prakash et al [16-21], Nakano [22], Lindenstrauss and Tzafriri [23], Altin et al [24], Et et al [25], Esi et al [26-27], Tripathy and Mahanta [28-29], Tripathy and Dutta [30-31],Tripathy and Goswami [32-35]* and many others.

Definition 2.2. A triple sequence $x = (x_{mnk})$ of real numbers is called almost $P-$ convergent to a limit 0 if

$$P - \lim_{p,q,u\to\infty} sup_{r,s,t\geq 0} \frac{1}{pqu} \sum_{m=r}^{r+p-1} \sum_{n=s}^{s+q-1} \sum_{k=t}^{t+u-1} \left((m+n+k)! \left|x_{mnk}\right|\right)^{1/m+n+k} \to 0.$$

that is, the average value of (x_{mnk}) taken over any rectangle $\{(m,n,k) : r \leq m \leq r+p-1, s \leq n \leq s+q-1, t \leq k \leq t+u-1\}$ tends to 0 as both p, q and u tend to ∞, and this $P-$ convergence is uniform in r, s and t. Let us denote set of sequences with this property as $\left[\widehat{\chi^3}\right]$.

Definition 2.3. Let $(q_{rst}), (\overline{q_{rst}}), (\overline{\overline{q_{rst}}})$ be sequences of positive numbers and

$$Q_r = \begin{bmatrix} q_{11} & q_{12} & ... & q_{1s} & 0... \\ q_{21} & q_{22} & ... & q_{2s} & 0... \\ . & & & & \\ . & & & & \\ . & & & & \\ q_{r1} & q_{r2} & ... & q_{rs} & 0... \\ 0 & 0 & ...0 & 0 & 0... \end{bmatrix} = q_{11} + q_{12} + \ldots + q_{rs} \neq 0,$$

$$\overline{Q}_s = \begin{bmatrix} \overline{q}_{11} & \overline{q}_{12} & ... & \overline{q}_{1s} & 0... \\ \overline{q}_{21} & \overline{q}_{22} & ... & \overline{q}_{2s} & 0... \\ . & & & & \\ . & & & & \\ . & & & & \\ \overline{q}_{r1} & \overline{q}_{r2} & ... & \overline{q}_{rs} & 0... \\ 0 & 0 & ...0 & 0 & 0... \end{bmatrix} = \overline{q}_{11} + \overline{q}_{12} + \ldots + \overline{q}_{rs} \neq 0,$$

$$\overline{\overline{Q}}_t = \begin{bmatrix} \overline{\overline{q}}_{11} & \overline{\overline{q}}_{12} & ... & \overline{\overline{q}}_{1s} & 0... \\ \overline{\overline{q}}_{21} & \overline{\overline{q}}_{22} & ... & \overline{\overline{q}}_{2s} & 0... \\ . & & & & \\ . & & & & \\ . & & & & \\ \overline{\overline{q}}_{r1} & \overline{\overline{q}}_{r2} & ... & \overline{\overline{q}}_{rs} & 0... \\ 0 & 0 & ...0 & 0 & 0... \end{bmatrix} = \overline{q}_{11} + \overline{q}_{12} + \ldots + \overline{q}_{rs} \neq 0.$$

Then the transformation which is given by

$T_{rst} = \frac{1}{Q_r \overline{Q}_s \overline{\overline{Q}}_t} \sum_{m=1}^{r} \sum_{n=1}^{s} \sum_{k=1}^{t} q_m \overline{q}_n \overline{\overline{q}}_k \left((m+n+k)! \left|x_{mnk}\right|\right)^{1/m+n+k}$ is called the Riesz mean of triple sequence $x = (x_{mnk})$. If $P - lim_{rst} T_{rst}(x) = 0, 0 \in \mathbb{R}$, then the sequence $x = (x_{mnk})$ is said to be Riesz convergent to 0. If $x = (x_{mnk})$ is Riesz convergent to 0, then we write $P_R - lim x = 0$.

Definition 2.4. The triple sequence $\theta_{i,\ell,j} = \{(m_i, n_\ell, k_j)\}$ is called triple lacunary if there exist three increasing sequences of integers such that

$$m_0 = 0, h_i = m_i - m_{r-1} \to \infty \text{ as } i \to \infty \text{ and}$$
$$n_0 = 0, \overline{h_\ell} = n_\ell - n_{\ell-1} \to \infty \text{ as } \ell \to \infty.$$
$$k_0 = 0, \overline{h_j} = k_j - k_{j-1} \to \infty \text{ as } j \to \infty.$$

