J. Chosun Natural Sci.
Vol. 11, No. 2 (2018) pp. 76 – 81
https://doi.org/10.13160/ricns.2018.11.2.76

Robust Design Using Desirability Function in Product-Array

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Abstract

Robust design is an approach to reducing performance variation of quality characteristic values in quality engineering. Product array approach which is used in the Taguchi parameter design has a number of advantages by considering the noise factor. Taguchi has an idea that mean and variation are handled simultaneously to reduce the expected loss in products and processes. Taguchi has used the signal-to-noise ratio (SN) to achieve the appropriate set of operating conditions where variability around target is low in the Taguchi parameter design. Many Statisticians criticize the Taguchi techniques of analysis, particularly those based on the SN. In this paper we propose a substantially simpler optimization procedure for robust design using desirability function without resorting to SN.

Keywords: Robust Design, Product Array, Optimization Procedure, Desirability Function

1. Introduction

Quality improvement activities at the design stage are absolutely necessary to dramatically improve quality. Taguchi Quality Engineering has contributed greatly to the statistical field widely used to improve the quality throughout product design stage[1].

In the Taguchi parameter design, the product array using the orthogonal array table was subjected to data analysis using the SN by performing experimental setup considering all the interaction effects of the control factor and the noise factor. In the product array, the noise factor plays a role in reducing quality variation of quality characteristics, which makes it possible to design a parameter which can find the optimum condition of control factor approaching the target value while the average of quality characteristics is insensitive to variation. Products and their manufacturing processes are influenced both by control factors that can be controlled by designers and by noise factors that are difficult or expensive to control such as environmental conditions. The basic idea of parameter design is to identify, through exploiting interactions between control factors and noise factors, appropriate settings of control factors that make the system's performance robust to changes in the noise factors. Parameter design is a quality improvement technique proposed by the Japanese quality expert Taguchi[2], which was described by Kackar[3] and others.

Although Taguchi quality engineering has made a great contribution to improving quality, many problems have been pointed out in the use of SN in analyzing data, and alternatives have been studied by various scholars. In this regard, the analysts such as Box[4] proposed an analytical method through data transformation. Vining and Myers[5] used regression analysis from repeated measurement data instead of using SN, For the first time as an alternative method to obtain the optimum condition of quality characteristics. They analyzed the experimental data using the optimization technique for the dual response function of Myers and Carter[6]. Since then, scholars such as Copeland and Nelson[7] have proposed different optimization methods for three different characteristics.

The purpose of this paper is to use the alternative method to find the optimum condition of the quality characteristic by separating the estimated mean model and the variance model without using SN, An example is illustrated to show the proposed method.

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(Received : March 20, 2018, Revised : May 5, 2018
Accepted : May 10, 2018)

2. Estimated Mean and Variance Models in a Product Array

In analyzing the data in the parameter design, problems have arisen due to the use of SN, a performance measure that combines mean and variation(or variance). Suppose that the quality characteristics ($\underline{y}$) are determined by the control factors ($\underline{x}$) and the noise factors ($\underline{z}$). The product array is possible to obtain repeated quality characteristics ($\underline{y}$) by the noise factor under the different experimental condition of the control factors.

The sample mean ($\bar{y}$) and sample variance(or standard deviation (v)) can be obtained from these data. Therefore, instead of using the SN, we can consider an alternative method to obtain the optimal condition of the quality characteristic by separating the estimated mean model and the standard deviation model from the repeated measurement data. From the repeated measurement data, the estimated mean model $\hat{m}(\underline{x})$ by the least squares method for the sample mean of the quality characteristics. the estimated standard deviation model $\hat{v}(\underline{x})$ by the least squares method for the sample standard deviation of the quality characteristics.

3. Desirability Function of the Mean and Variance Models

3.1. Desirability Function of the Mean Models

Derringer and Suich[8] presented three types of desirability functions and proposed a method to optimize them simultaneously in case of multiple quality characteristics. Taguchi[2] classifies all the quality characteristics into three categories, namely, the larger-is-better characteristics, the smaller-is-better characteristics and the nominal-is-best characteristics. We can apply three kinds of quality characteristics to three kinds of desirability functions in parameter design. Therefore, we propose three kinds of desirability functions for the mean model as follows. We propose a new optimization method by converting the mean model of quality into the larger-is-better characteristics, the smaller-is-better characteristics and the nominal-is-best characteristics. The desirability function involves transformation of each estimated mean response $\hat{m}(\underline{x})$ to a desirability value $d_m(\underline{x})$, where $(0 \le d_m(\underline{x}) \le 1)$.

