# Probabilistic free vibration analysis of Goland wing

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#### Abstract

In this paper, the probabilistic free vibration analysis of a geometrically coupled cantilever wing with uncertain material properties is carried out using stochastic finite element (SFEM) based on first order perturbation technique. Here, both stiffness and damping of the system are considered as random parameters. The bending and torsional rigidities are assumed as spatially varying second order Gaussian random fields and represented by Karhunen Loeve (K-L) expansion. Here, the expected value, standard deviation, and probability distribution of random natural frequencies and damping ratios are computed. The results obtained from the present approach are also compared with Monte Carlo simulations (MCS). The results show that the uncertain bending rigidity has more influence on the damping ratio and frequency of modes 1 and 3 while uncertain torsional rigidity has more influence on the damping ratio and frequency of modes 2 and 3.

Key Words : Coupled beam, Free vibration, Proportional damping, SFEM, Random process, K-L expansion

#### 1. Introduction

The free vibration analysis is important for any dynamical system such as bridges, turbines, compressors, aircraft structures etc. In these structures, there is some difference in location of elastic axis (shear center) and centroid of the crosssection. Due to this difference or non-coincidence of shear center and centroid of the cross-section a coupling in the dynamical system arises known as geometrical coupling. The geometrical coupling occurs in both civil and aircraft structures, which is known as bending-torsion inertia coupling. Friberg [1] considered geometrically coupled beam, and used Euler-Bernoulli-Saint Venant theory to develop a numerical procedure to solve coupled beam problem for eigen modes and eigen frequencies.

Received: Oct.13, 2018 Revised: Nov. 09, 2019 Accepted: Dec. 11, 2019 † Corresponding Author Tel: +91-80-2508-6490, E-mail: aeroamit@nal.res.in © The Society for Aerospace System Engineering Banarjee [2] and Banarjee et al. [3] developed an exact explicit expression which gives the natural frequencies and mode shapes of bending torsion coupled beam, and equivalent beam formulation of composite beams respectively. Hasheni and Richard [4] presented a dynamic finite element formulation for free vibration analysis of axially loaded bendingtorsion coupled beam, which uses the exact solution based on Euler-Bernoulli and St. Venant beam theories for finding the exact solution of axially loaded uniform beam. The solution is treated as the basis of dynamic finite element formulation. The flutter analysis of wing like structures was carried out by many researchers [5–7]. Cheng and Xiao [8, 9] carried out the probabilistic free vibration analysis of civil structures, axially loaded beams, and suspension bridges having both structural as well as geometric uncertainty modeled as random variables using response surface method. The coefficients of response surface polynomial were determined using design of experiment technique. Recently, Sepahvand

and Marburg [10] considered probabilistic dynamic analysis of cantilever beam (without geometrical coupling) with spatial random variation in bending rigidity. The proportional damping was considered as random fields, and for solution purpose generalized polynomial chaos conjunction with nonintrusive technique was used. From the literature, it is noted that much attention has been given by researchers on deterministic free vibration analysis of geometric coupled beam with various types of application irrespective of discipline but the study of probabilistic free vibration characteristics of geometrically coupled beam is not stressed.

In the present work, the probabilistic free vibration of a geometric coupled wing with uncertain bending and torsional rigidities is studied. Here, both bending and torsional rigidities are modelled as second order Gaussian random fields with isotropic exponential covariance function. The proportional damping is also considered as random field in order to get more realistic eigenvalue solution. The first order perturbation technique is used for the probabilistic free vibration analysis of the wing, and the obtained response is validated through Monte Carlo simulations (MCS).

#### 2. Mathematical model

The schematic representation of geometric coupled cantilever wing [5] modelled as 1-D beam is shown in Fig. 1. Here, 0 is the origin of the axis system. C and P are the locations of center of mass and shear center respectively. The dimensionless parameters a and e ( $-1 \le a \le 1$  and  $-1 \le e \le 1$ ) determine the location of elastic and inertia axes respectively. The governing equations of motion of beam with random stiffness parameters can be expressed as:

$$m\ddot{w} - mx_{\alpha}b\ddot{\alpha} + \frac{\partial^2}{\partial^2 y} \left( EI \frac{\partial^2 w}{\partial^2 y} \right) = 0 \tag{1}$$

$$I_p \ddot{\alpha} - m x_{\alpha} b \ddot{w} - \frac{\partial}{\partial y} \left( G J \frac{\partial \alpha}{\partial y} \right) = 0$$
 (2)

Here, w(y,t) and  $\alpha(y,t)$  denote the out-of-plane deflection and rotation of beam about elastic axis respectively. *m* denotes mass per unit span and  $I_p = I_{\alpha} + m(x_{\alpha}b)^2$  is mass moment of inertia per unit span about elastic axis, where  $I_{\alpha}$  is the mass moment of inertia per unit span about inertia axis,  $x_{\alpha} = (e - a)$ is the dimensionless static unbalance, and *b* is the semi-reference chord of the wing. The dot over any quantity represents the time derivative.