Let $m_{i,\ell,j} = m_i n_\ell k_j, h_{i,\ell,j} = h_i \overline{h_\ell h_j}$, and $\theta_{i,\ell,j}$ be determined by $I_{i,\ell,j} = \{(m,n,k) : m_{i-1} < m < m_i \, and \, n_{\ell-1} < n \leq n_\ell \, and \, k_{j-1} < k \leq k_j\}, q_k = \frac{m_k}{m_{k-1}}, \overline{q_\ell} = \frac{n_\ell}{n_{\ell-1}}, \overline{q_j} = \frac{k_j}{k_{j-1}}$.
Let $\theta_{i,\ell,j} = \{(m_i, n_\ell, k_j)\}$ be a triple lacunary sequence and $q_m \overline{q}_n \overline{\overline{q}}_k$ be sequences of positive real numbers such that $Q_{m_i} = \sum_{m\in(0,m_i]} p_{m_i}, Q_{n_\ell} = \sum_{n\in(0,n_\ell]} p_{n_\ell}, Q_{n_j} = \sum_{k\in(0,k_j]} p_{k_j}$ and $H_i = \sum_{m\in(0,m_i]} p_{m_i}, \overline{H} = \sum_{n\in(0,n_\ell]} p_{n_\ell}, \overline{\overline{H}} = \sum_{k\in(0,k_j]} p_{k_j}$.
Clearly, $H_i = Q_{m_i} - Q_{m_{i-1}}, \overline{H}_\ell = Q_{n_\ell} - Q_{n_{\ell-1}}, \overline{\overline{H}}_j = Q_{k_j} - Q_{k_{j-1}}$. If the Riesz transformation of triple sequences is RH-regular, and $H_i = Q_{m_i} - Q_{m_{i-1}} \to \infty$ as $i \to \infty, \overline{H} = \sum_{n\in(0,n_\ell]} p_{n_\ell} \to \infty$ as $\ell \to \infty, \overline{\overline{H}} = \sum_{k\in(0,k_j]} p_{k_j} \to \infty$ as $j \to \infty$, then $\theta'_{i,\ell,j} = \{(m_i, n_\ell, k_j)\} = \left\{\left(Q_{m_i} Q_{n_j} Q_{k_k}\right)\right\}$ is a triple lacunary sequence. If the assumptions $Q_r \to \infty$ as $r \to \infty$, $\overline{Q}_s \to \infty$ as $s \to \infty$ and $\overline{\overline{Q}}_t \to \infty$ as $t \to \infty$ may be not enough to obtain the conditions $H_i \to \infty$ as $i \to \infty, \overline{H}_\ell \to \infty$ as $\ell \to \infty$ and $\overline{\overline{H}}_j \to \infty$ as $j \to \infty$ respectively.
Throughout the paper, we assume that $Q_r = q_{11} + q_{12} + \ldots + q_{rs} \to \infty \, (r \to \infty)$, $\overline{Q}_s = \overline{q}_{11} + \overline{q}_{12} + \ldots + \overline{q}_{rs} \to \infty \, (s \to \infty), \overline{\overline{Q}}_t = \overline{\overline{q}}_{11} + \overline{\overline{q}}_{12} + \ldots + \overline{\overline{q}}_{rs} \to \infty \, (t \to \infty)$, such that $H_i = Q_{m_i} - Q_{m_{i-1}} \to \infty$ as $i \to \infty, \overline{H}_\ell = Q_{n_\ell} - Q_{n_{\ell-1}} \to \infty$ as $\ell \to \infty$ and $\overline{\overline{H}}_j = Q_{k_j} - Q_{k_{j-1}} \to \infty$ as $j \to \infty$.
Let $Q_{m_i,n_\ell,k_j} = Q_{m_i} \overline{Q}_{n_\ell} \overline{\overline{Q}}_{k_j}, H_{i\ell j} = H_i \overline{H}_\ell \overline{\overline{H}}_j$,
$I'_{i\ell j} = \left\{(m,n,k) : Q_{m_{i-1}} < m < Q_{m_i}, \overline{Q}_{n_{\ell-1}} < n < Q_{n_\ell} \; and \; \overline{Q}_{k_{j-1}} < k < \overline{Q}_{k_j}\right\}$,
$V_i = \frac{Q_{m_i}}{Q_{m_{i-1}}}, \overline{V}_\ell = \frac{Q_{n_\ell}}{Q_{n_{\ell-1}}}$ and $\overline{\overline{V}}_j = \frac{Q_{k_j}}{Q_{k_{j-1}}}$. and $V_{i\ell j} = V_i \overline{V}_\ell \overline{\overline{V}}_j$.
If we take $q_m = 1, \overline{q}_n = 1 \; and \; \overline{\overline{q}}_k = 1$ for all m, n and k then $H_{i\ell j}, Q_{i\ell j}, V_{i\ell j}$ and $I'_{i\ell j}$ reduce to $h_{i\ell j}, q_{i\ell j}, v_{i\ell j}$ and $I_{i\ell j}$.

Let f be an Orlicz function and $p = (p_{mnk})$ be any factorable triple sequence of strictly positive real numbers, we define the following sequence spaces:

$$\left[\chi^3_R, \theta_{i\ell j}, q, f, p\right]$$
$$= \left\{P - \lim_{i,\ell,j\to\infty} \frac{1}{H_{i,\ell j}} \sum_{i\in I_{i\ell j}} \sum_{\ell\in I_{i\ell j}} \times \sum_{j\in I_{i\ell j}} q_m \overline{q}_n \overline{\overline{q}}_k \left[f\left((m+n+k)! \left|x_{m+i,n+\ell,k+j}\right|\right)^{p_{mnk}}\right] = 0\right\}$$

, uniformly in i, ℓ and j.

$$\left[\Lambda^3_R, \theta_{i\ell j}, q, f, p\right]$$

$$= \left\{ x = (x_{mnk}) : P - sup_{i,\ell,j} \frac{1}{H_{i,\ell j}} \sum_{i \in I_{i\ell j}} \sum_{\ell \in I_{i\ell j}} \times \sum_{j \in I_{i\ell j}} q_m \overline{q}_n \overline{\overline{q}}_k \left[f \left| x_{m+i,n+\ell,k+j} \right|^{p_{mnk}} \right] < \infty \right\}$$

, uniformly in i, ℓ and j.

Let f be an Orlicz function, $p = p_{mnk}$ be any factorable double sequence of positive real numbers and $q_m, \overline{q}_n$ and $\overline{\overline{q}}_k$ be sequences of positive numbers and $Q_r = q_{11} + \cdots q_{rs}$, $\overline{Q}_s = \overline{q}_{11} \cdots \overline{q}_{rs}$ and $\overline{\overline{Q}}_t = \overline{\overline{q}}_{11} \cdots \overline{\overline{q}}_{rs}$,

If we choose $q_m = 1, \overline{q}_n = 1$ and $\overline{\overline{q}}_k = 1$ for all m, n and k, then we obtain the following sequence spaces.

$$\left[\chi^3_R, q, f, p \right] = \left\{ P - \lim_{r,s,t \to \infty} \frac{1}{Q_r \overline{Q}_s \overline{\overline{Q}}_t} \sum_{m=1}^{r} \sum_{n=1}^{s} \times \sum_{k=1}^{t} q_m \overline{q}_n \overline{\overline{q}}_k \left[f \left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{p_{mnk}} \right] = 0 \right\}$$

, uniformly in i, ℓ and j.

$$\left[\Lambda^3_R, q, f, p \right] = \left\{ P - sup_{r,s,t} \frac{1}{Q_r \overline{Q}_s \overline{\overline{Q}}_t} \sum_{m=1}^{r} \sum_{n=1}^{s} \times \sum_{k=1}^{t} q_m \overline{q}_n \overline{\overline{q}}_k \left[f \left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{p_{mnk}} \right] < \infty \right\}$$

, uniformly in i, ℓ and j.