For the larger-is-better case, $d_m(\underline{x})$ increases as $\hat{m}(\underline{x})$ increases and is employed where $\hat{m}(\underline{x})$ is to be maximized. We shall consider the transformations given by

$$d_m(\underline{x}) = \begin{cases} 0 & \hat{m}(\underline{x}) \le m_* \\ \left[\dfrac{\hat{m}(\underline{x}) - m_*}{m^* - m_*}\right]^p & m_* < \hat{m}(\underline{x}) \le m^* \\ 1 & m^* \le \hat{m}(\underline{x}) \end{cases} \tag{1}$$

where m_* is the minimum acceptable value of $\hat{m}(\underline{x})$, and the value m^* is the maximum (or target) value of $\hat{m}(\underline{x})$ over the region of interest R_x, where $(-1 \le R_x \le 1)$. A large value of p would be specified if it were very desirable for the value of $\hat{m}(\underline{x})$ to increase rapidly above m_*. On the other hand, a small value of p would be specified if having values of $\hat{m}(\underline{x})$ considerably above m_* were not of critical importance. Fig. 1 shows the plot of the desirability function for the larger-is-better quality characteristics, Equation (1).

For the smaller-is-better case, $d_m(\underline{x})$ increases as $\hat{m}(\underline{x})$ decreases and is employed where $\hat{m}(\underline{x})$ is to be minimized. We shall consider the transformations given by

$$d_m(\underline{x}) = \begin{cases} 0 & m^* \le \hat{m}(\underline{x}) \\ \left[\dfrac{m^* - \hat{m}(\underline{x})}{m^* - m_*}\right]^q & m_* < \hat{m}(\underline{x}) \le m^* \\ 1 & \hat{m}(\underline{x}) \le m_* \end{cases} \tag{2}$$

where m_* is the minimum (or target) value of $\hat{m}(\underline{x})$ over the region of interest R_x, and m^* is the maximum acceptable value of $\hat{m}(\underline{x})$. A large value of q would be specified if it were very desirable for the value of $\hat{m}(\underline{x})$ to

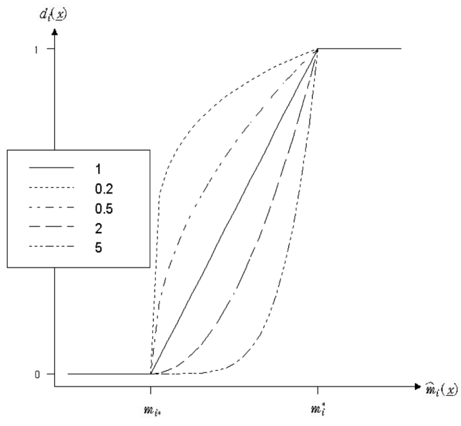

Fig. 1. Larger-the-better case.

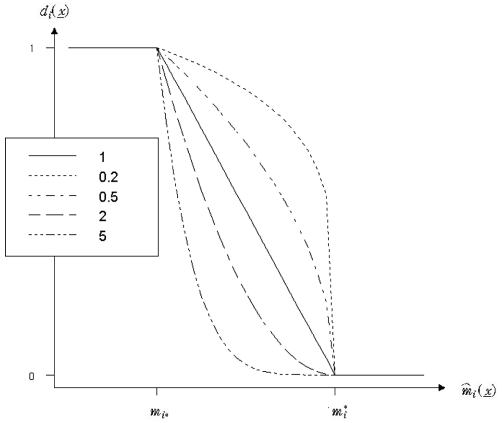

Fig. 2. Smaller-the-better case.