# Fig. 1 The schematic representation of geometric coupled cantilever beam

The beam bending rigidity (*E1*) and torsional rigidity (*GJ*) are assumed as random parameters and modeled as Gaussian random fields.

#### 2.1. Finite element discretization

The governing equations of motion of wing given in Eqs. 1 and 2 can be expressed in finite element form using weak formulation and choosing suitable interpolation functions for displacement fields. The discrete form of equation of motion can be expressed as:

$$M\{\ddot{q}\} + D\{\dot{q}\} + K\{q\} = \{0\}$$
(3)

where *M*, *D*, and  $K(=K_b + K_t)$  are the global mass, damping, and stiffness matrices respectively, and  $\{q\}$ and  $\{0\}$  denote the nodal displacement and null vectors respectively.  $K_b$  and  $K_t$  are global bending and torsional stiffness matrices respectively. Here, the distributed structural damping in the system is modeled using mass-stiffness proportional Rayleigh damping. The damping matrix *D* is formed by linear combination of the mass and stiffness matrices as  $D = \beta M + \gamma K$ , where  $\beta$  and  $\gamma$  are the mass and stiffness proportional damping constants.

#### 2.2. Random field discretization

In this paper, a spectral method is used to discretize the stochastic space for the treatment of randomness in physical quantities. In this method, random quantities are represented by spectral decomposition in terms of unknown coefficients and orthogonal basis functions. *EI* and *GJ* are treated as independent Gaussian random fields defined on the probability space  $(\Omega)$  over the physical domain *L*.

The random field is represented by mean and its covariance function [11], and discretized in a mean square convergent series known as K-L expansion [12] as:

$$EI(y,\theta) = \overline{EI} + \sum_{n=1}^{\infty} \sqrt{\lambda_n} \xi_n(\theta) f_n(y)$$
(4)

$$GJ(y,\theta) = \overline{GJ} + \sum_{n=1}^{\infty} \sqrt{\lambda_n} \xi_n(\theta) f_n(y)$$
(5)

Here,  $\overline{EI}$  and  $\overline{GJ}$  are the mean of random fields, and  $\lambda_n$  and  $f_n(y)$  are the eigenvalues and eigen functions of the covariance kernel [12].  $\xi_n(\theta)$  represents the zero mean and unit variance independent Gaussian random variables. The discrete form of Eq. 3 can be written with *N* truncated *K*-*L* expansion terms as:

$$M\{\ddot{q}\} + \left[\beta M + \gamma \left(\overline{K} + \sum_{n=1}^{N} \xi_n(\theta) K_{b,n} + \sum_{n=1}^{N} \xi_n(\theta) K_{t,n}\right)\right] \{\dot{q}\} + \left[\overline{K} + \sum_{n=1}^{N} \xi_n(\theta) K_{b,n} + \sum_{n=1}^{N} \xi_n(\theta) K_{t,n}\right] \{q\} = \{0\}$$

$$(6)$$

where  $\overline{K}$  is the global mean stiffness matrix containing bending and torsional stiffness terms, and  $\sum_{n=1}^{N} \xi_n(\theta) K_{b,n}$  and  $\sum_{n=1}^{N} \xi_n(\theta) K_{t,n}$  are stochastic bending and torsional stiffness matrices respectively. Let  $\{q(t)\} = \{\overline{q}\} exp(\lambda t)$ , where  $\lambda$  and  $\{\overline{q}\}$  are eigenvalue and eigenvector respectively,  $\lambda = -\zeta \omega + i\omega$ , where  $\zeta$  and  $\omega$  are damping ratio and damped natural frequency respectively. Upon substituting  $\{q\}$ in Eq. 6, we get the eigenvalue problem as:

$$\left[ \lambda^2 M + \lambda \left[ \beta M + \gamma \left( \overline{K} + \sum_{n=1}^N \xi_n(\theta) K_{b,n} + \sum_{n=1}^N \xi_n(\theta) K_{t,n} \right) \right] + \left[ \overline{K} + \sum_{n=1}^N \xi_n(\theta) K_{b,n} + \sum_{n=1}^N \xi_n(\theta) K_{t,n} \right] \left\{ \overline{q} \right\} = \{ 0 \}$$

$$(7)$$

The above equation is solved using perturbation technique in the next subsection.

