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η-RICCI SOLITONS ON KENMOTSU MANIFOLDS

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Abstract. The object of the present paper is to study the Kenmotsu manifolds which metric tensor is η-Ricci soliton. We bring out curvature conditions for which Ricci solitons in Kenmotsu manifolds are sometimes shrinking or expanding and some other times steady.

1. Introduction

Recently Baishya and Roy Chowdhury [1] have introduced and studied *generalized quasi-conformal curvature tensor in the context of* $N(k,\mu)$-manifold. The *generalized quasi-conformal curvature tensor for* $(2n+1)$-*dimensional manifold* (we suppose that $n>1$) is defined as

$$\begin{aligned} &W(X,Y)Z \\ =&\frac{2n-1}{2n+1}\left[(1+2na-b)-\{1+2n(a+b)\}c\right]C(X,Y)Z \\ &+[1-b+2na]\,E(X,Y)Z+2n(b-a)P(X,Y)Z \\ &+\frac{2n-1}{2n+1}(c-1)\{1+2\,n(a+b)\}\hat{C}(X,Y)Z \end{aligned} \tag{1.1}$$

for all X, Y, $Z\in\chi(M)$, the set of all smooth vector fields of the manifold M, where a,b,c are real constants. The beauty of *generalized quasi-conformal curvature tensor* lies in the fact that it has the flavour of Riemann curvature tensor R for $a=b=c=0$; conformal curvature tensor C [18] for $a=b=-\frac{1}{2n-1}$, $c=1$; conharmonic curvature tensor $\hat{C}$ [21] for $a=b=-\frac{1}{2n-1}$, $c=0$; concircular curvature tensor E for $a=b=0$, $c=1$; projective curvature tensor P for $a=-\frac{1}{2n}$, $b=0$,

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$c = 0$ and m-projective curvature tensor H [27], for $a = b = -\frac{1}{4n}$, $c = 0$ see also[3]. The equation (1.1) can be also written as

$$\begin{aligned} W(X,Y)Z =& R(X,Y)Z + a[S(Y,Z)X - S(X,Z)Y] \\ &+ b[g(Y,Z)QX - g(X,Z)QY] \\ &- \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)[g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{1.2}$$

The study of the Ricci solitons in contact geometry has begun with the work of Sharma [31]. During the last two decades, the geometry of Ricci solitons has been the focus of attention of many mathematicians, see [6], [13], [19], [33], [35], [15], [17] and also references cited therein. Ricci solitons are introduced as triples (M, g, V), where (M, g) is a Riemannian manifold and V is a vector field so that the following equation is satisfied:

$$\frac{1}{2}\pounds_V g + S + \lambda g = 0 \tag{1.3}$$

where $\pounds$ denotes the Lie derivative, S is the Ricci tensor and $\lambda \in \mathbb{R}$. A Ricci soliton is said to be shrinking, steady or expanding according as λ negative, zero and positive respectively.

In [17] Cho and Kimura studied the Ricci solitons of real hypersurfaces in a non-flat complex space form and they defined η-Ricci soliton, which satisfies the equation

$$\pounds_\xi g + 2S + 2\lambda g + 2\mu\eta \otimes \eta = 0. \tag{1.4}$$

where $\pounds_\xi$ is the Lie derivative operator along the vector field ξ, S is the Ricci curvature tensor field of the metric g, λ and μ are real constants. Thereafter η-Ricci soliton have been studied in ([7], [8], [9], [10], [11], [12], [20], [2]).