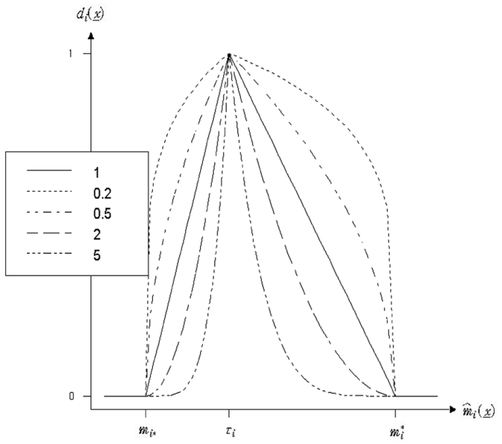

Fig. 3. Nominal-is-best case.

decrease rapidly above m_*. On the other hand, a small value of q would be specified if having values of $\hat{m}(\underline{x})$ considerably above m_* were not of critical importance. Fig. 2 shows the plot of the desirability function for the smaller-is-better quality characteristics, Equation (2).

For the nominal-is-best case, we shall consider the transformations given by

$$d_m(\underline{x}) = \begin{cases} \left[\dfrac{\hat{m}(\underline{x}) - m_*}{\tau - m_*}\right]^s & m_* \le \hat{m}(\underline{x}) \le \tau \\ \left[\dfrac{m^* - \hat{m}(\underline{x})}{m^* - \tau}\right]^t & \tau < \hat{m}(\underline{x}) \le m^* \\ 0 & \hat{m}(\underline{x}) \le m_* \text{ or } \hat{m}(\underline{x}) \ge m^* \end{cases} \quad (3)$$

where m_* is the minimum acceptable value of $\hat{m}(\underline{x})$ and

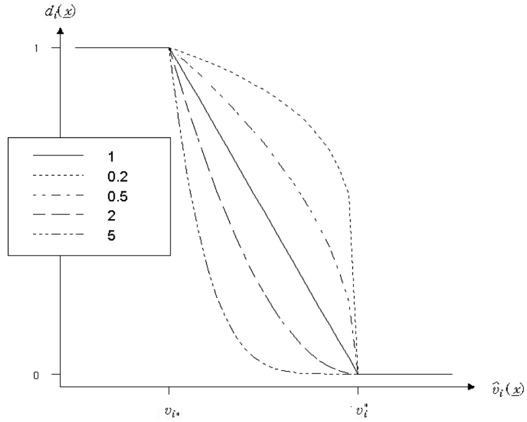

Fig. 4. Standard deviation case.

m^* is the maximum acceptable value of $\hat{m}(\underline{x})$. Values of $\hat{m}(\underline{x})$ outside these limits would make the entire product unacceptable. The value τ_i is the most-desirable (or target) value of $\hat{m}(\underline{x})$. The values of s and t play the same role as p and q in Equations (1) and (2). Fig. 3 shows the plot of the desirability function for the smaller-is-better quality characteristics, Equation (3).

3.2. Desirability Function of the Standard Deviation Model

Because the estimated standard deviation model, $\hat{v}(\underline{x})$ is the smaller-is-better case, $d_v(\underline{x})$ increases as $\hat{v}(\underline{x})$ decreases and is employed where $\hat{v}(\underline{x})$ is to be minimized. The desirability function involves transformation of each estimated mean response $\hat{v}(\underline{x})$ to a desirability value $d_v(\underline{x})$, where $(0 \le d_v(\underline{x}) \le 1)$. We shall consider the transformations given by

$$d_v(\underline{x}) = \begin{cases} 0 & v^* \le \hat{v}(\underline{x}) \\ \left[\dfrac{v^* - \hat{v}(\underline{x})}{v^* - v_*}\right]^w & v_* < \hat{v}(\underline{x}) \le v^* \\ 1 & \hat{v}(\underline{x}) \le v^* \end{cases} \quad (4)$$

where v_* is the minimum (or target) value of $\hat{v}(\underline{x})$ over the region of interest R_x, and v^* is the maximum acceptable value of $\hat{v}(\underline{x})$. On the other hand, a small value of w would be specified if having values of $\hat{v}(\underline{x})$ considerably above v_* were not of critical importance. Fig. 4 shows the plot of the desirability function for a set of estimated variance responses, Equation (4).

4. The Optimization Method for Robust Design in the Product Array

Let us find conditions on a set of control variables $\underline{x}$ which optimize simultaneously for the estimated mean model and the estimated standard deviation model. If all the estimated mean model and the estimated standard deviation model attain their individual optimum values at the same set $\underline{x}$ of operating conditions, then the problem of simultaneous optimization is obviously solved. This ideal optimum rarely occurs. In more general situations we might consider finding compromising conditions on the control variables that are somewhat favorable to the estimated mean model and the estimated standard deviation model. Such deviation of the compromising conditions from the ideal optimum condition can be formulated by means of the desirability function.

