East Asian Math. J.
Vol. 34 (2018), No. 3, pp. 295–303
http://dx.doi.org/10.7858/eamj.2018.021

APPLICATION OF PRODUCT OF THE MULTIVARIABLE A-FUNCTION AND THE MULTIVARIABLE SRIVASTAVA'S POLYNOMIALS

Dinesh Kumar, Frédéric Ayant, and Junesang Choi

Abstract. Gautam et al. [9] introduced the multivariable A-function, which is very general, reduces to yield a number of special functions, in particular, the multivariable H-function. Here, first, we aim to establish two very general integral formulas involving product of the general class of Srivastava multivariable polynomials and the multivariable A-function. Then, using those integrals, we find a solution of partial differential equations of heat conduction at zero temperature with radiation at the ends in medium without source of thermal energy. The results presented here, being very general, are also pointed out to yield a number of relatively simple results, one of which is demonstrated to be connected with a known solution of the above-mentioned equation.

1. Introduction and preliminaries

We consider a problem on outer heat conduction in a rod under certain boundary conditions. If the thermal coefficients are constants and there is no source of thermal energy, then the temperature $U(x,t)$ in one-dimensional rod $0 \le x \le L$ satisfies the following heat equation (see [14, p. 155, Eq. (6.7.1)]):

$$\frac{\partial U}{\partial t} = \mu \frac{\partial^2 U}{\partial x^2} \quad \left(t \in \mathbb{R}_0^+\right), \tag{1}$$

where $U(x,t)$ is the temperature distribution function of a thin bar, which has length L, and the positive constant $\mu = \frac{k}{c\rho}$ is called the thermal diffusivity of the material (k is thermal conductivity, c is heat capacity, ρ is density).

Here and in the following, let $\mathbb{C}$, $\mathbb{R}^+$, and $\mathbb{N}$ be the sets of complex numbers, positive real numbers, and positive integers, respectively, and let $\mathbb{R}_0^+ := \mathbb{R}^+ \cup \{0\}$

Received April 14, 2018; Accepted April 26, 2018.
2010 *Mathematics Subject Classification.* 33C05, 33C60.
Key words and phrases. A partial differential equation involving heat conduction, Multivariable A-function, multivariable Srivastava's polynomials, multivariable H-function.
*Corresponding author.

©2018 The Youngnam Mathematical Society
(pISSN 1226-6973, eISSN 2287-2833)

and $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. Let the following boundary conditions be satisfied

$$\frac{\partial U(0,t)}{\partial x} - hU(0,t) = 0, \quad \frac{\partial U(L,t)}{\partial x} + hU(L,t) = 0 \quad \left(h \in \mathbb{R}^+\right). \tag{2}$$

Here h is the heat transfer coefficient,

$$U(x,t) \text{ is finite as } t \to \infty, \tag{3}$$

and initial condition

$$U(x,0) = f(x). \tag{4}$$

The function $U(x,t)$ can be represented as the product of a special function $X(x)$ and a time function $T(t)$ with

$$U(x,t) = X(x)\, T(t). \tag{5}$$

Substituting the differentiated forms of (5) in (1) and separating the variables on either side of the equation results in

$$\frac{1}{T}\frac{\partial T}{\partial t} = \frac{1}{X}\frac{\partial^2 X}{\partial x^2}. \tag{6}$$

Putting each side equal to a negative constant $-\mu E_q^2$, it is possible to obtain solution for $T(t)$ and $X(x)$.

$$T(t) = A\, \mathsf{e}^{-\mu E_q^2 t}, \tag{7}$$

and

$$X(x) = B \cos E_q x + D \sin E_q x. \tag{8}$$

Using the equation (5), the general solution of (1) becomes

$$T(x,t) = \mathsf{e}^{-\mu E_q^2 t} \left(m \cos(bx) + n \sin(E_q x)\right), \tag{9}$$

where m and n are real constants.