Our work is structured as follows. Section 2 is a very brief review of Kenmotsu manifolds and η-Ricci solitons. In section 3, we investigate η-Ricci solitons in a Kenmotsu manifold admitting $\omega(\xi, X) \cdot W = 0$ where ω and W both stand for *generalized quasi-conformal curvature tensor* with the associated scalar triples $(\bar{a}, \bar{b}, \bar{c})$ and (a, b, c) respectively (two distinct notions have been used in order to study the nature of different semi-symmetric type curvature condition as shown in the table by taking the permutation and combination of the scalar triples into account), the dot means that $\omega(X, Y)$ acts as a derivation on W. It is observed that η-Ricci soliton (g, ξ, λ, μ) in a Kenmotsu manifold $M^{2n+1}(\phi, \xi, \eta, g)$ satisfying each of $R(\xi, X) \cdot W = 0$, $E(\xi, X) \cdot W = 0$ and $H(\xi, X) \cdot W = 0$ is always expanding provided $2n > \mu$. Further, we pointed out that η-Ricci soliton (g, ξ, λ, μ) in a Kenmotsu manifold $M^{2n+1}(\phi, \xi, \eta, g)$ satisfying each of $C(\xi, X) \cdot W = 0$ and $\hat{C}(\xi, X) \cdot W = 0$ is sometime expanding and some other time remains shrinking. In section 4, we study η-Ricci solitons in Kenmotsu manifold satisfying $W(\xi, X) \cdot S = 0$ and determine that Ricci solitons in Kenmotsu manifold satisfying $W(\xi, X) \cdot S = 0$ is either steady or expanding.

2. Preliminaries

Let M be a $(2n+1)$-dimensional connected differentiable manifold of class C^∞-covered by a system of coordinate neighborhoods (U, x) in which there are given a tensor field ϕ of type $(1,1)$, a contravariant vector field ξ and a 1-form η such that

$$\phi^2 X = -X + \eta(X)\xi, \tag{2.1}$$

$$\eta(\xi) = 1, \qquad \phi \cdot \xi = 0, \quad \eta(\phi X) = 0, \tag{2.2}$$

for any vector field X on M. Then the structure (ϕ, ξ, η) is called contact structure and the manifold M^{2n+1} equipped with such structure is said to be an almost contact manifold. If there is given a Riemannian compatible metric g such that

$$g(\phi X, Y) = -g(X, \phi Y), \qquad g(X, \xi) = \eta(X), \tag{2.3}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.4}$$

for all vector fields X and Y, then we say M is an almost contact metric manifold. An almost contact metric manifold M is called a *Kenmotsu manifold* if it satisfies [25],

$$(\nabla_X \phi)Y = -g(X, \phi Y)\xi \;-\; \eta(Y)\phi(X), \tag{2.5}$$

for all vector fields X and Y, where ∇ is the Levi-Civita connection of the Riemannian metric. From the above, it follows that

$$\nabla_X \xi = \; X - \eta(X)\xi, \tag{2.6}$$

$$(\nabla_X \; \eta)Y = g(X, Y) - \eta(X)\; \eta(Y), \tag{2.7}$$

In the Kenmotsu manifold the following relations hold ([23], [24], [26], [32], [29], [30]):

$$R(X,Y)\xi = \eta(X)\,Y - \eta(Y)X, \tag{2.8}$$

$$\eta(R(X,Y)Z) = \eta(Y)g(X,Z) - \eta(X)\,g(Y,Z). \tag{2.9}$$

Let (M,ϕ,ξ,η,g) be a Kenmotsu manifold satisfying (1.4). Writing $\pounds_\xi g$ in terms of the Levi-Civita connection ∇, we obtain from (1.4) that

$$2S(X,Y) = -g(\nabla_X\xi, Y) - g(X,\nabla_Y\xi) - 2\lambda g(X,Y) - 2\mu\eta(X)\eta(Y), \tag{2.10}$$

for any $X, Y \in \chi(M)$. In consequence of (2.6), one can easily bring out the followings:

$$S(X,Y) = -(\lambda+1)g(X,Y) + (1-\mu)\eta(X)\eta(Y), \tag{2.11}$$

$$QX = (1-\mu)\eta(X)\xi - (\lambda+1)X, \tag{2.12}$$

$$r = -(2n+1)(\lambda+1) + (1-\mu), \tag{2.13}$$

$$\begin{aligned} W(X,Y,Z,U) = {} & R(X,Y,Z,U) + a(1-\mu)\eta(Z)[\eta(Y)g(X,U) \\ & - \eta(X)g(Y,U)] + b(1-\mu)[g(Y,Z)\eta(X) \\ & - g(X,Z)\eta(Y)]\eta(U) - [(\lambda+1)(a+b) \\ & + \frac{cr}{2n+1}(\frac{1}{2n} + a + b)][g(Y,Z)g(X,U) - g(X,Z)g(Y,U)]. \end{aligned} \tag{2.14}$$

