

The magnetic dependence of 2-dimension quantum optical transition in electron-deformation potential phonon interaction systems in Ge

Hyenil Choi*, Hyunchul Cho**, Suho Lee***★

Abstract

In this work, we summarize the calculation processes of obtaining a scattering factor using with the equilibrium average projection scheme (EAPS), with moderately weak coupling (MWC) interaction, and obtain the line-shape formula of an electron-deformation phonon interacting system interested in the confinement of electrons by squarewell confinement potentials in quantum two dimensional system. Through the numerical analysis, we analysis the magnetic dependence of absorption power, $P(B)$ in several temperature and frequency difference dependence of absorption power $P(\Delta\omega)$, in several external field, where $\Delta\omega = \omega - \omega_0$ and $\omega(\omega_0)$ is the angular frequency (the cyclotron resonance frequency). The result of equilibrium average projection scheme (EAPS) in SER-MWC explains the properties of quantum transition quite well.

Key words : Ge, Quantum Optical Transition, Magnetic Dependence, Liouville equation, Equilibrium Average Projection Scheme

I. Introduction

Recently the approach to dynamical behavior of electrons interacting with acoustic phonons in semiconductors has received a great deal of attention among condensed matter physicists. The study of the quantum transport theories based on the Liouville equation is a useful tool for investigating the scattering mechanism of solids.

The Cyclotron Resonance absorption Line Shapes(CRLS) and the Cyclotron Resonance Line Widths(CRLW), which is the real part of scattering factor function of deformation potential semiconductors are very sensitive to the type of scattering mechanisms affecting the behavior of the carriers. There are several methods to obtain

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useful form of scattering factors of the electron-background particle correlation response function [1]-[9]. One of them is the Mori method using Kubo's inner product [1], which was applied to electron-impurity systems by Kawabata to obtain the CRLS[2]. Fujita expanded the propagator with the proper connected diagram method and applied this method to electron-phonon systems. Suzuki obtained the CRLS of electron-phonon systems with the Superoperator method [3]. Suzuki's theory well explains the result of Kobori group's experiment in Cyclotron Resonance(CR) transitions[3]. Zwanzig suggested a theory in which the time dependence of irrelevant part(non-diagonal part) is expressed in terms of relevant part(diagonal part) using the projection operator directly on the Liouville equation[1]. In similar ways, Kenkre suggested a response formula which contains non linear terms and the linear response term in the lowest order.

Using the projected Liouville equation method with the equilibrium average projection scheme (EAPS), we have suggested a new quantum transport theory of linear-nonlinear form [5]. The merit of using EAPS scheme is that the generalized susceptibility and scattering factor can be obtained in one step process of expanding the theory.

In 1974, Argyres and Sigel pointed out that the power series expansion in terms of the diagonal propagator may incur danger of divergence at the resonance peak [4]. They presented a way to evade the danger, by elaborately manipulating the line shape function. However, in order that the theory may be applied to real problems, some more manipulation may be inevitable. One way for this purpose appeared in the work of Badjou and Argyres [4].

It is well known that the CFR is another method to avoid the danger of divergence at the resonance peak, because of that the CFR is

not contained the diagonal propagator. In previous works, we expanded the propagators with the continued fractional representation (CFR) to avoid the problem of the divergence of linshape functions which arise at the resonance peak [5],[6].

In real system, we consider the problem of the divergence of linshape functions is not important dealing with cyclotron transition phenomena. We consider the divergence problem is restricted within the theoretical aspect to obtain the more rigorous mathematical representation of quantum transport theory. Because of that the CFR is very complicate formula, we need some more approximations to obtain the scattering factor functions and the numerical result in applying the CFR to real system[5]. We conclude that the CFR is limited itself to applying in real system. So, in this work, we expand the propagator with the conventional series expansion representation (SER)[17]-[19].

In this work, we have a more rigorous calculation with moderately weak coupling (MWC) interaction, while used the extremely weak coupling(EWC) interaction approximation in the most previous works[11],[13]-[16]. The difference between the MWC and EWC is the quite different type of the distribution functions part of the CRLW. In this work, we summarize the calculation processes of obtaining a scattering factor using EAPS with MWC and obtain the line-shape formula of an electron-deformation phonon interacting system.