The (9) satisfies the (1), (2) and (3) on the conditions that

$$E_q n - hm = 0 \tag{10}$$

and

$$E_q \left(n \cos E_q L - m \sin E_q L\right) + h \left(m \cos E_q L + n \sin E_q L\right) = 0. \tag{11}$$

From (10) and (11), we find

$$m/n = E_q/h, \tag{12}$$

and

$$\tan E_q L = \frac{2E_q h}{E_q^2 - h^2}. \tag{13}$$

Then the solution of (1) takes the following form (see [2]):

$$U(x,t) = \sum_{q=0}^{\infty} R_q \left(\cos E_q x + \frac{h}{E_q} \sin E_q\right) \mathsf{e}^{-\mu E_q^2 t}, \tag{14}$$

here the quantities $U(x,t)$ and R_q are real numbers.

We will take a formal initial condition

$$\boldsymbol{U}(x,0)=\boldsymbol{f}(x)=\left(\sin\frac{\pi x}{L}\right)^{w-1} \times S_{N_1,\ldots,N_u}^{M_1,\ldots,M_u}\begin{bmatrix} y_1\left(\sin\frac{\pi x}{L}\right)^{2\rho_1} \\ \vdots \\ y_u\left(\sin\frac{\pi x}{L}\right)^{2\rho_u}\end{bmatrix} A\begin{pmatrix} Z_1\left(\sin\frac{\pi x}{L}\right)^{2\eta_1} \\ \vdots \\ Z_s\left(\sin\frac{\pi x}{L}\right)^{2\eta_s}\end{pmatrix}, \tag{15}$$

here $\boldsymbol{U}(x,0)$ depends of several complex variables and parameters. Also, $S_{N_1,\cdots,N_u}^{M_1,\ldots,M_u}[y_1,\ldots,y_u]$ are the generalized polynomials defined by (see [18])

$$S_{N_1,\ldots,N_u}^{M_1,\ldots,M_u}[y_1,\ldots,y_u]=\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_u)_{M_uK_u}}{K_u!} \times\ B[N_1,K_1;\cdots;N_u,K_u]\ y_1^{K_1}\cdots y_u^{K_u}, \tag{16}$$

where $M_1,\ldots,M_u,\ N_1,\ldots,N_u\in\mathbb{N}$ and the coefficients $B[N_1,K_1;\cdots;N_u,K_u]$ are arbitrary real or complex constants. Gautam et al. [9] introduced the multivariable A-function defined by

$$\begin{aligned} &A(Z_1,\ldots,Z_s)\\ &:=A_{p,q:p_1,q_1;\cdots;p_s,q_s}^{m,n:m_1,n_1;\cdots;m_s,n_s}\left(\begin{matrix} Z_1\\ \vdots\\ Z_s\end{matrix}\ \middle|\ \begin{matrix}\left(a_j;A_j^{(1)},\cdots,A_j^{(s)}\right)_{1,p}:\left(c_j^{(1)},C_j^{(1)}\right)_{1,p_1};\cdots;\left(c_j^{(s)},C_j^{(s)}\right)_{1,p_s}\\ \left(b_j;B_j^{(1)},\cdots,B_j^{(s)}\right)_{1,q}:\left(d_j^{(1)},D_j^{(1)}\right)_{1,q_1};\cdots;\left(d_j^{(s)},D_j^{(s)}\right)_{1,q_s}\end{matrix}\right)\\ &:=\frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\phi'(t_1,\ldots,t_s)\prod_{i=1}^{s}\theta_i'(t_i)\ Z_i^{t_i}\ \mathrm{d}t_1\cdots\mathrm{d}t_s, \end{aligned} \tag{17}$$

where $\omega=\sqrt{-1}$; $\phi'(t_1,\ldots,t_s)$ and $\theta_j'(t_j)$ $(j=1,\ldots,s)$ are given as

$$\phi'(t_1,\cdots,t_s):=\frac{\prod_{j=1}^{m}\Gamma\left(b_j-\sum_{i=1}^{s}B_j^{(i)}t_i\right)\ \prod_{j=1}^{n}\Gamma\left(1-a_j+\sum_{i=1}^{s}A_j^{(i)}t_j\right)}{\prod_{j=n+1}^{p}\Gamma\left(a_j-\sum_{i=1}^{s}A_j^{(i)}t_j\right)\ \prod_{j=m+1}^{q}\Gamma\left(1-b_j+\sum_{i=1}^{s}B_j^{(i)}t_j\right)} \tag{18}$$