3. η-Ricci Soliton in Kenmotsu manifolds satisfying $\omega(\xi,X)\cdot W = 0$

Let us now consider a $(2n+1)$-dimensional Kenmotsu manifold M satisfying the condition

$$\omega(\xi,X)\cdot W(Y,Z)U = 0 \tag{3.1}$$

where ω and W stands for *generalized quasi-conformal curvature tensor* with the associated scalar triples $(\bar{a},\bar{b},\bar{c})$ and (a,b,c) respectively, the dot means that $\omega(X,Y)$ acts as a derivation on W. The foregoing equation is equivalent to

$$\begin{aligned} & g(\omega(\xi,X)W(Y,Z)U,\xi) - g(W(\omega(\xi,X)Y,Z)U,\xi) \\ & - g(W(Y,\omega(\xi,X)Z)U,\xi) - g(W(Y,Z)\omega(\xi,X)U,\xi) = 0. \end{aligned} \tag{3.2}$$

In view of (2.9), (2.11) and (1.1), we have

$$\begin{aligned}&W(\xi,X)Y\\&=\Big[1-\bar{a}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}\\&\quad+\bar{b}\big)\Big]\eta(Y)X-\Big[1-\bar{b}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})\\&\quad+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]g(X,Y)\xi+(\bar{a}-\bar{b})\eta(X)\eta(Y)\xi,\end{aligned}\tag{3.3}$$

$$\begin{aligned}&\eta(W(X,Y)Z)\\&=\Big[1-\bar{b}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]\times\\&\Big[g(X,Z)\eta(Y)-g(Y,Z)\eta(X)\Big],\end{aligned}\tag{3.4}$$

$$\begin{aligned}&\sum_{i=1}^{2n+1}W(e_i,Z,U,e_i)\\&=(1-\bar{b}+2n\bar{a})(1-\mu)\eta(U)\eta(Z)\\&+\Big[\bar{b}(1-\mu)-2n\big(1+\lambda+\frac{\bar{c}r}{2n+1}\big)\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]g(Z,U),\end{aligned}\tag{3.5}$$

$$\begin{aligned}&\sum_{i=1}^{2n+1}\eta\big(W(e_i,Z)e_i\big)\\&=2n\Big[1-\bar{b}(1-\mu)+(1+\lambda)(\bar{a}+\bar{b})+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]\eta(Z).\end{aligned}\tag{3.6}$$

In view of (3.3) and (3.4), we obtain from (3.2)

$$\begin{aligned}&\Big[1-\bar{a}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})\\&\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]\times\Big[1-b(1-\mu)+(\lambda+1)(a+b)\\&+\frac{cr}{2n+1}\big(\frac{1}{2n}+a+b\big)\Big]\times\Big[g(X,Z)\eta(U)\eta(Y)-g(X,Y)\eta(U)\eta(Z)\Big]\\&-\Big[1-\bar{b}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]W(X,Y,Z,U)\\&+\Big[1-\bar{b}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]\times\Big[1-b(1-\mu)\\&+(\lambda+1)(a+b)+\frac{cr}{2n+1}\big(\frac{1}{2n}+a+b\big)\Big]\times\Big[g(X,Y)\eta(U)\eta(Z)\\&-g(X,Y)g(Z,U)+g(X,Z)g(Y,U)-g(X,Z)\eta(U)\eta(Y)\Big]=0.\end{aligned}\tag{3.7}$$

Contracting (3.7) over X and Y and then using (3.5) and (3.6), we get

$$\begin{aligned}
& -2n\Big[1-\bar{a}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]\times \\
& \Big[1-b(1-\mu)+(\lambda+1)(a+b)+\frac{cr}{2n+1}\big(\frac{1}{2n}+a+b\big)\Big]\eta(U)\eta(Z) \\
& -\Big[1-\bar{b}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]\times \\
& \Big[(1-b+2na)(1-\mu)\eta(U)\eta(Z)+\big\{b(1-\mu)-2n\big(1+\lambda+\frac{cr}{2n+1}\big)\times \\
& \big(\frac{1}{2n}+a+b\big)\big\}g(Z,U)\Big]+\Big[1-\bar{b}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b}) \\
& +\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]\times\Big[1-b(1-\mu)+(\lambda+1)(a+b) \\
& +\frac{cr}{2n+1}\big(\frac{1}{2n}+a+b\big)\Big]\times[2n\eta(U)\eta(Z)-2ng(Z,U)]=0.
\end{aligned} \tag{3.8}$$