Definition 2.5. Let f be an Orlicz function and $p = (p_{mnk})$ be any factorable triple sequence of positive real numbers, we define the following sequence space: $\theta_{i,\ell,j} = \{(m_i, n_\ell, k_j)\}$ be a triple lacunary sequence

$$\chi^3_f \left[AC_{\theta_{i,\ell,j}}, p \right] = \left\{ P - lim_{i,\ell,j} \frac{1}{h_{i\ell j}} \sum_{m \in I_{i,\ell,j}} \sum_{n \in I_{i,\ell,j}} \times \sum_{k \in I_{i,\ell,j}} \left[f \left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} = 0 \right\}$$

, uniformly in i, ℓ and j.

We shall denote $\chi^3_f \left[AC_{\theta_{i,\ell,j}}, p \right]$ as $\chi^3 \left[AC_{\theta_{i,\ell,j}}, p \right]$ when $p_{mnk} = 1$ for all m, n

and k. If x is in $\chi^3\left[AC_{\theta_{i,\ell,j}},p\right]$, we shall say that x is almost lacunary χ^3 strongly $p-$convergent with respect to the Orlicz function f. Also note if $f(x)=x, p_{mnk}=1$ for all m,n and k then $\chi^3_f\left[AC_{\theta_{i,\ell,j}},p\right]=\chi^3\left[AC_{\theta_{i,\ell,j}}\right]$ which are defined as follows:

$$\chi^3\left[AC_{\theta_{i,\ell,j}}\right] = \left\{P-\lim_{i,\ell,j}\frac{1}{h_{i\ell j}}\sum_{m\in I_{i,\ell,j}}\sum_{n\in I_{i,\ell,j}} \times \sum_{k\in I_{i,\ell,j}}\left[f\left((m+n+k)!\left|x_{m+i,n+\ell,k+j}\right|\right)^{1/m+n+k}\right]=0\right\}$$

, uniformly in i,ℓ and j.
Again note if $p_{mnk}=1$ for all m,n and k then $\chi^3_f\left[AC_{\theta_{i,\ell,j}},p\right]=\chi^3_f\left[AC_{\theta_{i,\ell,j}}\right]$. we define

$$\chi^3_f\left[AC_{\theta_{i,\ell,j}},p\right] = \left\{P-\lim_{i,\ell,j}\frac{1}{h_{i\ell j}}\sum_{m\in I_{k,\ell,j}}\sum_{n\in I_{i,\ell,j}} \times \sum_{k\in I_{i,\ell,j}}\left[f\left((m+n+k)!\left|x_{m+i,n+\ell,k+j}\right|\right)^{1/m+n+k}\right]^{p_{mnk}}=0\right\}$$

, uniformly in i,ℓ and j.

Definition 2.6. Let f be an Orlicz function $p=(p_{mnk})$ be any factorable triple sequence of strictly positive real numbers, we define the following sequence space:

$$\chi^3_f[p] = \left\{P-\lim_{r,s,t\to\infty}\frac{1}{rst}\sum_{m=1}^{r}\sum_{n=1}^{s} \times \sum_{k=1}^{t}\left[f\left((m+n+k)!\left|x_{m+i,n+\ell,k+j}\right|\right)^{1/m+n+k}\right]^{p_{mnk}}=0\right\}$$

, uniformly in i,ℓ and j.

If we take $f(x)=x, p_{mnk}=1$ for all m,n and k then $\chi^3_f[p]=\chi^3$.

Definition 2.7. Let $\theta_{i,\ell,j}$ be a triple lacunary sequence; the triple number sequence x is $\widehat{S_{\theta i,\ell,j}}-p-$ convergent to 0 then

$$P-\lim_{i,\ell,j}\frac{1}{h_{i,\ell,j}}max_{i,\ell,j} \times \left|\left\{(m,n,k)\in I_{i,\ell,j}: f\left((m+n+k)!\left|x_{m+i,n+\ell,k+j}-0\right|\right)^{1/m+n+k}\right\}\right|=0.$$

In this case we write $\widehat{S_{\theta i,\ell,j}} - lim \left(f\left(m+n+k\right)! \left|x_{m+i,n+\ell,k+j} - 0\right|\right)^{1/m+n+k} = 0.$

3. Main results

Theorem 3.1. *If f is any Orlicz function and a bounded factorable positive triple number sequence (p_{mnk}), then $\chi^3_f\left[AC_{\theta_{i,\ell,j}}, P\right]$ is a linear space.*

Proof. The proof is easy. Theorefore omit the proof. □

Theorem 3.2. *For any modulus function f, we have $\chi^3\left[AC_{\theta_{i,\ell,j}}\right] \subset \chi^3_f\left[AC_{\theta_{i,\ell,j}}\right]$*

Proof. Let $x \in \chi^3\left[AC_{\theta_{i,\ell,j}}\right]$ so that for each i, ℓ and j

$$\chi^3\left[AC_{\theta_{i,\ell,j}}\right] = \left\{\lim_{i,\ell,j} \frac{1}{h_{i\ell j}} \sum_{m \in I_{i,\ell,j}} \sum_{n \in I_{i,\ell,j}} \times \sum_{k \in I_{i,\ell,j}} \left[\left((m+n+k)! \left|x_{m+i,n+\ell,k+j}\right|\right)^{1/m+n+k}\right] = 0\right\}.$$

Since f is continuous at zero, for $\varepsilon > 0$ and choose δ with $0 < \delta < 1$ such that $f(t) < \epsilon$ for every t with $0 \leq t \leq \delta$. We obtain the following,

$$\frac{1}{h_{i\ell j}}\left(h_{i\ell j}\epsilon\right) + \frac{1}{h_{i\ell j}} \sum_{m \in I_{i,\ell,j}} \times \sum_{n \in I_{i,\ell,j}\, and\, \left|x_{m+i,n+\ell,k+j}-0\right| > \delta} f\left[\left((m+n+k)! \left|x_{m+i,n+\ell,k+j}\right|\right)^{1/m+n+k}\right] \frac{1}{h_{i\ell j}}\left(h_{i\ell j}\epsilon\right) + \frac{1}{h_{i\ell j}} K\delta^{-1} f(2)\, h_{i\ell j}\, \chi^3\left[AC_{\theta_{i,\ell,j}}\right].$$

