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ON HELICES AND SLANT HELICES IN THE LIGHTLIKE CONE

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Abstract. In this paper, we investigate the notions of helix and slant helices in the lightlike cone. Using the asymptotic orthonormal frame we present some characterizations of helices and slant helices.

1. Introduction

From the point of mathematics, curve theory has been a fascination for differential geometers and so it has been a compeletely studied subject. Curves of constant slope, or so-called general helices are well-known curves in the classical differential geometry of space curves. Helices are characterized by the feature that the tangent makes a constant angle with a fixed straight line (the axis of the general helix), [7]. And also, it is known that a curve α is called a slant helix if the principal normal lines of α make a constant angle with a fixed direction. The concept of slant helix defined by Izumiya and Takeuchi [8]. The geometry of helix and slant helices have been represented in a different ambient spaces by many mathematicians. In [1], the authors studied timelike B_2-slant helices in Minkowski 4-space E_1^4. Ahmad studied the position vectors of a spacelike general helix with respect to the standart frame in Minkowski 4-space E_1^3, [2]. In [3], the authors introduced the notion of a k-type slant helix in Minkowski 4-space E_1^4. In [4], they gave necessary and sufficient conditions to be a slant helix in the Euclidean n-space and they expressed some integral characterizations of such curves in terms of curvature functions. Camcı and et al. examined some characterizations for a non degenerate curve α to be a generalized helix by using its harmonic curvatures, [5]. Ferrandez and et al. obtained a Lancret-type theorem

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for null generalized helices in Lorentz-Minkowski space L^n, [6]. Ilarslan and et al. studied the position vectors of a timelike and a null helix in Minkowski 3-space E_1^3. Also, they gave some characterizations for timelike and a null helix by using the position vectors of the curve, [9]. In [10], the authors examined spherical images the tangent indicatrix and binormal indicatrix of slant helix. In [15], the authors defined a new kind of slant helix in Euclidean 4-space E^4 which they called B_2-slant helix and they found some characterizations of these curves. In [16], they gave some characterizations for spacelike helices in Minkowski space-time E_1^4. Furthermore, they found the differential equations characterizating the spacelike helices in Minkowski space-time E_1^4. In [18], Turgut and et al. they examined the concept of a slant helix in Minkowski spacetime. Also, they defined type-3 slant helices whose trinormal lines make a constant angle with a fixed direction. Turgut and et al. defined 3-type slant helices whose trinormal lines make a constant angle with a fixed direction in E^4 and examined some characterizations of such curves and other forms, [19]. Yilmaz and et al. studied a new version of Bishop frame using a common vector field as binormal vector field of a regular curve and called that frame as type-2 Bishop frame, [20]. In this paper, we study helix and slant helices according to asymptotic orthonormal frame in the lightlike cone. We give some characterizations of these curves.

2. Preliminaries

Let E_1^3 be the $3-$dimensional Minkowski space with the metric

$$g(X,Y) = \langle X,Y\rangle = x_1y_1 + x_2y_2 - x_3y_3$$

for all $X = (x_1, x_2, x_3)$, $Y = (y_1, y_2, y_3) \in E_1^3$. E_1^3 is a Minkowski space of signature $(2,1)$, [17].

The lightlike cone is defined by

$$\mathbf{Q}^2 = \left\{x \in E_1^3 : g(x,x) = 0\right\},\ [13].$$

Let $x : I \to \mathbb{Q}^2 \subset \mathbb{E}_1^3$ be a spacelike curve in $\mathbf{Q}^2$ with arc length parameter s. Then $y(s)$ defined by

$$y(s) = -x''(s) - \frac{1}{2}\left\langle x'', x''\right\rangle x,$$

and also called dual curve of the curve $x(s)$.

Using $x'(s) = \alpha(s)$, we know that $\{x, \alpha, y\}$ form an asymptotic orthonormal frame along the curve x and the cone frenet formulas of x

are given by

$$(1)\qquad \begin{aligned} x' &= \alpha \\ \alpha' &= \kappa x - y \\ y' &= -\kappa\alpha \end{aligned}$$

where

$$(2)\qquad \begin{aligned} &\langle x, x\rangle = \langle x, \alpha\rangle = \langle y, \alpha\rangle = \langle y, y\rangle = 0 \\ &\langle x, y\rangle = \langle \alpha, \alpha\rangle = 1, \end{aligned}$$

[11, 12, 13].