Again putting $Z=U=\xi$ in (3.8) and taking the summation over i, $1\le i\le 2n+1$, we get

$$\begin{aligned}
& -2n\Big[1-\bar{a}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]\times \\
& \Big[1-b(1-\mu)+(\lambda+1)(a+b)+\frac{cr}{2n+1}\big(\frac{1}{2n}+a+b\big)\Big] \\
& -\Big[1-\bar{b}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]\times \\
& \Big[(1+2na)(1-\mu)-2n\big(1+\lambda+\frac{cr}{2n+1}\big)\big(\frac{1}{2n}+a+b\big)\Big]=0.
\end{aligned} \tag{3.9}$$

Based on (3.9), we can state the following:

Theorem 3.1. *In a Kenmotsu manifold $M^{2n+1}(\phi,\xi,\eta,g)$ which metric tensor is η-Ricci soliton (g,ξ,λ,μ) satisfies the curvature condition in the following table:*

Curvature condition	Value of λ
$R(\xi, X)\cdot W = 0$ (Obtain by $\bar{a}=\bar{b}=\bar{c}=0$)	$\lambda = 2n-1+(1-\mu)[1+2n(a-b)]$
$E(\xi, X)\cdot W = 0$ (Obtain by $\bar{a}=\bar{b}=0$, $\bar{c}=1$)	$\lambda = (2n+\frac{r}{2n+1})\left\{(1-\frac{\mu}{2n})+(1-\mu)(a-b)\right\}$
$C(\xi, X)\cdot W = 0$ (Obtain by $\bar{a}=\bar{b}=-\frac{1}{2n-1}, \bar{c}=1$)	$\lambda = n-1-\frac{\mu}{2}-\frac{r}{4n}$ or $\lambda = (2n+\frac{r}{2n+1})\left\{(1-\frac{\mu}{2n})+(1-\mu)(a-b)\right\}$
$\hat{C}(\xi, X)\cdot W = 0$ (Obtain by $\bar{a}=\bar{b}=-\frac{1}{2n-1}, \bar{c}=0$)	$\lambda = n-1-\frac{\mu}{2}$ or $\lambda = (2n+\frac{r}{2n+1})\left\{(1-\frac{\mu}{2n})+(1-\mu)(a-b)\right\}$
$P(\xi, X)\cdot W = 0$ (Obtain by $\bar{a}=-\frac{1}{2n}, \bar{b}=\bar{c}=0$)	$\lambda = 2n-1$ for $\mu = 1$
$H(\xi, X)\cdot W = 0$ (Obtain by $\bar{a}=\bar{b}=-\frac{1}{4n}$, $\bar{c}=0$)	$\lambda = 2n-\frac{1}{2}-\frac{\mu}{2}$ or $\lambda = (2n+\frac{r}{2n+1})\left\{(1-\frac{\mu}{2n})+(1-\mu)(a-b)\right\}$.

Theorem 3.2. *The η-Ricci soliton (g, ξ, λ, μ) in a Kenmotsu manifold $M^{2n+1}(\phi, \xi, \eta, g)$ satisfying each of $R(\xi, X)\cdot W = 0$, $E(\xi, X)\cdot W = 0$ and $H(\xi, X)\cdot W = 0$ is always expanding provided $2n > \mu$.*

Theorem 3.3. *The η-Ricci soliton (g, ξ, λ, μ) in a Kenmotsu manifold $M^{2n+1}(\phi, \xi, \eta, g)$ satisfying each of $C(\xi, X)\cdot W = 0$ and $\hat{C}(\xi, X)\cdot W = 0$ is sometime expanding and some other time remains shrinking.*