Hence i, ℓ and j go to infinity, so we are granted $x \in \chi^3_f\left[AC_{\theta_{i,\ell,j}}\right]$. □

Theorem 3.3. *Let $\theta_{i,\ell,j} = \{m_i, n_\ell, k_j\}$ be a triple lacunary sequence with $liminf_i q_i > 1$, $liminf_\ell \overline{q_\ell} > 1$ and $liminf_j q_j > 1$ then for any Orlicz function f, $\chi^3_f(P) \subset \chi^3_f\left(AC_{\theta_{i,\ell,j}}, P\right)$*

Proof. Suppose $liminf_i q_i > 1$, $liminf_\ell \overline{q_\ell} > 1$ and $liminf_j q_j > 1$ then there exists $\delta > 0$ such that $q_i > 1+\delta$, $\overline{q_\ell} > 1+\delta$ and $q_j > 1+\delta$. This implies $\frac{h_i}{m_i} \geq \frac{\delta}{1+\delta}$, $\frac{h_\ell}{n_\ell} \geq \frac{\delta}{1+\delta}$ and $\frac{h_j}{k_j} \geq \frac{\delta}{1+\delta}$ Then for $x \in \chi^3_f(P)$, we can write for each r, s and u.

$$B_{i,\ell,j} = \frac{1}{h_{i\ell j}} \sum_{m \in I_{i,\ell,j}} \sum_{n \in I_{i,\ell,j}} \sum_{k \in I_{i,\ell,j}} f\left[\left((m+n+k)! \left|x_{m+i,n+\ell,k+j}\right|\right)^{1/m+n+k}\right]^{p_{mnk}}$$

$$= \frac{1}{h_{i\ell j}} \sum_{m=1}^{m_i} \sum_{n=1}^{n_\ell} \sum_{k=1}^{k_j} f\left[\left((m+n+k)! \left|x_{m+i,n+\ell,k+j}\right|\right)^{1/m+n+k}\right]^{p_{mnk}}$$

$$- \frac{1}{h_{i\ell j}} \sum_{m=1}^{m_{i-1}} \sum_{n=1}^{n_{\ell-1}} \sum_{k=1}^{k_{i-1}} f\left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}}$$

$$- \frac{1}{h_{i\ell j}} \sum_{m=m_{i-1}+1}^{m_i} \sum_{n=1}^{n_{\ell-1}} \sum_{k=1}^{k_{j-1}} f\left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}}$$

$$- \frac{1}{h_{i\ell j}} \sum_{k=k_j+1}^{k_j} \sum_{n=n_{\ell-1}+1}^{n_\ell} \sum_{m=1}^{m_{k-1}} f\left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}}$$

$$= \frac{m_i n_\ell k_j}{h_{i\ell j}} \left(\frac{1}{m_i n_\ell k_j} \sum_{m=1}^{m_i} \sum_{n=1}^{n_\ell} \times \sum_{k=1}^{k_j} f\left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} \right)$$

$$- \frac{m_{k-1} n_{\ell-1} k_{j-1}}{h_{i\ell j}} \left(\frac{1}{m_{i-1} n_{\ell-1} kj-1} \sum_{m=1}^{m_{i-1}} \sum_{n=1}^{n_{\ell-1}} \times \sum_{k=1}^{k_{j-1}} f\left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} \right)$$

$$- \frac{k_{j-1}}{h_{i\ell j}} \left(\frac{1}{k_{j-1}} \sum_{m=m_{i-1}+1}^{m_i} \times \sum_{n=1}^{n_{\ell-1}} \sum_{k=1}^{k_j} f\left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} \right)$$

$$- \frac{n_{\ell-1}}{h_{i\ell j}} \left(\frac{1}{n_{\ell-1}} \sum_{m=m_{k-1}+1}^{m_k} \sum_{n=1}^{n_{\ell-1}} \times \sum_{k=1}^{k_j} f\left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} \right)$$

$$- \frac{m_{k-1}}{h_{i\ell j}} \left(\frac{1}{m_{k-1}} \sum_{k=1}^{k_j} \sum_{n=n_{\ell-1}+1}^{n_\ell} \times \sum_{m=1}^{m_{k-1}} f\left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} \right).$$

Since $x \in \chi_f^3(P)$ the last three terms tend to zero uniformly in m, n, k. Thus, for each r, s and u

$$B_{i,\ell,j} = \frac{m_i n_\ell k_j}{h_{i\ell j}} \left(\frac{1}{m_i n_\ell k_j} \sum_{m=1}^{m_i} \sum_{n=1}^{n_\ell} \times \sum_{k=1}^{k_j} f \left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} \right)$$
$$- \frac{m_{i-1} n_{\ell-1} k_{j-1}}{h_{i\ell j}} \left(\frac{1}{m_{i-1} n_{\ell-1} k_{j-1}} \sum_{m=1}^{m_{i-1}} \sum_{n=1}^{n_{\ell-1}} \times \sum_{k=1}^{k_{j-1}} f \left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} \right) + O(1).$$

Since $h_{i\ell j} = m_i n_\ell k_j - m_{i-1} n_{\ell-1} k_{j-1}$ we are granted for each i, ℓ and j the following

$$\frac{m_i n_\ell k j}{h_{i\ell j}} \leq \frac{1+\delta}{\delta} \text{ and } \frac{m_{i-1} n_{\ell-1} k_{j-1}}{h_{i\ell j}} \leq \frac{1}{\delta}.$$

The terms $\left(\frac{1}{m_i n_\ell k_j} \sum_{m=1}^{m_i} \sum_{n=1}^{n_\ell} \sum_{k=1}^{k_j} f \left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} \right)$ and $\left(\frac{1}{m_{i-1} n_{\ell-1} k_{j-1}} \sum_{m=1}^{m_{i-1}} \sum_{n=1}^{n_{\ell-1}} \sum_{k=1}^{k_{j-1}} f \left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} \right)$ are both gai sequences for all i, ℓ and j. Thus $B_{i\ell j}$ is a gai sequence for each i, ℓ and j. Hence $x \in \chi_f^3 \left(AC_{\theta_{i,\ell,j}}, P \right)$. □