Let E_1^4 be the 4-dimensional Minkowski space with the metric

$$\langle \widetilde{x}, \widetilde{y}\rangle = x_1y_1 + x_2y_2 + x_3y_3 - x_4y_4$$

where $\widetilde{x} = (x_1, x_2, x_3, x_4)$, $\widetilde{y} = (y_1, y_2, y_3, y_4) \in \mathbb{E}_1^4$. $\mathbb{E}_1^4$ is a flat pseudo-Riemannian manifold of signature (3.1).

Let M be a submanifold of $\mathbb{E}_1^4$. If the pseudo-Riemannian metric $\widetilde{g}$ of $\mathbb{E}_1^4$ induces a pseudo-Riemannian metric $\widetilde{g}$ on M, then M is called a timelike submanifold of $\mathbb{E}_1^4$.

The 4-dimensional lightlike cone is defined by

$$\mathbf{Q}^3 = \left\{x \in \mathbb{E}_1^4 : \langle x, x\rangle = 0\right\}.$$

Let $x = I \to \mathbf{Q}^3 \subset \mathbb{E}_1^4$ be a spacelike curve in $\mathbf{Q}^3$ with arc length parameter s. Then $y(s)$ defined by

$$y(s) = x'' - \frac{1}{2}\left\langle x'', x''\right\rangle x$$

also called dual curve of the curve $x(s)$.

Using $x'(s) = \alpha(s)$, we can write $\{x, \alpha, \beta, y\}$ form an asymptotic orthonormal frame along the curve x and the cone frenet formulas of x are given by

$$(3)\qquad \begin{aligned} x' &= \alpha \\ \alpha' &= \kappa x + \lambda\beta - y \\ \beta' &= \tau x - \lambda\alpha \\ y' &= -\kappa\alpha - \tau\beta, \end{aligned}$$

where

$$(4)\qquad \begin{aligned} &\langle x, x\rangle = \langle y, y\rangle = \langle x, \alpha\rangle = \langle x, \beta\rangle = \langle y, \beta\rangle = \langle y, \alpha\rangle = 0 \\ &\langle x, y\rangle = 1, \langle \alpha, \alpha\rangle = \langle \beta, \beta\rangle = 1, \end{aligned}$$

and the functions κ, λ, τ are called cone curvature functions of the curve x, [13].

3. Helix and Slant Helix in $\mathbf{Q}^2$

Definition 3.1. *Let x be a spacelike curve with arc length according to the asymptotic orthonormal frame $\{x, \alpha, y\}$ in $\mathbf{Q}^2$. If there exists a constant $V_\alpha \neq 0$ in $\mathbf{Q}^2$ such that*

$$\langle \alpha(s), V_\alpha(s) \rangle = cons\tan t,$$

*for $\forall s \in I$, then x is called a **helix** in $\mathbf{Q}^2$ and $V_\alpha(s)$ is said to be an **axis** of $x(s)$,* [8].

Theorem 3.2. *Let x be a helix in $\mathbf{Q}^2$. Then the axes of x are given as*

$$V_\alpha(s) = \left(-A \int \kappa(s)\, ds + e\right) x(s) + A\alpha(s) + A\left(\frac{1}{\kappa(s)} \int \kappa(s)\, ds\right) y(s)$$

or

$$V_\alpha(s) = \left(-A \int \kappa(s)\, ds + e\right) x(s) + A\alpha(s) + (A\ s\ + c)\, y(s), \tag{5}$$

where $A, c, e \in \mathbb{R}_0^+$.

Proof. Let $V_\alpha(s)$ be an axis of helix x with the asymptotic orthonormal frame $\{x, \alpha, y\}$. Then, $V_\alpha(s)$ can be written as

$$V_\alpha = \eta_1 x + \eta_2 \alpha + \eta_3 y, \tag{6}$$

where η_1, η_2, η_3 are differentiable functions and from definition 3.1 of the helix, we have $\langle \alpha(s), V_\alpha(s) \rangle = A, A \in \mathbb{R}_0^+$.