and

$$\theta_i'(t_i):=\frac{\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+C_j^{(i)}t_i\right)\ \prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-D_j^{(i)}t_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma\left(c_j^{(i)}-C_j^{(i)}t_i\right)\ \prod_{j=m_i+1}^{q_i}\Gamma\left(1-d_j^{(i)}+D_j^{(i)}t_i\right)} \tag{19}$$

$$\Big(m,\ n,\ p,\ q,\ m_i,\ n_i,\ p_i,\ q_i\in\mathbb{N}_0 \text{ with } m\le q,\ n\le p,\ m_i\le q_i,\ n_i\le p_i \text{ and } z_i\neq 0\quad(i=1,\ldots,s);\\ a_j,\ b_j,\ c_j^{(i)},\ d_j^{(i)},\ A_j^{(i)},\ B_j^{(i)},\ C_j^{(i)},\ D_j^{(i)}\in\mathbb{C}\Big).$$

The multiple integral defining the A-function of r variables converges absolutely if

$$|\arg(\Omega'_i)\, Z_i| < \frac{1}{2}\,\eta'_i\pi, \quad \xi'^*_i = 0, \quad \eta'_i \in \mathbb{R}^+ \quad (i = 1, \ldots, s), \tag{20}$$

where

$$\Omega'_i := \prod_{j=1}^{p} \left\{A_j^{(i)}\right\}^{A_j^{(i)}} \prod_{j=1}^{q} \left\{B_j^{(i)}\right\}^{-B_j^{(i)}} \prod_{j=1}^{q_i} \left\{D_j^{(i)}\right\}^{D_j^{(i)}} \prod_{j=1}^{p_i} \left\{C_j^{(i)}\right\}^{-C_j^{(i)}}, \tag{21}$$

$$\xi'^*_i := \Im\left(\sum_{j=1}^{p} A_j^{(i)} - \sum_{j=1}^{q} B_j^{(i)} + \sum_{j=1}^{q_i} D_j^{(i)} - \sum_{j=1}^{p_i} C_j^{(i)}\right), \tag{22}$$

$$\begin{aligned}\eta'_i := \Re\Bigg(&\sum_{j=1}^{n} A_j^{(i)} - \sum_{j=n+1}^{p} A_j^{(i)} + \sum_{j=1}^{m} B_j^{(i)} - \sum_{j=m+1}^{q} B_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)} \\ &- \sum_{j=m_i+1}^{q_i} D_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)}\Bigg).\end{aligned} \tag{23}$$

For details about the nature of contours $L_1, \cdots, L_s$ and other special cases of the multivariable A-function, we refer to the papers [9, 16, 17].

In this paper, first, we aim to establish two very general integral formulas involving product of the general class of Srivastava multivariable polynomials and the multivariable A-function. Then, using those integrals, we find a solution of the partial differential equation (1). The results presented here, being very general, are also pointed out to yield a number of relatively simple results, one of which is demonstrated to be connected with a known solution of the above-mentioned equation. In this regard, for simplicity, in what follows, we use the following notations.

$$X := m_1, n_1; \cdots; m_s, n_s; \quad Y := p_1, q_1; \cdots; p_s, q_s; \tag{24}$$

$$\mathbb{A} := \left(a_j; A_j^{(1)}, \cdots, A_j^{(s)}\right)_{1,p} : \left(c_j^{(1)}, C_j^{(1)}\right)_{1,p_1}; \cdots; \left(c_j^{(s)}, C_j^{(s)}\right)_{1,p_s}; \tag{25}$$

$$\mathbb{B} := \left(b_j; B_j^{(1)}, \cdots, B_j^{(s)}\right)_{1,q} : \left(d_j^{(1)}, D_j^{(1)}\right)_{1,q_1}; \cdots; \left(d_j^{(s)}, D_j^{(s)}\right)_{1,q_s}; \tag{26}$$

$$B' := \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_s)_{M_sK_s}}{K_s!}\, B\left[N_1, K_1; \cdots; N_s, K_s\right]. \tag{27}$$

2. Integral formulas

We begin by recalling some known integral formulas in the following lemma (see, e.g., [10, p. 614, Eq. (1) and p. 615, Eq. (8)], and [4, 5, 6]).