Theorem 3.4. *Let $\theta_{i,\ell,j} = \{m, n, k\}$ be a triple lacunary sequence with $limsup_\eta q_\eta < \infty$ and $limsup_i \overline{q_i} < \infty$ then for any Orlicz function f, $\chi_f^3 \left(AC_{\theta_{i,\ell,j}}, P \right) \subset \chi_f^3 (p)$.*

Proof. Since $limsup_i q_i < \infty$ and $limsup_i \overline{q_i} < \infty$ there exists $H > 0$ such that $q_i < H$, $\overline{q_\ell} < H$ and $q_j < H$ for all i, ℓ and j. Let $x \in \chi_f^3 \left(AC_{\theta_{i,\ell,j}}, P \right)$. Also there exist $i_0 > 0, \ell_0 > 0$ and $j_0 > 0$ such that for every $a \geq i_0$ $b \geq \ell_0$ and $c \geq j_0$ and i, ℓ and j.

$$A'_{abc} = \frac{1}{h_{abc}} \sum_{m \in I_{a,b,c}} \sum_{n \in I_{a,b,c}} \sum_{k \in I_{a,b,c}} f \left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} \to 0$$

as $m, n, k \to \infty$. Let $G' = max \left\{ A'_{a,b,c} : 1 \leq a \leq i_0, \;\; 1 \leq b \leq \ell_0 \;\; and \;\; 1 \leq c \leq j_0 \right\}$ and p, q and t be such that $m_{i-1} < p \leq m_i$, $n_{\ell-1} < q \leq n_\ell$ and $m_{j-1} < t \leq m_j$. Thus we obtain the following:

$$\frac{1}{pqt} \sum_{m=1}^{p} \sum_{n=1}^{q} \sum_{k=1}^{t} \left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}}$$

$$\leq \frac{1}{m_{i-1}n_{\ell-1}k_{j-1}} \sum_{m=1}^{m_i} \sum_{n=1}^{n_\ell} \sum_{k=1}^{k_j} \left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}}$$

$$\leq \frac{1}{m_{i-1}n_{\ell-1}k_{j-1}} \sum_{a=1}^{i} \sum_{b=1}^{\ell}$$

$$\times \sum_{c=1}^{j} \left(\sum_{m \in I_{a,b,c}} \sum_{n \in I_{a,b,c}} \sum_{k \in I_{a,b,c}} \left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} \right)$$

$$= \frac{1}{m_{i-1}n_{\ell-1}k_{j-1}} \sum_{a=1}^{i_0} \sum_{b=1}^{\ell_0} \sum_{c=1}^{j_0} h_{a,b,c} A'_{a,b,c}$$

$$+ \frac{1}{m_{k-1}n_{\ell-1}k_{j-1}} \sum_{(i_0 < a \leq i) \bigcup (\ell_0 < b \leq \ell) \bigcup (j_0 < c \leq j)} h_{a,b,c} A'_{a,b,c}$$

$$\leq \frac{G'}{m_{i-1}n_{\ell-1}k_{j-1}} \sum_{a=1}^{i_0} \sum_{b=1}^{\ell_0} \sum_{c=1}^{j_0} h_{a,b,c}$$

$$+ \frac{1}{m_{i-1}n_{\ell-1}k_{j-1}} \sum_{(i_0 < a \leq i) \bigcup (\ell_0 < b \leq \ell) \bigcup (j_0 < c \leq j)} h_{a,b,c} A'_{a,b,c}$$

$$\leq \frac{G' m_{i_0} n_{\ell_0} k_{j_0} i_0 \ell_0 j_0}{m_{i-1}n_{\ell-1}k_{j-1}} + \frac{1}{m_{i-1}n_{\ell-1}j_{j-1}} \sum_{(i_0 < a \leq i) \bigcup (\ell_0 < b \leq \ell) \bigcup (j_0 < c \leq j)} h_{a,b,c} A'_{a,b,c}$$

$$\leq \frac{G' m_{i_0} n_{\ell_0 k_{j_0}} i_0 \ell_0 j_0}{m_{i-1}n_{\ell-1}k_{j-1}}$$

$$+ \left(sup_{a \geq i_0 \bigcup b \geq \ell_0 \bigcup c \geq j_0} A'_{a,b,c} \right) \frac{1}{m_{i-1}n_{\ell-1}k_{j-1}} \sum_{(i_0 < a \leq i) \bigcup (\ell_0 < b \leq \ell) \bigcup (j_0 < c \leq j)} h_{a,b,c}$$

$$\leq \frac{G' m_{i_0} n_{\ell_0 k_{j_0}} i_0 \ell_0 j_0}{m_{i-1}n_{\ell-1}k_{j-1}} + \frac{\epsilon}{m_{i-1}n_{\ell-1}k_{j-1}} \sum_{(i_0 < a \leq i) \bigcup (\ell_0 < b \leq \ell) \bigcup (j_0 < c \leq j)} h_{a,b,c}$$

$$\leq \frac{G' m_{i_0} n_{\ell_0 k_{j_0}} i_0 \ell_0 j_0}{m_{i-1}n_{\ell-1}k_{j-1}} + \epsilon H^3.$$

Since m_i, n_ℓ and k_j both approache to infinity as both p, q and t approache to infinity, it follows that

$$\frac{1}{pqt} \sum_{m=1}^{p} \sum_{n=1}^{q} \sum_{k=1}^{t} \left[\left((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right| \right)^{1/m+n+k} \right]^{p_{mnk}} = 0$$