4. η-Ricci soliton in a Kenmotsu manifold with $W \cdot S = 0$

Let $M^{2n+1}(\phi, \xi, \eta, g)$ be a Kenmotsu manifold, satisfying the condition

$$W \cdot S = 0, \tag{4.1}$$

i.e.,

$$W(\xi, X)S(Y, Z) - S(W(\xi, X)Y, Z) - S(Y, W(\xi, X)Z) = 0,$$

i.e.,

$$S(W(\xi, X)Y, Z) + S(Y, W(\xi, X)Z) = 0. \tag{4.2}$$

In view of (2.9) and (1.1), we have

$$\begin{aligned}
&\eta(W(\xi, X)Y)\\
&= \Big[\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big) - \bar{b}(1-\mu) + (\lambda+1)(\bar{a}+\bar{b}) + 1\Big] g(X, Y)\\
&+ \Big[1 - \bar{b}(1-\mu) + (\lambda+1)(\bar{a}+\bar{b}) + \frac{\bar{c}r}{2n+1}\big(\frac{1}{2\,n}+\bar{a}+\bar{b}\big)\Big]\eta(X)\eta(Y).
\end{aligned} \tag{4.3}$$

Using (4.3) in (4.2), we have

$$\begin{aligned}
&2\Big[\big\{1-\bar{a}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\big\}\\
&-\lambda(\bar{a}-\bar{b})\Big]\eta(X)\eta(Y)\eta(Z)+\lambda\big\{1-\bar{b}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})\\
&+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\big\}[g(X,Y)\eta(Z)+g(X,Z)\eta(Y)]\\
&-(\lambda+1)\big\{1-\bar{a}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})+\frac{\bar{c}r}{2n+1}\big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\big\}\\
&[g(X,Y)\eta(Z)+g(X,Z)\eta(Y)]=0.
\end{aligned} \tag{4.4}$$

Putting $Z=\xi$ and $X=Y=e_i$ in (4.4), where $\{e_1, e_2, e_3, \cdots, e_{2n}, e_{2n+1}=\xi\}$ is an orthonormal basis of the tangent space at each point of the manifold M, and taking the summation over i, $1\leq i\leq 2n+1$, we get

$$\begin{aligned}
&\lambda\Big[1-\bar{b}(1-\mu)+(\lambda+1)(\bar{a}+\bar{b})\\
&\bar{c}\big(\frac{\mu\ +\lambda(2n+1\ )+2n}{2n+1}\big)\ \big(\frac{1}{2n}+\bar{a}+\bar{b}\big)\Big]=0.
\end{aligned}$$

From the foregoing equation, one can easily bring out the following:

Theorem 4.1. *Any Kenmotsu manifold $M^{2n+1}(\phi,\xi,\eta,g)$ whose metric tensor is η-Ricci soliton (g,ξ,λ,μ) satisfies the curvature condition in the following table:*

Curvature condition	Value of λ
$R(\xi,X)\cdot S=0$ (Obtain by $\bar{a}=\bar{b}=\bar{c}=0$)	$\lambda=0$
$E(\xi,X)\cdot S=0$ (Obtain by $\bar{a}=\bar{b}=0,\ \bar{c}=1$)	$\lambda=0,\frac{1-\mu}{2n+1}$
$C(\xi,X)\cdot S=0$ (Obtain by $\bar{a}=\bar{b}=-\frac{1}{2n-1},\bar{c}=1$)	$\lambda=0,2n-\mu$
$\hat{C}(\xi,X)\cdot S=0$ (Obtain by $\bar{a}=\bar{b}=-\frac{1}{2n-1},\bar{c}=0$)	$\lambda=0,n-1-\frac{\mu}{2}$
$P(\xi,X)\cdot S=0$ (Obtain by $\bar{a}=-\frac{1}{2n},\bar{b}=\bar{c}=0$)	$\lambda=0,2n-1$
$H(\xi,X)\cdot S=0$ (Obtain by $\bar{a}=\bar{b}=-\frac{1}{4n},\ \bar{c}=0$)	$\lambda=0,\frac{4n-1-\mu}{2}$.

From the above table, we can state the following

Theorem 4.2. *η-Ricci soliton (g,ξ,λ,μ) in the Kenmotsu manifold admitting $W\cdot S=0$ is sometime expanding and some other time remains steady provided $\mu<1$.*

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