Furthermore, from (2) and (6), we can write

$$\eta_1 = \langle y, V_\alpha(s) \rangle, \eta_2 = \langle \alpha, V_\alpha(s) \rangle = A, \eta_3 = \langle x, V_\alpha(s) \rangle, \forall s \in I. \tag{7}$$

By differentiating on both sides of (6) and using (2), we get

$$\eta_1' + \eta_2 \kappa = 0,\ \eta_1 + \eta_2' - \eta_3 \kappa = 0,\ -\eta_2 + \eta_3' = 0. \tag{8}$$

Substituting $\eta_2 = A$ into (8), we have

$$\eta_1' + A\kappa = 0,\ \eta_1 - \kappa\eta_3 = 0,\ -A + \eta_3' = 0. \tag{9}$$

From (9), solving calculations necassary, we obtain

$$\begin{aligned} \eta_1 &= \ -A \int \kappa(s)\, ds + e, \\ \eta_3 &= A\ s\ + c \text{ or } \eta_3 = \frac{A}{\kappa(s)} \int \kappa(s)\, ds; c, e \in \mathbb{R}_0^+. \end{aligned} \tag{10}$$

Hence, using (6) and (10), we have

$$V_\alpha = \left(-A\int \kappa(s)ds + e\right)\overrightarrow{x} + A\overrightarrow{\alpha} + (A\ s\ + c)\,\overrightarrow{y},$$
$$V_\alpha(s) = \left(e - A\int \kappa(s)\,ds\right)\ x(s) + A\alpha(s) + A\left(\frac{1}{\kappa(s)}\int \kappa(s)\,ds\right)y(s).$$

□

Definition 3.3. *Let x be a curve with asymptotic orthonormal frame $\{x,\alpha,y\}$ in $\mathbf{Q}^2$. If there is a constant vector field $V_y \neq 0$ in $\mathbf{Q}^2$ such that*

$$< y, V_y >= d; d \in \mathbb{R}_0^+, \forall s \in I.$$

*Then x is called y-**type slant helix** and V_y is called y-**axis** of x.*

Theorem 3.4. *Let x be y-type slant helix in $\mathbf{Q}^2$ then the axis of x are given as*

$$V_y = d\left(\overrightarrow{x} + \frac{1}{\kappa}\overrightarrow{y}\right), \tag{11}$$

where $d \in \mathbb{R}_0^+$.

Proof. From definition 3.3 and (7), we have

$$\eta_1 = g(y, V_y) = d, d \in \mathbb{R}_0^+.$$

Substituting $\eta_1 = d$ into (8), we obtain

$$\eta_2 = 0, \eta_3 = \frac{d}{\kappa},$$

where $d \in \mathbb{R}_0^+$. □

4. Helix in $\mathbf{Q}^3$

Definition 4.1. *Let $x(s)$ be a spacelike curve with the asymptotic orthonormal frame $\{x,\alpha,\beta,y\}$ in $\mathbf{Q}^3$. If there exists a constant vector field W_α in $\mathbf{Q}^3$ such that*

$$< \alpha, W_\alpha >= m, m \in \mathbb{R}_0^+,$$

*for $\forall s \in I$. Then x is called a **helix** and W_α is called the $\alpha-$**axis** of x.*

Theorem 4.2. *Let x be a helix. Then x is a helix if and only if*

$$(\lambda' + \tau)(s)\int((ms+c)\,\tau(s) - m\lambda(s))\,ds + (\lambda\tau - \kappa')(s)(ms+c) - m\lambda^2(s) = 0, \tag{12}$$

where $c, m \in \mathbb{R}_0^+$ and κ, τ, λ are cone curvature functions.

Proof. Let W_α be an axis of helix x with the asymptotic orthonormal frame $\{x, \alpha, \beta, y\}$. Then, W_α can be written as follows:

$$W_\alpha = \theta_1 \overrightarrow{x} + \theta_2 \overrightarrow{\alpha} + \theta_3 \overrightarrow{\beta} + \theta_4 \overrightarrow{y}, \tag{13}$$

where θ_1, θ_2, θ_3, θ_4 are differentiable functions. Hence, from (13) we get

$$\theta_1 = \langle W_\alpha, y \rangle, \; \theta_2 = \langle W_\alpha, \alpha \rangle = m, \; \theta_3 = \langle W_\alpha, \beta \rangle, \; \theta_4 = \langle W_\alpha, x \rangle, \tag{14}$$

where $m \in \mathbb{R}_0^+$.