Through the numerical analysis, we compare the response function of SER-MWC and the response function of CFR-EWC. We also compare the CRLW(which is the real part of the scattering factor function) of SER-MWC and the CRLW of CFR-EWC. We analysis the magnetic dependence of absorption power, $P(B)$ in several temperature and frequency difference dependence of absorption power $P(\Delta\omega)$, in several external field, where $\Delta\omega = \omega - \omega_0$ and $\omega(\omega_0)$ is the angular frequency (the cyclotron resonance

frequency). The result of equilibrium average projection scheme (EAPS) in SER-MWC explains the properties of quantum transition quite well.

II. The System and the Current Formula

When a static magnetic field $\vec{B}=B_z\hat{Z}$ is applied to an electron system, the single electron energy state is quantized to the Landau levels. We select a system of electrons confined in an infinite square well potential (SQWP) between $z=0$ and $z=L_z$ in the z -direction. We use the eigenvalue and eigenstate of Ref.[10] of the square well potential system. We suppose that an oscillatory electric field $E(t)=E_0e^{j\omega t}$ is applied along the z -axis, which gives the absorption power delivered to the system as $P(\omega)=(E_0^2/2)Re[\sigma(\omega)]$, where "Re" denotes the real component and $\sigma(\omega)$ is the optical conductivity tensor which is the coefficient of the current formula. Here the absorption power represents the optical QTLS, and the scattering factor function represents the optical QTLW. We consider the electron-phonon interacting system and then we have the Hamiltonian of the system as

$$H_s = H_e + H_p + V = \sum_{\beta} \langle \beta | h_0 | \beta \rangle \alpha_{\beta}^{\dagger} \alpha_{\beta} \quad (1)$$

$$+ \sum_q \eta \omega_q b_q^{\dagger} b_q + \sum_q \sum_{\alpha, \mu} C_{\alpha, \mu}(q) a_{\alpha}^{\dagger} a_{\mu} (b_q + b_{-q}^{\dagger})$$

Here H_e is the electron Hamiltonian, h_0 is a single-electron Hamiltonian, H_p is the phonon Hamiltonian and V is the electron-phonon interaction Hamiltonian. The $b_1(b_2^{\dagger})$ are the annihilation operator(creation operator) of boson particle, and \vec{q} is phonon(or impurity) wave vector. The interaction Hamiltonian of electron-phonon (or impurity)-interacting system is $V \equiv \sum_q \sum_{\alpha, \mu} C_{\alpha, \mu}(q) a_{\alpha}^{\dagger} a_{\mu} (b_q + b_{-q}^{\dagger})$ where the coupling matrix element of electron-phonon interaction

$C_{\alpha, \mu}(q)$ is $C_{\alpha, \mu}(q) \equiv V_q \langle \alpha | \exp(i\vec{q} \cdot \vec{r}) | \mu \rangle$, \vec{r} is the position vector of electron and V_q is coupling coefficient of the materials. Recently, we suggested the absorption power formula in Ref.[12] in confining potential systems. With the continuous approximation, in a right circularly polarized external field, the absorption power formula (or the QTLS formula) is obtained finally as

$$P(\omega) \propto \left(\frac{e^2 \omega_c^2}{\pi^2 \eta \omega} \right) \left\{ \frac{\gamma_{total}(\omega_c) \sum_{N_{\alpha}} \int_{-\infty}^{\infty} dk_{z\alpha} (N_{\alpha} + 1) (f_{\alpha} - f_{\alpha+1})}{(\omega - \omega_c)^2 + (\gamma_{total}(\omega_c))^2} \right\} \quad (2)$$

where the scattering factor function(or QTLW) is given by

$$\gamma_{total}(\omega) \equiv Re \Xi_M(\omega) \equiv \sum_{\mu} \sum_{N_{\alpha}=0} \sum_{N_{\beta}=0} \gamma_{\alpha, \beta}^{\mu}$$