, uniformly in $i, \ell \, and \, j$. Hence $x \in \chi_f^3 (P)$. □

Theorem 3.5. *Let $\theta_{i,\ell,j}$ be a triple lacunary sequence. Then*
(i) $(x_{mnk}) \xrightarrow{P} \chi^3\left(\widehat{S_{\theta_{i,\ell,j}}}\right)$
(ii)$\left(AC_{\theta_{i,\ell,j}}\right)$ *is a proper subset of* $\left(\widehat{S_{\theta_{i,\ell,j}}}\right)$
(iii) If $x \in \Lambda^3$ *and* $(x_{mnk}) \xrightarrow{P} \chi^3\left(\widehat{S_{\theta_{i,\ell,j}}}\right)$ *then* $(x_{mnk}) \xrightarrow{P} \chi^3\left(AC_{\theta_{i,\ell,j}}\right)$
(iv) $\chi^3\left(\widehat{S_{\theta_{i,\ell,j}}}\right) \bigcap \Lambda^3 = \chi^3\left[AC_{\theta_{i,,\ell,j}}\right] \bigcap \Lambda^3.$

Proof. For all r, s and u

$$\left|\left\{(m,n,k) \in I_{i,\ell,j} : \left((m+n+k)!\,|x_{m+i,n+\ell,k+j} - 0|\right)^{1/m+n+k}\right\} = 0\right|$$
$$\leq \sum_{m \in I_{i,\ell,j}} \sum_{n \in I_{i,\ell,j}} \times \sum_{k \in I_{i,\ell,j} \; and \; |x_{m+i,n+\ell,k+j}|=0} \left((m+n+k)!\,|x_{m+i,n+\ell,k+j} - 0|\right)^{1/m+n+k}$$
$$\leq \sum_{m \in I_{i,\ell,j}} \sum_{n \in I_{i,\ell,j}} \sum_{k \in I_{i,\ell,j}} \left((m+n)!\,|x_{m+i,n+\ell,k+j} - 0|\right)^{1/m+n+k}$$

, for all r, s and u

$$P - \lim_{i,\ell,j} \frac{1}{h_{i\ell j}} \sum_{m \in I_{i,\ell,j}} \sum_{n \in I_{i,\ell,j}} \times \sum_{k \in I_{i,\ell,j}} \left((m+n+k)!\,|x_{m+i,n+\ell,k+j} - 0|\right)^{1/m+n+k} = 0$$

This implies, for all i, ℓ and j

$$P - \lim_{i,\ell,j} \frac{1}{h_{i,\ell,j}} \left|\left\{(m,n,k) \in I_{i,\ell,j} : \left((m+n+k)!\,|x_{m+i,n+\ell,k+j} - 0|\right)^{1/m+n+k} = 0\right\}\right| = 0.$$

(ii) Let $x = (x_{mnk})$ be defined as follows:

$$(x_{mnk}) = \begin{bmatrix} 1 & 2 & 3 & ...\frac{\left[\sqrt[4]{h_{i,\ell,j}}\right]^{m+n+k}}{(m+n+k)!} & 0 & \ldots \\ 1 & 2 & 3 & ...\frac{\left[\sqrt[4]{h_{i,\ell,j}}\right]^{m+n+k}}{(m+n+k)!} & 0 & \ldots \\ . \\ . \\ . \\ 1 & 2 & 3 & ...\frac{\left[\sqrt[4]{h_{i,\ell,j}}\right]^{m+n+k}}{(m+n+k)!} & 0 & \ldots \\ . \\ . \\ . \\ 0 & 0 & 0 & ...0 & 0 & \ldots \\ . \\ . \\ . \end{bmatrix};$$

Here x is an triple sequence and for all i, ℓ and j

$$P - lim_{i,\ell,j} \frac{1}{h_{k,\ell,j}} \left| \left\{ (m,n,k) \in I_{i,\ell,j} : ((m+n+k)! \left| x_{m+i,n+\ell,k+j} - 0 \right|)^{1/m+n+k} = 0 \right\} \right|$$
$$= P - lim_{i,\ell,j} \frac{1}{h_{i,\ell,j}} \left(\frac{(m+n+k)! \left[\sqrt[4]{h_{i,\ell,j}}\right]^{m+n+k}}{(m+n+k)!} \right)^{1/m+n+k}$$
$$= 0.$$

Therefore $(x_{mnk}) \xrightarrow{P} \chi^3 \left(\widehat{S_{\theta_{i,\ell,j}}} \right)$. Also

$$P - lim_{i,\ell,j} \frac{1}{h_{i\ell j}} \sum_{m \in I_{i,\ell,j}} \sum_{n \in I_{i,\ell,j}} \sum_{k \in I_{i,\ell,j}} ((m+n+k)! \left| x_{m+i,n+\ell,k+j} \right|)^{1/m+n+k}$$
$$= P - \frac{1}{2} \Bigg(\lim_{i,\ell,j} \frac{1}{h_{i,\ell,j}}$$
$$\times \left(\frac{(m+n+k)! \left[\sqrt[4]{h_{i,\ell,j}}\right]^{m+n+k} \left[\sqrt[4]{h_{i,\ell,j}}\right]^{m+n+k} \left[\sqrt[4]{h_{i,\ell,j}}\right]^{m+n+k}}{(m+n+k)!} \right)^{1/m+n+k} + 1 \Bigg)$$
$$= \frac{1}{2}.$$

Therefore $(x_{mnk}) \overset{P}{\nrightarrow} \chi^3 \left(AC_{\theta_{i,\ell,j}} \right)$.
(**iii**) If $x \in \Lambda^3$ and $(x_{mnk}) \xrightarrow{P} \chi^3 \left(\widehat{S_{\theta_{i,\ell,j}}} \right)$ then $(x_{mnk}) \xrightarrow{P} \chi^3 \left(AC_{\theta_{i,\ell,j}} \right)$.
Suppose $x \in \Lambda^3$ then for all r, s and u, $((m+n+k)! \left| x_{m+i,n+\ell,k+j} - 0 \right|)^{1/m+n+k}$ $\leq M$ for all m, n, k. Also for given $\epsilon > 0$ and i, ℓ and j large for all r, s and u we

obtain the following:

$$\frac{1}{h_{i\ell j}} \sum_{m \in I_{i,\ell,j}} \sum_{n \in I_{i,\ell,j}} \sum_{k \in I_{i,\ell,j}} \left((m+n+k)! \left|x_{m+i,n+\ell,k+j} - 0\right|\right)^{1/m+n+k}$$

$$= \frac{1}{h_{i\ell j}} \sum_{m \in I_{k,\ell}} \sum_{n \in I_{i,\ell,j}}$$

$$\times \sum_{k \in I_{k,\ell,j} \; and \; |x_{m+i,n+\ell,k+j}| \geq 0} \left((m+n+k)! \left|x_{m+i,n+\ell,k+j} - 0\right|\right)^{1/m+n+k}$$