We propose a simultaneous optimization for the estimated mean model and the estimated standard deviation model over the region of interest R_x. From Equations (1) and (4), the proposed optimization measure can be written as

$$P_d = \max_{\underline{x} \in R_x} [\lambda d_m(\underline{x}) + (1-\lambda) d_v(\underline{x})] \quad (5)$$

where R_x is the region of interest on a set of control variables $\underline{x}$ and $0 \le \lambda \le 1$. λ is a criterion in which $d_m(\underline{x})$ and $d_v(\underline{x})$ take on different degrees of importance. The larger λ, the greater the importance of $d_m(\underline{x})$. The smaller λ, the greater the importance of $d_v(\underline{x})$.

If a simultaneous optimum value is much different from its corresponding individual optimum value, we may choose a bound on it and then reoptimize P_d. Also we may analyze P_d sequentially as the acceptable values for λ is varied.

We used the genetic algorithm in the Optimization toolbox of MATLAB to find the optimal solution of the control factors by the proposed optimization formula. The optimization process is described in the next chapter.

5. Numerical Example

Box and Draper[9] conducted an experiment on the printing process. Table 1 shows that the factorial experiment was repeated three times at different locations.

Table 1. Printing process data

Run	x_1	x_2	x_3	y_{z_1}	y_{z_2}	y_{z_3}	$\overline{y}$	v^2
1	-1	-1	-1	34	10	28	24.0	12.49
2	0	-1	-1	115	116	130	120.3	8.39
3	1	-1	-1	192	186	263	213.7	42.80
4	-1	0	-1	82	88	88	86.0	3.46
5	0	0	-1	44	178	188	136.7	80.41
6	1	0	-1	322	350	350	340.7	16.17
7	-1	1	-1	141	110	86	112.3	25.57
8	0	1	-1	259	251	259	256.3	4.62
9	1	1	-1	290	280	245	271.7	23.63
10	-1	-1	0	81	81	81	81.0	0.00
11	0	-1	0	90	122	93	101.7	17.67
12	1	-1	0	319	376	376	357.0	32.91
13	-1	0	0	180	180	154	171.3	15.01
14	0	0	0	372	372	372	372.0	0.00
15	1	0	0	541	568	396	501.7	92.50
16	-1	1	0	288	192	312	264.0	63.50
17	0	1	0	432	336	513	427.0	88.61
18	1	1	0	713	725	754	730.7	21.08
19	-1	-1	1	364	99	199	220.7	133.80
20	0	-1	1	232	221	266	239.7	23.46
21	1	-1	1	408	415	443	422.0	18.52
22	-1	0	1	182	233	182	199.0	29.45
23	0	0	1	507	515	434	485.3	44.64
24	1	0	1	846	535	640	673.7	158.20
25	-1	1	1	236	126	168	176.7	55.51
26	0	1	1	660	440	403	501.0	138.90
27	1	1	1	878	991	1161	1010.0	142.50

This can be seen as a product array data from repeated measurements with three levels of noise factor, z_1, z_2 and z_3.

The mean model estimated by the least squares method for the sample mean ($\overline{y}$) of the quality characteristics from the experimental data on the printing process is as follows.

$$\hat{m}(\underline{x}) = 327.6 + 117.0x_1 + 109.4x_2 + 131.5x_3 + 32.0x_1^2 - 22.4x_2^2 - 29.1x_3^2 + 66.0x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3 \quad (6)$$

The estimated standard deviation model for the sample standard deviation (v) of the quality characteristics is as follows.