$$= \left(\frac{\Omega}{4\pi \eta^2 v_s} \right) \left(\frac{\pi}{L_z} (2 + \delta(n_{\alpha}, n_{\beta})) \right) \left\{ \frac{\sum_{\mu} \sum_{N_{\alpha}=0} \sum_{N_{\beta}=0} \int_{-\infty}^{\infty} dk_{z\alpha} \int_{-\infty}^{\infty} dq_z Y_{\alpha, \beta}^{\mu}}{\sum_{N_{\alpha}=0} \int_{-\infty}^{\infty} dk_{z\alpha} (N_{\alpha} + 1) (f_{\alpha+1} - f_{\alpha})} \right\} \quad (3)$$

Recently, we suggested the final derivation of the integrand $Y_{\alpha, \beta}^{\mu}$ of the scattering factor in Ref.[12]. We use the result equations from Eq.(18) to Eq.(23) in Ref.[12].

III. The Absorption Powers and Line Width in Ge

In this section, through the numerical calculation of Eq.(2)-Eq.(3), we analyze absorption power and linewidths of Ge(germanium). It is well known that the deformation-potential scattering is dominant for pure Ge. The band structures of Ge can be approximated to be ellipsoidal. We use $\bar{m}=0.22m_0$ and $m^*=0.35m_0$ which are the effective masses of Ge. Here the m_0 is the free-electron mass. The other constants of Ge are $\rho=5.36g/cm^3$, $v_s=5.94 \times 10^5 cm/s$, $\varepsilon_g(0)=0.744eV$, $k=4.77 \times 10^{-4} eV/K$ and $\xi=235K$. We use the well known value of the deformation potential coupling parameter. We use $E_1=13.2eV$ for Ge.

The shape of $P(B)$ resembles the experimental

shape of $P(B)$ performed by Kobori et al, in an arbitrary unit. In fig. 1, shows the magnetic field dependence of absorption power $P(B)$ and $P(\Delta\omega)$ of Ge with $\lambda = 84\mu m \sim \lambda = 550\mu m$, at $T=20K$. Here, the relativity difference frequency $\Delta\omega$ is $\Delta\omega = \omega - \omega_0$. From the graph of $P(\Delta\omega)$, we can see the broadening effects near the

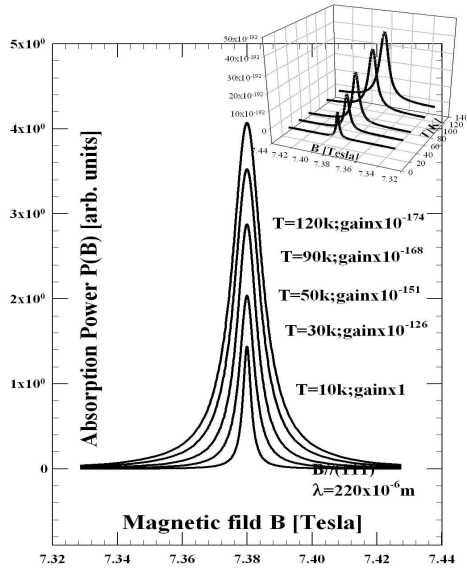


Fig.1. The Magnetic field dependence of absorption power, $P(B)$ of Ge with $\lambda = 220\mu m$ at $T=10, 30, 50, 90, 120K$.

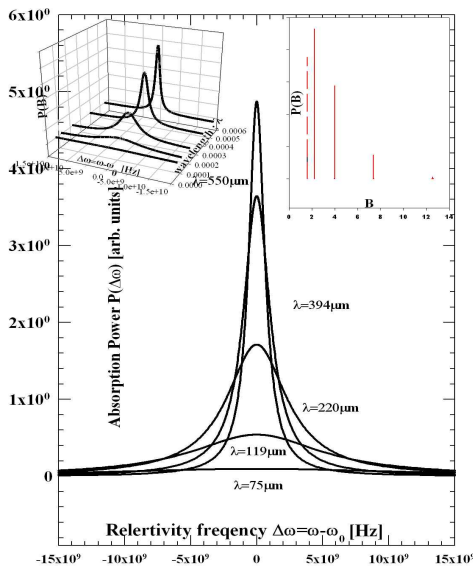


Fig.2. The relativity frequency dependence of absorption power $P(B)$ of Ge with $\lambda = 75, 119, 220, 394, 550\mu m$ at $T=30K$.

resonance peaks for various external fields. The merit of this research is easier to analysis

the power absorptions and half line widths than the experimental analysis and other theories, in the several cases.