$$+ \frac{1}{h_{i\ell j}} \sum_{m \in I_{i,\ell,j}} \sum_{n \in I_{i,\ell,j}}$$

$$\times \sum_{k \in I_{i,\ell,j} \; and \; |x_{m+i,n+\ell,k+j}| \leq 0} \left((m+n+k)! \left|x_{m+i,n+\ell,k+j} - 0\right|\right)^{1/m+n+k}$$

$$\leq \frac{M}{h_{i\ell j}} \left| \left\{ (m,n,k) \in I_{i,\ell,j} : \left((m+n+k)! \left|x_{m+i,n+\ell,k+j} - 0\right|\right)^{1/m+n+k} \right\} = 0 \right| + \epsilon.$$

Therefore $x \in \Lambda^3$ and $(x_{mnk}) \xrightarrow{P} \chi^3 \left(\widehat{S_{\theta_{i,\ell,j}}}\right)$ this implies $(x_{mnk}) \xrightarrow{P} \chi^3 \left(AC_{\theta_{i,\ell,j}}\right)$.

(iv)$\chi^3 \left(\widehat{S_{\theta_{i,\ell,j}}}\right) \bigcap \Lambda^3 = \chi^3 \left[AC_{\theta_{i,\ell,j}}\right] \bigcap \Lambda^3$. Follows from (i),(ii) and (iii). □

Theorem 3.6. *If f be any Orlicz function then* $\chi^3_f \left[AC_{\theta_{i,\ell,j}}\right] \notin \chi^3 \left(\widehat{S_{\theta_{i,\ell,j}}}\right)$

Proof. Let $x \in \chi^3_f \left[AC_{\theta_{i,\ell,j}}\right]$, for all i, ℓ and j.
Therefore we have

$$\frac{1}{h_{i\ell j}} \sum_{m \in I_{i,\ell,j}} \sum_{n \in I_{i,\ell,j}} \sum_{k \in I_{i,\ell,j}} f\left[\left((m+n+k)! \left|x_{m+i,n+\ell,k+j} - 0\right|\right)^{1/m+n+k}\right]$$

$$\geq \frac{1}{h_{i\ell j}} \sum_{m \in I_{i,\ell,j}} \sum_{n \in I_{i,\ell,j}}$$

$$\times \sum_{k \in I_{i,\ell,j} \; and \; |x_{m+r,n+s,k+u}| = 0} f\left[\left((m+n+k)! \left|x_{m+i,n+\ell,k+j} - 0\right|\right)^{1/m+n+k}\right]$$

$$> \frac{1}{h_{i\ell j}} f(0) \left| \left\{ (m,n,k) \in I_{i,\ell,j} : \left((m+n+k)! \left|x_{m+i,n+\ell,k+j} - 0\right|\right)^{1/m+n+k} \right\} = 0 \right|.$$

Hence $x \notin \chi^3 \left(\widehat{S_{\theta_{i,\ell,j}}}\right)$. □

References

1. T. Apostol, *Mathematical Analysis*, Addison-Wesley London **1** , 978.
2. A. Esi , *On some triple almost lacunary sequence spaces defined by Orlicz functions*, Research and Reviews:Discrete Mathematical Structures **1**(2) (2014), 16-25.
3. A. Esi and M. Necdet Catalbas, *Almost convergence of triple sequences*, Global Journal of Mathematical Analysis **2**(1) (2014), 6-10.

4. A. Esi and E. Savas, *On lacunary statistically convergent triple sequences in probabilistic normed space*,Appl.Math.and Inf.Sci. **9**(5) (2015), 2529-2534.
5. A. Esi , *Statistical convergence of triple sequences in topological groups*, Annals of the University of Craiova, Mathematics and Computer Science Series **40**(1) (2013), 29-33.
6. E. Savas and A. Esi, *Statistical convergence of triple sequences on probabilistic normed space* ,Annals of the University of Craiova, Mathematics and Computer Science Series **39**(2) (2012), 226-236.
7. G.H. Hardy, *On the convergence of certain multiple series*, Proc. Camb. Phil. Soc. **19** (1917), 86-95.
8. A. Sahiner, M. Gurdal and F.K. Duden, *Triple sequences and their statistical convergence*, Selcuk J. Appl. Math.**8**(2) (2007), 49-55.
9. Deepmala, N. Subramanian and Vishnu Narayan Misra, *Double almost* $(\lambda_m \mu_n)$ *in* χ^2- *Riesz space*, Southeast Asian Bulletin of Mathematics **35** (2016), 1-11.
10. N. Subramanian, B.C. Tripathy and C. Murugesan, *The double sequence space of* Γ^2, Fasciculi Math. **40** (2008), 91-103.
11. N. Subramanian, B.C. Tripathy and C. Murugesan, *The Cesáro of double entire sequences*, International Mathematical Forum **4**(2) (2009), 49-59.
12. N. Subramanian and A. Esi, *The generalized triple difference of* χ^3 *sequence spaces* , Global Journal of Mathematical Analysis **3**(2) (2015), 54-60.
13. N. Subramanian and A. Esi, *The study on* χ^3 *sequence spaces*, Songklanakarin Journal of Science and Technology **38**(5) (2016), 581-590.
14. N. Subramanian and A. Esi, *Characterization of Triple* χ^3 *sequence spaces* , Mathematica Moravica **20**(1) (2016), 105-114.
15. N. Subramanian and A. Esi, *Some New Semi-Normed Triple Sequence Spaces Defined By A Sequence Of Moduli*, Journal of Analysis & Number Theory **3**(2) (2015), 79-88.
16. T.V.G. Shri Prakash, M. Chandramouleeswaran and N. Subramanian , *The Triple Almost Lacunary* Γ^3 *sequence spaces defined by a modulus function*, International Journal of Applied Engineering Research **10**(80) (2015), 94-99.
17. T.V.G. Shri Prakash, M. Chandramouleeswaran and N. Subramanian , *The triple entire sequence defined by Musielak Orlicz functions over* $p-$ *metric space*, Asian Journal of Mathematics and Computer Research,International Press **5**(4) (2015), 196-203.
18. T.V.G. Shri Prakash, M. Chandramouleeswaran and N. Subramanian , *The Random of Lacunary statistical on* Γ^3 *over metric spaces defined by Musielak Orlicz functions*, Modern Applied Science **10**(1) (2016), 171-183 .
19. T.V.G. Shri Prakash, M. Chandramouleeswaran and N. Subramanian , *The Triple* Γ^3 *of tensor products in Orlicz sequence spaces*, Mathematical Sciences International Research Journal **4**(2) (2015), 162-166.
20. T.V.G. Shri Prakash, M. Chandramouleeswaran and N. Subramanian , *The strongly generalized triple difference* Γ^3 *sequence spaces defined by a modulus* , Mathematica Moravica ,in press.
21. T.V.G. Shri Prakash, M. Chandramouleeswaran and N. Subramanian , *Lacunary Triple sequence* Γ^3 *of Fibonacci numbers over probabilistic* $p-$ *metric spaces* , International Organization of Scientific Research **12**(I-IV) (2016), 10-16.
22. H. Nakano, *Concave modulars*, Journal of the Mathematical Society of Japan **5** (1953), 29-49.
23. J. Lindenstrauss and L. Tzafriri, *On Orlicz sequence spaces*, Israel J. Math. **10**(1971), 379-390.
24. Y. Altin, M. Et and B.C. Tripathy, *The sequence space* $\left|\overline{N}_p\right|(M,r,q,s)$ *on seminormed spaces*, Applied Math.Comput. **154**(2004), 423-430.
25. M. Et, P.Y. Lee and B.C. Tripathy, *Strongly almost* $(V,\lambda)(\Delta^r)-$*summable sequences defined by Orlicz function*, Hokkaido Math.Jour. **35**(2006), 197-213.