#### 2.3. Perturbation approach

In the perturbation technique, eigenvalue and eigenvector are expanded via Taylor series expansion about the mean values of random variables obtained from *K*-*L* expansion as:

$$\lambda = \lambda_0 + \sum_{n=1}^N \frac{\partial \lambda}{\partial \xi_n} |_{\xi_n = 0} \xi_n(\theta)$$
(8)

$$\{\bar{q}\} = \{\bar{q}_0\} + \sum_{n=1}^N \frac{\partial\{\bar{q}\}}{\partial\xi_n}|_{\xi_n=0}\xi_n(\theta)$$
(9)

Here,  $\lambda_0$  and  $\{\bar{q}_0\}$  are the mean values of  $\lambda$  and  $\{\bar{q}\}$  respectively. Upon substitution of Eqs. 8 and 9 into Eq. 7, and separating zeroth order and first order terms, the following equations are obtained.

Zeroth order:

$$[\lambda_o^2 M + \lambda_0 (\beta M + \gamma \overline{K}) + \overline{K}] \{\overline{q}_0\} = \{0\}$$
(10)

First order:

$$\begin{split} \left[\lambda_o^2 M + \lambda_0 (\beta M + \gamma \overline{K}) + \overline{K}\right] \frac{\partial \{\overline{q}\}}{\partial \xi_n} + \left[\lambda_0 \left(\gamma (K_{b,n} + K_{t,n})\right) + \left(K_{b,n} + K_{t,n}\right)\right] \{\overline{q}_0\} + \frac{\partial \lambda}{\partial \xi_n} [2\lambda_0 M + (\beta M + \gamma \overline{K})] \{\overline{q}_0\} = \{0\} \end{split}$$
(11)

The mean eigenvalues are evaluated from zeroth order equation. To find the derivative of eigenvalue, multiply mean adjoint eigenvector transpose  $\{\bar{q}_{o_{adj}}\}^{T}$  or left eigenvector transpose [13-15] in Eq. 11. We get the eigenvalue derivative with respect to random variables as:

$$\frac{\partial\lambda}{\partial\xi_n} = -\frac{\left\{\bar{q}_{o_{adj}}\right\}^T \left[\lambda_0 \left(\gamma(K_{b,n} + K_{t,n})\right) + (K_{b,n} + K_{t,n})\right] \left\{\bar{q}_0\right\}}{\left\{\bar{q}_{o_{adj}}\right\}^T \left[2\lambda_0 M + (\beta M + \gamma \bar{K})\right] \left\{\bar{q}_0\right\}}$$
(12)

The eigenvalue  $(\lambda)$  consists of real and imaginary parts. The real part consists of damping ratio and frequency, and imaginary part consists of frequency of the damped system. The variance of damping ratio can be expressed as:

$$\sigma_{\zeta}^{2} = \frac{1}{\omega_{o}^{2}} \sum_{n=1}^{N} \left( \frac{\partial Re(\lambda)}{\partial \xi_{n}} + \zeta_{o} \frac{\partial Im(\lambda)}{\partial \xi_{n}} \right)^{2}$$
(13)

and variance of frequency as:

$$\sigma_{\omega}^2 = \sum_{n=1}^{N} \left( \frac{\partial Im(\lambda)}{\partial \xi_n} \right)^2 \tag{14}$$

where  $\zeta_o$  and  $\omega_o$  are the mean damping ratio and frequency respectively.

#### 3. Results and discussion

The mean properties of cantilever beam [2] used for the analysis are  $EI = 9.75 \times 10^6$  Nm<sup>2</sup>,  $GJ = 9.88 \times$  $10^5$  Nm<sup>2</sup>, m = 35.75 kg/m,  $I_p = 8.65$  kg-m,  $x_{\alpha} = 0.36$ , a = 0, b = 0.5 m and L= 6 m. First, the validation of mean natural frequencies of cantilever beam is performed. Table 1 shows the comparison of the first two mean frequencies obtained from the present approach with those given by Banerjee [2] which match well with each other.