The fig.3 shows the temperature dependence of the half width $\gamma(T)$ of Ge with $\lambda = 75, 119, 172, 220, 295, 394, 513, 550, 720\mu m$.

They show that the temperature dependence of the half width $\gamma(T)$ of Ge increases as temperature increase.

This result implies that the scattering effect of the phonon increase as the temperature increase.

The fig.4 shows the magnetic dependence of the half width $\gamma(B)$ of Ge at $T=10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140K$. They show that the magnetic dependence of the half width $\gamma(B)$ of Ge increases as magnetic field increase. They also show that the magnetic dependence of the half width $\gamma(B)$ of Ge increases as the temperature increase. This result implies that the scattering effect of the phonon increase as the magnetic field and the temperature increase.

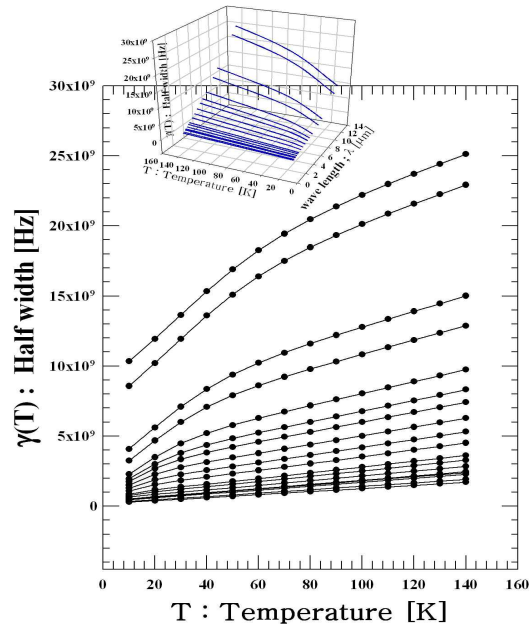


Fig.3. Temperature dependence of half width $\gamma(T)$ of Ge with $\lambda = 75, 119, 172, 220, 295, 394, 513, 550, 720\mu m$.

We compare the temperature dependence of the total half width $\gamma(T)_{total} \equiv \gamma(T)_{inter} + \gamma(T)_{intra}$

of Ge, $\gamma(T)_{inter}$ of inter Landau level transition process and $\gamma(T)_{intra}$ of inter Landau level transition process in fig.5. We also compare the absorption power $P(B)$ of Ge with the total

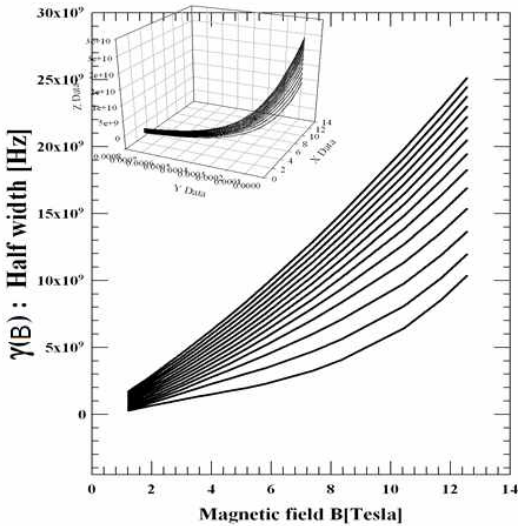


Fig.4. Magnetic dependence of half width $\gamma(B)$ of Ge with $T=10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140K$ with $\lambda=119\mu m$.

half width $\gamma(T)_{total}$ in fig.6, $P(B)$ of Ge only with $\gamma(T)_{inter}$ and $P(B)$ of Ge only with $\gamma(T)_{intra}$. We see a good agreement between $\gamma(T)$ of fig.5 and the broadening of the power absorptions $P(B)$ of fig.6.