By differentiating on both sides of (13), we obtain

$$\begin{aligned} &\theta_1' + \kappa\theta_2 + \tau\theta_3 = 0, \\ &\theta_1 + \theta_2' - \lambda\theta_3 - \kappa\theta_4 = 0, \\ &\lambda\theta_2 + \theta_3' - \tau\theta_4 = 0, \\ &-\theta_2 + \theta_4' = 0. \end{aligned} \tag{15}$$

From definition of the helix and substituting $\theta_2 = m$ into (15), we obtain

$$\begin{aligned} &\theta_1(s) = \lambda \int ((ms + c)\,\tau(s) - m\lambda(s))\,ds - \kappa(s)(ms + c), \\ &\theta_3(s) = \int ((ms + c)\,\tau(s) - m\lambda(s))\,ds, \\ &\theta_4(s) = ms + c, \end{aligned} \tag{16}$$

where m, $c \in \mathbb{R}_0^+$.

Using (16) and equation $\theta_1' + \kappa\theta_2 + \tau\theta_3 = 0$, we obtain

$$\left(\lambda' + \tau\right) \int ((ms + c)\,\tau - m\lambda)\,ds + \left(\lambda\tau - \kappa'\right)(ms + c) - m\lambda^2 = 0. \tag{17}$$

Conversely, assume that (17) holds. We can define a vector field W_α as (18). Since $W_\alpha' = 0$, we have $\langle W_\alpha, \alpha \rangle = m$. Hence, the theorem is proved. □

Theorem 4.3. *Let x be a helix in $\mathbf{Q}^3$. Then the axes of x are given as*

$$\begin{aligned} W_\alpha = &\left(\lambda \int ((ms + c)\,\tau - \lambda m)\,ds - \kappa(ms + c)\right) x + m\alpha \\ &+ \left(\int ((ms + c)\,\tau - \lambda m)\,ds\right) \beta + (ms + c)\,y, \end{aligned} \tag{18}$$

where m, $c \in \mathbb{R}_0^+$.

Proof. From (13) and (15), we have (18). □

5. β-type Slant Helix in $\mathbf{Q}^3$

Definition 5.1. *Let x be a spacelike curve with the asymptotic orthonormal frame $\{x, \alpha, \beta, y\}$ in $\mathbf{Q}^3$. If there is a constant vector field $W_\beta \neq 0$ in $\mathbf{Q}^3$ such that $< \beta, W_\beta >= \ell$, $\ell \in \mathbb{R}_0^+$, for $\forall s \in I$. Then x is called β-**type slant helix** and W_β is called the β-**axis of** x.*

Theorem 5.2. *Let x be a β-type slant helix in $\mathbf{Q}^3$. Then the β-axis of x are given as*

$$\begin{aligned} W_\beta = &\left(-\int \tau.\left(n\frac{\kappa}{\lambda}e^{\int \frac{\tau}{\lambda}ds} + \ell\right) ds + p\right) x(s) \\ &+ n\left(\frac{\tau}{\lambda}e^{\int \frac{\tau}{\lambda}ds}\right)\alpha(s) + \ell\beta(s) + ne^{\int \frac{\tau}{\lambda}ds}y(s), \end{aligned} \tag{19}$$

where $n, \ell, p \in \mathbb{R}_0^+$.

Proof. Let W_β be an axis of β-type helix x with the asymptotic orthonormal frame $\{x, \alpha, \beta, y\}$. From definition 5.1 and equations (15), we get

$$\begin{aligned} \theta_1 &= -\int \tau.\left(n\frac{\kappa}{\lambda}e^{\int \frac{\tau}{\lambda}ds} + \ell\right) ds + p \\ \theta_2 &= n\frac{\tau}{\lambda}e^{\int \frac{\tau}{\lambda}ds} \\ \theta_4 &= ne^{\int \frac{\tau}{\lambda}ds}, \end{aligned} \tag{20}$$

where $n, \ell, p \in \mathbb{R}_0^+$. Considering (20) and (13), we obtain (19). $\square$

Theorem 5.3. *Let x be a β-type slant helix. Then x is a β-slant helix if and only if*

$$n\left(\left(\frac{\tau}{\lambda}\right)' + \left(\frac{\tau}{\lambda}\right)^2 - \kappa\right)e^{\int \frac{\tau}{\lambda}ds} - \int \tau\left(n\frac{\kappa}{\lambda}e^{\int \frac{\tau}{\lambda}ds} + \ell\right) ds - \ell\lambda + p = 0, \tag{21}$$

where n, ℓ, $p \in \mathbb{R}_0^+$.