Table 1 Mean natural frequencies of cantilever beam

Mode	Present FEM	Analytical results <sup>2</sup>		
Number	solution (rad/s)	(rad/s)		
$\omega_1$	49.62	49.6		
ω2	97.13	97.0		

Next, the probabilistic free vibration analysis of geometric coupled beam is carried out using first order perturbation technique. The mean properties of

Parameters	Description	Values	
EI	Span-wise bending stiffness	$9.77 \times 10^6 \text{ Nm}^2$	
GJ	Span-wise torsion stiffness	$0.988 \times 10^{6} \text{ Nm}^{2}$	
m	Mass per unit length	35.719 kg/m	
$x_{\alpha}$	Dimensionless static balance	0.33	
а	Elastic axis location parameter	- 0.2	
b	Semi-reference chord	0.9144 m	
L	L Span		
Ip	<i>I<sub>p</sub></i> Mass moment of inertia per unit length		
β, γ	Damping coefficients	$1.58, 1.33 \times 10^{-4}$	

 Table 2 Mean properties of cantilever wing

**Table 3** Comparison of Mean and SD of frequencies and damping ratios of geometric coupled wing using variousapproaches for C.O.V of EI = 0.05

No of	Mode	Perturbation			MCS		
K-L	Nos						
terms		$(\omega_0,\zeta_0)$	$\sigma_{\omega}$	$\sigma_{\zeta}$	$(\omega_0,\zeta_0)$	$\sigma_{\omega}$	$\sigma_{\zeta}$
N=2	Mode 1	(46.6810, 0.0200)	0.9320	2.7592e-04	(46.6779, 0.0200)	0.9255	2.7489e-04
	Mode 2	(141.3769, 0.0149)	0.4551	1.2288e-05	(141.3794,0.0149)	0.4518	1.2199e-05
	Mode 3	(253.2289, 0.0199)	2.4215	1.3135e-04	(253.1812,0.0199)	2.4065	1.3048e-04
N=4	Mode 1	(46.6810, 0.0200)	0.9400	2.7829e-04	(46.6747,0.0200)	0.9340	2.7749e-04
	Mode 2	(141.3769, 0.0149)	0.4570	1.2339e-05	(141.3779,0.0149)	0.4540	1.2260e-05
	Mode 3	(253.2289, 0.0199)	2.4218	1.3136e-04	(253.1766,0.0199)	2.4070	1.3051e-04

**Table 4** Comparison of Mean and SD of frequencies and damping ratios of geometric coupled wing using variousapproaches for C.O.V of GJ = 0.05

No of	Mode	Perturbation			MCS		
K-L	Nos						
terms		$(\omega_0,\zeta_0)$	$\sigma_{\omega}$	$\sigma_{\zeta}$	$(\omega_0,\zeta_0)$	$\sigma_{\omega}$	$\sigma_{\zeta}$
N=2	Mode 1	(46.6810, 0.0200)	0.1280	3.7892e-05	(46.6768, 0.0200)	0.1276	3.7801e-05
	Mode 2	(141.3769, 0.0149)	2.7362	7.3871e-05	(141.3800, 0.0149)	2.7155	7.3242e-05
	Mode 3	(253.2289, 0.0199)	2.2825	1.2381e-04	(253.1896, 0.0199)	2.2672	1.2294e-04
N=4	Mode 1	(46.6810, 0.0200)	0.1281	3.7931e-05	(46.6764, 0.0200)	0.1278	3.7868e-05
	Mode 2	(141.3769, 0.0149)	2.7444	7.4094e-05	(141.3697, 0.0149)	2.7259	7.3512e-05
	Mode 3	(253.2289, 0.0199)	2.2871	1.2406e-04	(253.1827, 0.0199)	2.2737	1.2329e-04

cantilever wing [7] used in the present analysis are given in Table 2. For the present problem exponential covariance function is assumed as  $C(y, y_1) = \sigma^2 e^{-c|y-y_1|}$ , where  $\sigma^2$  is the variance of the field and *c* is the reciprocal of the correlation length. The correlation length considered here is span length (*L*), and for the probabilistic model validation coefficient of variation (C.O.V) in bending and torsional rigidities are taken as 0.05. The expected value and standard deviation (SD) of the first three natural frequencies and damping ratios of the geometric coupled beam obtained from perturbation approach and MCS (with 5000 Samples) are shown in Tables 3 and 4. The effect of number of terms in the K-L expansion (N = 2 and 4) on the SD of natural frequencies and damping ratios are also shown in Tables 3 and 4. From Table 3, it is observed that the mean and SD of damping ratio and frequency of



Fig. 2 The C.O.V of damping ratio and frequency of Mode 1, Mode 2 and Mode 3 vs C.O.V of EI

various modes obtained using perturbation approach agree well with MCS. The convergence of the SD of natural frequencies and damping ratios are also shown for increasing terms in the K-L expansion, and found to be converging well with four terms of K-L expansion. It is also observed that the mean and SD of the second mode damping ratio is slightly lower compared to other modes in the case C.O.V of *EI*. The SD of frequency of mode 2 is less than other two modes, so C.O.V of *EI* has very less effect on mode 2 frequency as well. From Table 4, in the case of uncertain torsional rigidity, it is also observed that mean and SD of damping ratio and frequency of various modes obtained from perturbation approach agree well with MCS, which validates perturbation approach.