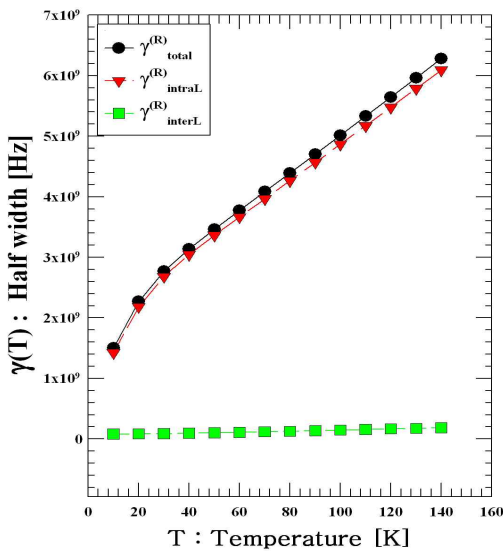


Fig.5. Comparisons of temperature dependence of half width of Ge in the cases of total half width $\gamma(T)_{total}$, half width of total inter Landau level transition process $\gamma(T)_{inter}$ and half width of total intra Landau level $\gamma(T)_{intra}$ transition process.

We compare the temperature dependence of the total half width $\gamma(T)_{total}$ of Ge, $\gamma(T)_{total} \equiv \gamma(T)_{total}^{em} + \gamma(T)_{total}^{ab}$ of total inter Landau level transition process, $\gamma(T)_{total}^{em}$ of phonon emission inter Landau level transition process and $\gamma(T)_{total}^{ab}$ of phonon absorption inter Landau level transition process in fig.7.

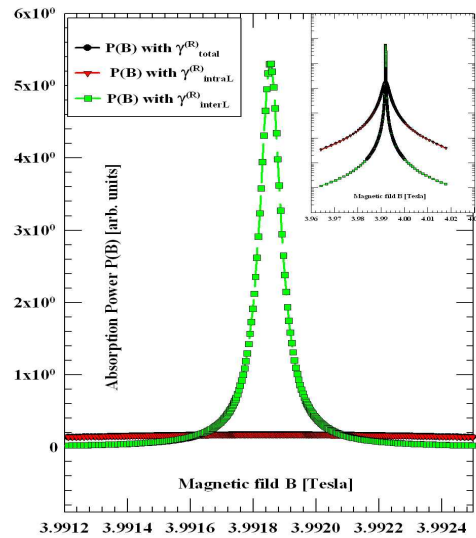


Fig.6. The magnetic field dependence of absorption power, $P(B)$ of Ge in the cases $P(B)$ with γ_{total} , $P(B)$ with γ_{inter} , $P(B)$ with γ_{intra} , at $T=20K$ with $\lambda=394\mu m$.

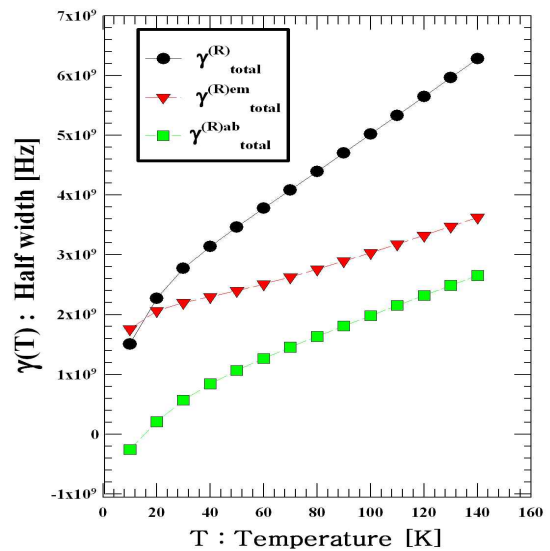


Fig.7. Comparisons of temperature dependence of half width of Ge in the cases of total half width γ_{total} , half width of total phonon emission transition process $\gamma(T)_{total}^{em}$ and half width of total phonon absorption transition process $\gamma(T)_{total}^{ab}$.