Proof. Using (20) and the second equation in (15), we get (21). Conservely, assume that (21) holds, we can define a vector field W_β as (19). Hence, from definition 5.1, we can write $W_\beta' = 0, < \beta, W_\beta >= \ell$, $\ell \in \mathbb{R}_0^+$. $\square$

6. y−type Slant Helix in $\mathbf{Q}^3$

Definition 6.1. *Let x be a spacelike curve with the asymptotic orthonormal frame $\{x, \alpha, \beta, y\}$ in $\mathbf{Q}^3$. If there is a $W_y \neq 0$ constant field in $\mathbf{Q}^3$ such that*

$$< y, W_y >= i, i \in \mathbb{R}_0^+.$$

*Then x is called y-**type slant helix** and W_y is called the y-**axis of** x.*

Theorem 6.2. *Let x be a y-type slant helix in $\mathbf{Q}^3$. Then the y-axis of x are given as follows*

$$W_y(s) = i\ x(s) + \theta_4^{y\prime}\alpha(s) + \left(-\frac{\kappa}{\tau}\right)\theta_4^{y\prime}\beta(s) + \theta_4^y y(s), \tag{22}$$

where

$$\theta_4^y = c_1 e^{t_1 s} + c_2 e^{t_2 s},$$

$$t_{1,2} = \frac{\tau}{2\kappa}\left(\lambda - \left(\frac{\kappa}{\tau}\right)'\right) \pm \frac{\tau}{2\kappa}\sqrt{\left(\lambda - \left(\frac{\kappa}{\tau}\right)'\right)^2 - 4\kappa},$$

i, c_1, $c_2 \in \mathbb{R}_0^+$.

Proof. From definition of the y-type slant helix and (15), we get

$$\begin{aligned} \theta_2^y &= c_1\left(t_1 + t_1' s\right)e^{t_1 s} + c_2\left(t_2 + t_2 s\right)e^{t_2 s}, \\ \theta_3^y &= -\frac{\kappa}{\tau}\theta_2^y \\ \theta_4^y &= c_1 e^{t_1 s} + c_2 e^{t_2 s}, \end{aligned} \tag{23}$$

where $c_1, c_2 \in \mathbb{R}_0^+$. Considering (23) and (13), we have (22). □

Theorem 6.3. *Let x be a y-type slant helix. Then x is a y-type slant helix if and only if*

$$\frac{-\kappa}{\tau}\theta_4^{y\prime\prime} + \left(\lambda - \left(\frac{\kappa}{\tau}\right)'\right)\theta_4^{y\prime} - \tau\theta_4^y = 0, \tag{24}$$

where

$$\theta_4^y = c_1 e^{t_1 s} + c_2 e^{t_2 s}$$

$$t_{1,2} = \frac{\tau}{2\kappa}\left(\lambda - \left(\frac{\kappa}{\tau}\right)'\right) \pm \frac{\tau}{2\kappa}\sqrt{\left(\lambda - \left(\frac{\kappa}{\tau}\right)'\right)^2 - 4\kappa},$$

c_1, $c_2 \in \mathbb{R}_0^+$.

Proof. From definition 6.1, we write equation $\lambda\theta_2^y - \theta_3^{y\prime} - \tau\theta_4^y = 0$ and using (15), we get (24).

Conversely, assume that (24) holds, we can write a vector field W_y as (22). Since $W_y' = 0$, we obtain $< y, W_y >= i$. Hence, the theorem is provided. □

Theorem 6.4. *Let x be a y-type slant helix. Then x is a y-slant helix if and only if*

- *if $\lambda = constant$, we have*

$$\theta_4^y = c_1^* e^{\kappa s} + c_2^* e^{\kappa s} + \frac{\ell}{\kappa}(-\lambda + 1)$$

- *if $\lambda = bs + a$, we have*

$$\theta_4^y = c_1^* e^{\kappa s} + c_2^* e^{\kappa s} + \frac{\ell}{\kappa}(-bs - a + 1)$$

- *if $\lambda = as^2 + bs + c$, we have*

$$\theta_4^y = c_1^* e^{\kappa s} + c_2^* e^{\kappa s} + \frac{\ell}{\kappa}\left(-as^2 - bs - c - 2\frac{a}{\kappa} + 1\right).$$

where a, b, c, c_1^*, $c_2^* \in \mathbb{R}_0^+$.

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