26. A. Esi, N. Subramanian and A. Esi, *Triple rough statistical convergence of sequence of Bernstein operators*, Int. J. Adv. Appl. Sci. **4**(2) (2017), 28-34.
27. A. Esi, N. Subramanian and A. Esi, *The multi rough ideal convergence of difference strongly of χ^2 in p-metric spaces defined by Orlicz functions*, Turkish Journal of Analysis and Number Theory **5**(3) (2017), 93-100.
28. B.C. Tripathy and S. Mahanta, *On a class of vector valued sequences associated with multiplier sequences*, Acta Math.Applicata Sinica (Eng.Ser.) **20**(3) (2004), 487-494.
29. B.C. Tripathy and S. Mahanta, *On a class of difference sequences related to the λ^p space defined by Orlicz functions*, Mathematica Slovaca **57**(2) (2007), 171-178.
30. B.C. Tripathy and H.Dutta, *On some new paranormed difference sequence spaces defined by Orlicz functions*, Kyungpook Mathematical Journal **50**(1) (2010), 59-69.
31. B.C. Tripathy and H. Dutta, *On some lacunary difference sequence spaces defined by a sequence of Orlicz functions and q-lacunary Δ_m^n-statistical convergence*, Analele Stiintifice ale Universitatii Ovidius, Seria Matematica **20**(1) (2012), 417-430.
32. B.C. Tripathy and R. Goswami, *On triple difference sequences of real numbers in probabilistic normed spaces*, Proyecciones Jour.Math. **33**(2) (2014), 157-174.
33. B.C. Tripathy and R. Goswami, *Vector valued multiple sequences defined by Orlicz functions*, Boletim de Sociedade Paranaense De Matematica **33**(1) (2015), 67-79.
34. B.C. Tripathy and R. Goswami, *Multiple sequences in probabilistic normed spaces*, Afrika Matematika **26**(5-6) (2015), 753-760.
35. B.C. Tripathy and R. Goswami, *Statistically convergent multiple sequences in probabilistic normed spaces*, U.P.B.Sci.Bull, Ser.A **78**(4) (2016), 83-94.

N. Subramanian is a professor in SASTRA Deemed to be university, Thanjavur. He received the PhD degree in Mathematics for Alagappa University at Karaikudi, Tamil Nadu, India and also getting Doctor of Science (D.Sc) degree in Mathematics for Berhampur University,Berhampur, Odissa, India. His research interests are in the areas of summability through functional analysis of applied mathematics and pure mathematics. He has published research articles 280 in reputed international journals of mathematical and engineering sciences.

Department of Mathematics, SASTRA Deemed to be University, Thanjavur-613 401, India.
e-mail: nsmaths@gmail.com

Ayhan Esi is a Professor in Turkey Department of Mathematics at the University of Adiyaman .
He received his Ph.D. Degree in Mathematics from Firat University in 1995 and earned his postdoctoral from university of Firat. Then worked as a full time mathematician in many different Institute/University for 31 Years. He is having 31 Years of Experience in the field of Mathematics. His Area of Expertise includes: Summability Theory, Sequences, Series, Fuzzy Sequence Spaces. He is the author of many international published papers.

Department of Mathematics,Adiyaman University Adiyaman-02040, Turkey
e-mail: aesi23@hotmail.com

M. Aiyub has received his Ph. D. degree in pure Mathematics from Aligarh Muslim University, Aligarh India in 2000. His Ph.D. thesis is on Matrix transformation and Sequence Spaces.Currently working as an assistant Professor in the Department of Mathematics College of Science in the University of Bahrain, Kingdom of Bahrain.He has 16 years of TeachingExperience. He has also worked as Assistant Professor through united Nations Development Programs in Alemaya University Ethiopia since October 2002 to June 2006. Then he joined in the Bahrain University on September 1, 2006 and has been working till date .He taught somany courses of Mathematics of 100 level, 200 level and 300 level . He also worked in many departmental committees. His research interest is Matrix Transformations and sequence spaces, Sequence and Series and sumability. He has published 25 papers in national and International Journals. He has also reviewed so many research papers for the national and international journals. He is a member of Editorial Board of International Journal of Applied & Experimental Mathematics.

Department of Mathematics,College of Science University of Bahrain,P.O.Box-32038 Manam, Kingdom of Bahrain.
e-mail: maiyub2002@gmail.com