We also compare the absorption power $P(B)$ of Ge with the total half width $\gamma(T)_{total}$ in fig.8, $P(B)$ of Ge only with $\gamma(T)_{total}$, $P(B)$ of Ge only with $\gamma(T)^{em}_{total}$ and $P(B)$ of Ge only with $\gamma(T)^{ab}_{total}$. We see a good agreement between $\gamma(T)$ of fig.7 and the broadening of the power absorptions $P(B)$ of fig.8.

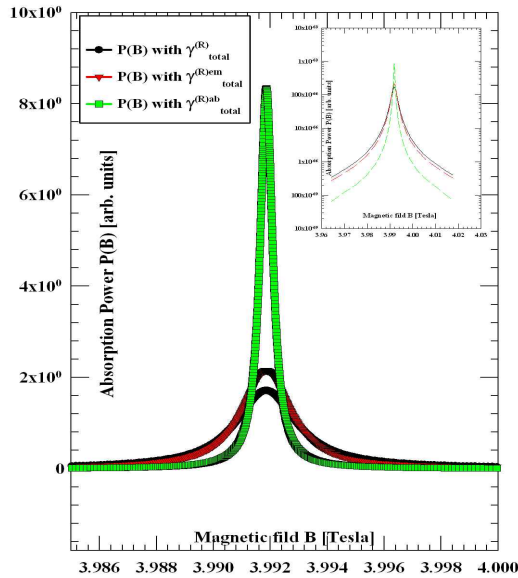


Fig.8. The Magnetic field dependence of absorption power, $P(B)$ of Ge in the cases $P(B)$ with γ_{total} , $P(B)$ with γ^{em}_{total} , $P(B)$ with γ^{ab}_{total} , at $T=20K$ with $\lambda=394\mu m$.

The fig.10 shows the magnetic dependence of the half width $\gamma(B)$ of Ge at $T=30K$. They show that the magnetic dependence of the half width $\gamma(B)$ of Ge increases as magnetic field increase. They also show that the magnetic dependence of the half width $\gamma(B)$ of Ge increases as the temperature increase. This result implies that the scattering effect of the phonon increase as the magnetic field and the temperature increase.

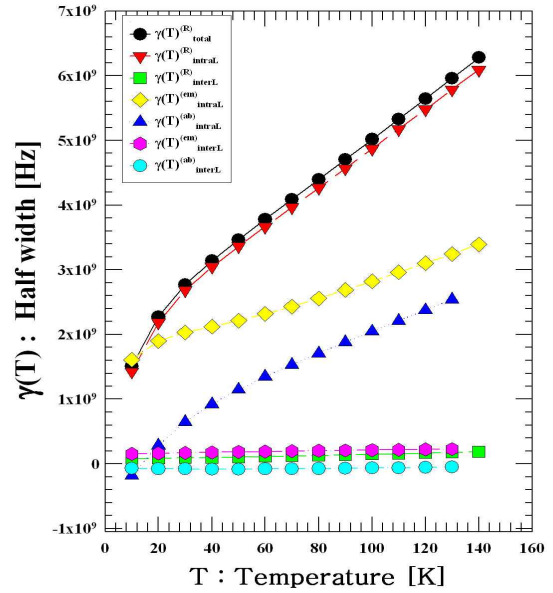


Fig.9. Comparisons of temperature dependence of half width of Ge

We compare the magnetic dependence of the total half width $\gamma(T)_{total}$ of Ge, $\gamma(T)_{inter} \equiv \gamma(T)^{em}_{inter} + \gamma(T)^{ab}_{inter}$ of total inter Landau level transition process, $\gamma(T)^{em}_{inter}$ of phonon emission inter Landau level transition process and $\gamma(T)^{ab}_{inter}$ of phonon absorption inter Landau level transition process in fig.11.