Table 2. Results of optimization

Quality Characteristics	λ	x_1	x_2	x_3	m	v	d_m	d_v	P_d
Larger-the-better	0.2	-0.19	-1.00	-0.31	161.61	14.96	0.11	0.98	0.81
	0.4	-1.00	0.98	-0.69	181.63	15.63	0.14	0.97	0.64
	0.6	1.00	1.00	0.25	691.44	85.69	0.79	0.41	0.64
	0.8	1.00	1.00	1.00	851.10	137.50	1.00	0.00	0.80
Smaller-the-better	0.2	-1.00	1.00	-1.00	134.91	12.50	0.92	1.00	0.98
	0.4	0.26	-1.00	-1.00	74.63	19.94	1.00	0.94	0.96
	0.6	0.30	-1.00	-1.00	74.34	19.99	1.00	0.94	0.98
	0.8	0.34	-1.00	-1.00	74.17	20.05	1.00	0.94	0.99
Nominal-is-best	0.2	-0.10	-1.00	-0.25	169.25	15.37	0.20	0.98	0.82
	0.4	1.00	0.94	-0.28	550.00	59.08	1.00	0.63	0.78
	0.6	0.98	0.72	-0.15	550.00	59.62	1.00	0.62	0.85
	0.8	0.85	0.70	-0.03	550.00	61.34	1.00	0.61	0.92

$$\hat{v}(\underline{x}) = 34.9+11.5x_1+15.3x_2+29.2x_3+4.2x_1^2 -1.3x_2^2+16.8x_3^2+7.7x_1x_2+5.1x_1x_3+14.1x_2x_3 \quad (7)$$

On the other hand, the interest region of the three control factors is $-1 \le x_1, x_2, x_3 \le 1$.

We will apply equation (5), which is an optimization formula using the above-mentioned desirability function. In addition, weights q, p, s, t and w can be used in various ways as needed in the equations (1), (2), (3) and (4). Let us assume that all expected functions are linear functions and that 1 is used.

In Table 2, if the quality characteristics are Larger-the-better, estimated mean model equation (6) uses the desirability function equation (1) as the Larger-the-better characteristics. If the quality characteristics are Smaller-the-better, estimated mean model equation (6) uses the desirability function equation (2) as the Smaller-the-better characteristics. If the quality characteristics are Nominal-is-best, estimated mean model equation (6) uses the desirability function equation (3) as the Nominal-is-best characteristics. The estimated standard deviation model (7) uses the desirability function equation (4). In the estimated mean model equation (6), the minimum value m_* is 74.11 and the maximum value m^* is 851.10. In the estimated standard deviation model (7), the minimum value v_* is 12.50 and the maximum value v^* is 137.50. For Nominal-is-best characteristics, the target value τ is 550.00.

In Table 2, if the quality characteristic is a Larger-the-better characteristic, when the weighting factor λ is 0.2, the optimal point of the control factor is x_1=-0.19, x_2=-1.00 and x_3=-0.31 as a result of applying the optimization equation (5). At this time, the value of the estimated mean model equation (6), m is 161.61 and the estimated standard deviation model equation (7), v is 14.96, the optimization equation (5), P_d is 0.81, the desirability function of mean model equation (1), d_m is 0.11 and the desirability function of standard deviation model equation (4), d_v is 0.98. Then, as the weight λ increases, the importance of the quality mean increases, so that the value of the mean model equation (6), m is increases. On the other hand, as the importance of quality variation decreases, the value of standard deviation equation (7), v tends to increase. If we look at the case where the quality characteristic is the Smaller-the-better characteristic, as the weight λ increases, the importance of the quality mean increases, so that the value of the mean model equation (6), m is decreases. On the other hand, as the importance of quality variation decreases, the value of standard deviation equation (7), v tends to increase. If we look at the case where the quality characteristic is the Nominal-is-best characteristic, as the weight λ increases, the importance of the quality mean increases, so that the value of the mean model equation (6), m approaches the target value τ= 550.00. On the other hand, as the importance of quality variation decreases, the value of standard deviation equation (7), v tends to increase.

6. Conclusion

The proposed product array approach allows one to provide separate estimates for the mean response and for the variance response. Accordingly, we can apply the primary goal of the Taguchi methodology which is to obtain a target condition on the mean while achieving the variance, or to minimize the variance while constraining the mean. In this study we proposed the optimization measure P_d. The proposed concept of P_d measure is minimizing the deviation of the mean response from the target value constraining the mean and variance responses. The P_d measure is easy to apply, and permits the user to make subjective judgements on the importance (or desirability).

In the case of Taguchi's parameter design, if the optimum level is obtained by using the SN ratio, the proposed optimization method has a great advantage that the optimum condition can be obtained within the range of interest by using the regression model. Of course, the problem in this paper depends on the premise that regression fit by regression analysis should be done well by relying on regression model.

Acknowledgments

This study was supported(in part) by research funds from Chosun University, 2016.

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