Due to uncertainty in *GJ*, the SD of damping ratio increases with increasing mode number (i.e. increasing frequency), and mode 2 frequency has maximum SD, which shows that uncertainty in the



Fig. 3 The C.O.V of damping ratio and frequency of Mode 1, Mode 2 and Mode 3 vs C.O.V of GJ

torsional rigidity has more effect on mode 2 frequency. The convergence study is also carried out by considering two and four terms of K-L expansion. From the table, it is observed that, the SD of damping ratios and frequencies are converging with four terms of K-L expansion. The number of K-L expansion terms i.e. N = 4 is fixed for further studies on the basis of convergence studies presented above. The C.O.V of damping ratio and frequency of various modes for the variation in bending rigidity (*EI*) is shown in Fig. 2. From the figure, it is observed that

the C.O.V of damping ratio and frequency of all three modes obtained from both perturbation approach and MCS match well except for mode 1 where the C.O.V of damping ratio obtained from perturbation approach starts deviating from MCS results at C.O.V of EI = 7%. The C.O.V of damping ratio and frequency of various modes for the variation in torsional rigidity (*GJ*) is shown in Fig. 3. From the figure, it is observed that the C.O.V of damping ratio of first two modes (mode 1 and mode 2) and frequency of mode



Fig. 4 The probability density function of damping ratio and frequency of Mode 1, Mode 2 and Mode 3 for C.O.V of EI = 5 %

1 obtained from the perturbation approach and MCS for variation in the torsional rigidity match up to C.O.V of GJ = 7%.

For the C.O.V of EI = 0.05, the probability density functions (pdfs) of damping ratio and frequency for various modes obtained from MCS and perturbation approaches are shown in Fig. 4. From the figure, it is observed that the nature of pdf for damping ratio and frequency of various modes of geometric coupled wing obtained from MCS and perturbation approach possess Gaussian characteristics. From the figure, it is also observed that for mode 2, the pdfs of damping ratio and frequency show narrow band compare to other two modes, which indicates that the



Fig. 5 The probability density function of damping ratio and frequency of Mode 1, Mode 2 and Mode 3 for C.O.V of GJ = 5 %

uncertainty in bending rigidity has less effect on mode 2.

Next, we consider the case of C.O.V of GJ = 0.05, the pdfs of damping ratio and frequency of various modes are shown in Fig. 5. From the Fig. 5, it is observed that the pdfs of damping ratio and frequency of various modes obtained from MCS and perturbation approach are essentially Gaussian. The pdf of frequency of mode 2 shows wide band, which shows that the variation in GJ has more effect on frequency of mode 2 in comparison of other modes under consideration.

### 4. Conclusions

In this paper, a probabilistic free vibration problem has been studied using first order perturbation technique. For this purpose, a geometrically coupled cantilever wing has been considered with bending and torsional rigidities as second order Gaussian random fields. The damping has been modeled by proportional damping which depends on bending as well as torsional rigidities. By virtue of randomness in bending and torsional rigidity, damping also becomes a random parameter. The probabilistic free vibration response is obtained in the form of mean and SD, and pdf of damping ratio and frequency of various modes. The mean and SD of damping ratios and frequencies for the C.O.V of EI and GJ obtained from the perturbation approach match well with corresponding MCS results. In the case of variation in EI, the C.O.V of the damping ratio of mode 1 obtained from perturbation approach agrees well with MCS up to C.O.V of EI = 7%. Similarly, in the case of variation in GJ, damping ratio and frequency of mode 1, and damping ratio of mode 2 agree well with MCS up to C.O.V of GJ = 7%. The pdfs of damping ratio and frequency of various modes obtained from perturbation approach and MCS have Gaussian characteristics for the C.O.V of EI and GJ =0.05. In the case of C.O.V of EI, the pdf of damping ratio and frequency of mode 2 show narrow band while in the case of C.O.V of GJ, the pdf of frequency has wide band. This indicates that the effect of uncertainty in bending rigidity on frequency of mode 2 is much less than torsional rigidity.

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# **Biography**



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