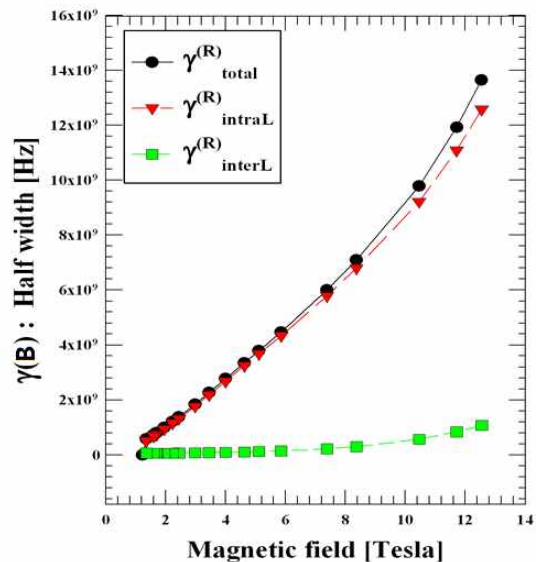


Fig.10. Comparisons of the magnetic field dependence of half width of Ge in the cases of total half width γ_{total} , half width of total inter Landau level transition process γ_{inter} and half width of total intra Landau level γ_{intra} transition process (in small box).

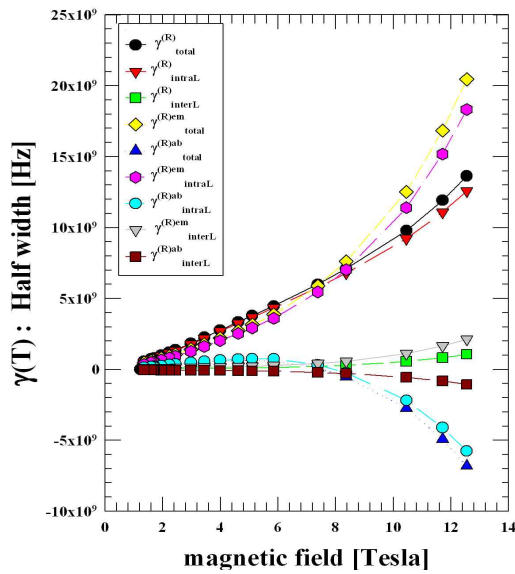


Fig.11. Comparisons of magnetic field dependence of half width of Ge

IV. Conclusion

In this work, we studied scattering factor functions of deformation potential semiconductors in moderately weak coupling(MWC) interaction scheme. We used the equilibrium average projection scheme(EAPS) based on the projected Liouville equation method[5]. The merit of using EAPS scheme is that the generalized susceptibility and scattering factor can be obtained in one step process of expanding the theory. We expanded the propagator with the conventional series expansion representation(SER).

We obtained the power absorptions, $P(B)$ of Ge with $\lambda=119\mu\text{m}$ at $T=10,20,30,40,50,60,70,80,90,100,110,120,130,140K$, in fig.4. The shape of $P(B)$ resembles the experimental shape of $P(B)$ performed by Kobori et al, in an arbitrary unit. We obtained the magnetic field dependence of absorption power $P(B)$ and $P(\Delta\omega)$ of various external fields, in fig. 6. From the graph of $P(\Delta\omega)$, we can see the broadening effects near the resonance peaks for various external fields. They showed increases as temperature increase. This result implied that the scattering effect of the phonon increase as the temperature increase.

In this paper, we investigate the temperature (the magnetic) dependence of the broadening effects between the Landau levels. We see the dominant broadening effect of Ge is the inter Landau level with the emission of phonon energy ($\gamma_{\text{inter}}^{\text{em}}$) in these analysis. We also see the power absorption lines are quit well consistence the properties of the broadening effects. Through the analysis of the broadening effects between the Landau levels, we conclude the dominant broadening effects arise the inter-Landau level transition between the $\alpha=1$ state and the $\beta=0$ state with the emission of phonon energy ($\gamma_{\text{inter}}^{\text{em}}$).

Through these analysis, we consider that SER-MWC of the EAPS theory is quite useful to understand the scattering mechanism in many body transition system. The merits of this research is the easier scheme to analysis the power absorptions and broadening effects in the various cases than other theories and the experimental analysis. We expect it can be applied to analyses of other condensed material systems.

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