Lemma 2.1. *Each of the following integral formulas holds.*

$$\int_0^L \left(\sin\frac{\pi x}{L}\right)^{w-1} \sin\frac{\pi x E_p}{L}\,\mathrm{d}x = \frac{\pi\Gamma(w)\,L\,\sin\left(\frac{\pi E_p}{2}\right)}{2^{w-1}\,\Gamma\left(\frac{w\pm E_p+1}{2}\right)} \quad (\Re(w)>0)\,; \tag{28}$$

$$\int_0^L \left(\sin\frac{\pi x}{L}\right)^{w-1} \cos\frac{\pi x E_p}{L}\,\mathrm{d}x = \frac{\pi\Gamma(w)\,L\cos\left(\frac{\pi E_p}{2}\right)}{2^{w-1}\,\Gamma\left(\frac{w\pm E_p+1}{2}\right)} \quad (\Re(w)>0)\,; \tag{29}$$

$$\begin{aligned}&\int_0^L \left(\cos E_q x + \frac{h}{E_q}\sin E_q x\right)\left(\cos E_p x + \frac{h}{E_p}\sin E_p x\right)\mathrm{d}x\\ &\quad = 2E_q^{-2}\left[\left(E_q^2+h^2\right)L+2l\right]\delta_{pq},\end{aligned} \tag{30}$$

where $\delta_{pq}=1$ $(p=q)$ *and* $\delta_{pq}=0$ *otherwise and* E_p $($*or* $E_q)$ *is a positive root of the following transcendental equation* $($*see* (13)$)$

$$\tan EL = \frac{2Eh}{E^2-h^2}. \tag{31}$$

We establish two integral formulas, which are useful to find a solution of the partial differential equation (1), in the following theorems.

Theorem 2.2. *Let all involved notations and conditions be given as above. Then*

$$\begin{aligned}&\int_0^L \left(\sin\frac{\pi x}{L}\right)^{w-1}\sin\frac{E_p\pi x}{L}\, S^{M_1,\ldots,M_u}_{N_1,\ldots,N_u}\begin{pmatrix} y_1\left(\sin\frac{\pi x}{L}\right)^{2\rho_1}\\ \vdots\\ y_u\left(\sin\frac{\pi x}{L}\right)^{2\rho_u}\end{pmatrix} A\begin{pmatrix} Z_1\left(\sin\frac{\pi x}{L}\right)^{2\eta_1}\\ \vdots\\ Z_s\left(\sin\frac{\pi x}{L}\right)^{2\eta_s}\end{pmatrix}\mathrm{d}x\\
&= \frac{L\sin\left(\frac{E_p\pi}{2}\right)}{2^{w-1}}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]} B'\,\frac{y_1^{K_1}\cdots y_u^{K_u}}{4^{\sum_{i=1}^u\rho_i K_i}}\\
&\times\; A^{m,n+1;X}_{p+1,q+2;Y}\left(\begin{matrix} Z_1 4^{-\eta_1}\\ \vdots\\ Z_s 4^{-\eta_s}\end{matrix}\;\middle|\;\begin{matrix}\left(1-w-2\sum_{i=1}^u\rho_i K_i : 2\eta_1,\cdots,2\eta_s\right),\,\mathbb{A}\\ \left(\frac{1}{2}\left(1-w-2\sum_{i=1}^u\rho_i K_i \pm E_p\right) : \eta_1,\cdots,\eta_s\right),\,\mathbb{B}\end{matrix}\right),\end{aligned} \tag{32}$$

provided

$$\Re(w)>0,\ \min\{\rho_i,\eta_l\}>0 \quad (i=1,\ldots,u;\ l=1,\ldots,s),$$

$$\left|\arg(\Omega_i')\,Z_i\,\sin\left(\frac{\pi x}{L}\right)^{2\eta_i}\right| < \frac{1}{2}\eta_i'\pi,\ \xi_i'^{*}=0,\ \ \eta_i'\in\mathbb{R}^+\ \ (i=1,\ldots,s),$$

and

$$\Re\left(w+\sum_{i=1}^u K_i\rho_i\right)+\sum_{i=1}^s\eta_i\min_{1\leqslant j\leq m_i}\Re\left(\frac{d_j^{(i)}}{D_j^{(i)}}\right)>0.$$

Proof. Firstly, express the class of multivariable polynomials $S_{N_1,\ldots,N_u}^{M_1,\ldots,M_u}[\cdot]$ in the left sided integral of (32) as a series with the help of (16) and interchange the order of summations and x-integral (which is permissible under the conditions stated). Secondly, in the resulting expression, expressing the A-function of s-variables in the Mellin-contour integral and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved. Thirdly, collect the power of $\sin\left(\frac{\pi x}{L}\right)$ and evaluate the inner x-integral with the help of (28). Finally, interpreting the Mellin-Barnes contour integral in the last resulting expression in the multivariable A-function, we obtain the desired result (32). □

Theorem 2.3. *Let all involved notations and conditions be given as above. Then*

$$\int_0^L \left(\sin\frac{\pi x}{L}\right)^{w-1} \cos\frac{E_p \pi x}{L}\, S_{N_1,\ldots,N_u}^{M_1,\ldots,M_u}\begin{pmatrix} y_1\left(\sin\frac{\pi x}{L}\right)^{2\rho_1} \\ \vdots \\ y_u\left(\sin\frac{\pi x}{L}\right)^{2\rho_u}\end{pmatrix} A\begin{pmatrix} Z_1\left(\sin\frac{\pi x}{L}\right)^{2\eta_1} \\ \vdots \\ Z_s\left(\sin\frac{\pi x}{L}\right)^{2\eta_s}\end{pmatrix} \mathrm{d}x$$

$$= \frac{L\cos\left(\frac{E_p\pi}{2}\right)}{2^{w-1}} \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_u=0}^{[N_u/M_u]} B'\,\frac{y_1^{K_1}\cdots y_u^{K_u}}{4^{\sum_{i=1}^u \rho_i K_i}}$$

$$\times\, A_{p+1,q+2;Y}^{m,n+1;X}\left(\begin{matrix} Z_1 4^{-\eta_1} \\ \vdots \\ Z_s 4^{-\eta_s}\end{matrix}\;\middle|\; \begin{matrix}\left(1-w-2\sum_{i=1}^u \rho_i K_i : 2\eta_1,\ldots,2\eta_s\right),\, \mathbb{A} \\ \left(\frac{1}{2}\left(1-w-2\sum_{i=1}^u \rho_i K_i \pm E_p\right) : \eta_1,\ldots,\eta_s\right),\, \mathbb{B}\end{matrix}\right), \tag{33}$$

provided

$$\Re(w) > 0,\ \min\{\rho_i, \eta_l\} > 0 \quad (i = 1,\ldots,u;\ \ l = 1,\ldots,s),$$

$$\left|\arg\left(\Omega_i'\right) Z_i \sin\left(\frac{\pi x}{L}\right)^{2\eta_i}\right| < \frac{1}{2}\eta_i'\pi,\ \ \xi_i'^{*} = 0,\ \ \eta_i' \in \mathbb{R}^+ \quad (i = 1,\ldots,s),$$

and

$$\Re\left(w + \sum_{i=1}^u K_i\rho_i\right) + \sum_{i=1}^s \eta_i \min_{1\leqslant j\leq m_i} \Re\left(\frac{d_j^{(i)}}{D_j^{(i)}}\right) > 0.$$

Proof. A similar argument as in the proof of Theorem 2.2 will establish the result here. We omit the details. □

3. A solution of (1)

Here we use the results in Section 2 to give a solution of (1) in the following theorem.

Theorem 3.1. *Let all involved notations and conditions be given as above. Then a solution of* (1) *is given as follows:*

$$\boldsymbol{U}(x,t) = \frac{L}{2^{w-1}} \sum_{p=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} B' \, \frac{y_1^{K_1} \cdots y_u^{K_u}}{4^{\sum_{i=1}^{u} \rho_i K_i}}$$

$$\times \frac{E_p^2}{\left[\left(E_p^2 + h^2\right) L + 2h\right]} \left(\cos E_p x + \frac{h}{E_p} \sin E_p x\right) \tag{34}$$

$$\times \, \mathrm{e}^{-kE_p^2 t} \, A_{p+1,q+2;Y}^{m,n+1;X} \left(\begin{array}{c} Z_1 4^{-\eta_1} \\ \vdots \\ Z_s 4^{-\eta_s} \end{array} \middle| \begin{array}{c} \left(1 - w - 2\sum_{i=1}^{u} \rho_i K_i \,:\, 2\eta_1, \cdots, 2\eta_s\right), \mathbb{A} \\ \left(\frac{1}{2}\left(1 - w - 2\sum_{i=1}^{u} \rho_i K_i \pm E_p\right) \,:\, \eta_1, \cdots, \eta_s\right), \mathbb{B} \end{array} \right).$$

Proof. Setting $t = 0$ in (14) and using (15), we have

$$\left(\sin \frac{\pi x}{L}\right)^{w-1} S_{N_1,\ldots,N_u}^{M_1,\ldots,M_u} \left(\begin{array}{c} y_1 \left(\sin \frac{\pi x}{L}\right)^{2\rho_1} \\ \vdots \\ y_u \left(\sin \frac{\pi x}{L}\right)^{2\rho_u} \end{array} \right) A \left(\begin{array}{c} Z_1 \left(\sin \frac{\pi x}{L}\right)^{2\eta_1} \\ \vdots \\ Z_s \left(\sin \frac{\pi x}{L}\right)^{2\eta_s} \end{array} \right)$$

$$= \sum_{q=0}^{\infty} \boldsymbol{R}_q \left(\cos E_q x + \frac{h}{E_q} \sin E_q x\right). \tag{35}$$

Multiply both sides of (35) by $\cos E_p x + \frac{h}{E_p} \sin E_p x$ and integrating each side of the resulting identity with respect to x from 0 to L, with the aid of (30), (32) and (33), we obtain

$$\boldsymbol{R}_q = \frac{1}{2^{w-1}} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} B' \, \frac{y_1^{K_1} \cdots y_u^{K_u}}{4^{\sum_{i=1}^{u} \rho_i K_i}} \, \frac{2LE_q^2}{\left[\left(E_q^2 + h^2\right) L + 2h\right]}$$

$$A_{p+1,q+2;Y}^{m,n+1;X} \left(\begin{array}{c} Z_1 4^{-\eta_1} \\ \vdots \\ Z_s 4^{-\eta_s} \end{array} \middle| \begin{array}{c} \left(1 - w - 2\sum_{i=1}^{u} \rho_i K_i \,:\, 2\eta_1, \ldots, 2\eta_s\right), \mathbb{A} \\ \left(\frac{1 - w - 2\sum_{i=1}^{u} \rho_i K_i \pm E_p}{2} \,:\, \eta_1, \ldots, \eta_s\right), \mathbb{B} \end{array} \right). \tag{36}$$

Finally, substituting the $\boldsymbol{R}_q$ in (36) for the $\boldsymbol{R}_q$ in (14), we find the desired solution (34). □

4. Special cases and remarks

The results presented here, being very general, can be reduced to yield a number of relatively simple formulas and solutions. For example, when $A_j^{(i)}$, $B_j^{(i)}$, $C_j^{(i)}$, $D_j^{(i)} \in \mathbb{R}$ and $m = 0$ and $m_j = 0$, the multivariable A-function reduces to the multivariable H-function (see [19, 20]). Then we have a known solution of (1) (see, Chandel and Singh [3]) which is recorded in the following theorem.

Theorem 4.1. *Let all involved notations and conditions be given as above. Then a solution of* (1) *is given as follows:*

$$\boldsymbol{U}(x,t) = \frac{L}{2^{w-1}} \sum_{p=1}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} B' \, \frac{y_1^{K_1} \cdots y_u^{K_u}}{4^{\sum_{i=1}^{u} \rho_i K_i}} \, \frac{2L\, E_p^2}{\left[\left(E_p^2 + h^2\right) L + 2h\right]}$$

(37)

$$\times \; \mathrm{e}^{-kE_p^2 t} \, H_{p+1,q+2;Y}^{0,n+1;X} \left(\begin{array}{c} Z_1 4^{-\eta_1} \\ \vdots \\ Z_s 4^{-\eta_s} \end{array} \middle| \begin{array}{c} \left(1 - w - 2\sum_{i=1}^{u} \rho_i K_i \,:\, 2\eta_1, \cdots, 2\eta_s\right), \mathbb{A} \\ \left(\frac{1-w-2\sum_{i=1}^{u} \rho_i K_i \pm E_p}{2} \,:\, \eta_1, \cdots, \eta_s\right), \mathbb{B} \end{array} \right),$$

where $A_j^{(i)}$, $B_j^{(i)}$, $C_j^{(i)}$, $D_j^{(i)} \in \mathbb{R}$ *and* $m = 0$ *and* $m_j = 0$.

Acknowledgments The author (Dinesh Kumar) would like to express his deep thanks to NBHM (National Board of Higher Mathematics), India, for granting a Post-Doctoral Fellowship (sanction no.2/40(37)/2014/R&D-II/14131).

References

[1] D. Baleanu, D. Kumar and S. D. Purohit, Generalized fractional integrals of product of two H-functions and a general class of polynomials, *Int. J. Comput. Math.* **93**(8) (2016), 1320–1329.

[2] H. S. Carslaw and J. C. Jarger, *Conduction of Heat in Solids*, second edition, Oxford University Press, 1959.

[3] R. C. Chandel and Y. K. Singh, Two boundary value problems, *Indian J. Theor. Phys.* **53**(4) (2005), 339–351.

[4] V. B. L. Chaurasia, Application of the multivariable H-function in Heat conduction in a rod under the Robin condition, *Jnanabha* **21** (1991), 77–82.

[5] V. B. L. Chaurasia and N. Gupta, A solution of the partial differential equation of heat conduction in a rod under the robin condition, *Jnanabha* **27** (1997), 31–45.

[6] V. B. L. Chaurasia and Lal, Use of the H-function of several complex variables and a general class of polynomials in heat conduction in a rod under the Robin condition, *Indian J. Pure Appl. Math.* **23**(7) (1992), 489–496.

[7] J. Choi and D. Kumar, Certain unified fractional integrals and derivatives for a product of Aleph function and a general class of multivariable polynomials, *J. Inequ. Appl.* **2014** (2014), 15 pages.

[8] J. Daiya, J. Ram and D. Kumar, The multivariable H-function and the general class of Srivastava polynomials involving the generalized Mellin-Barnes contour integrals, *FILOMAT* **30**(6) (2016), 1457–1464.

[9] B. P. Gautam, A. S. Asgar and A. N. Goyal, On the multivariable A-function, *Vijnana Parishas Anusandhan Patrika* **29**(4) (1986), 67–81.

[10] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, Academic Press, New York, 1996.

[11] D. Kumar, Fractional calculus formulas involving H-function and Srivastava polynomials, *Commun. Korean Math. Soc.* **31**(4) (2016), 827–844.

[12] D. Kumar, P. Agarwal and S. D. Purohit, Generalized fractional integration of the H-function involving general class of polynomials, *Walailak J. Sci. Technol.* **11**(12) (2014), 1019–1030.

[13] D. Kumar, S. D. Purohit and J. Choi, Generalized fractional integrals involving product of multivariable H-function and a general class of polynomials, *J. Nonlinear Sci. Appl.* **9** (2016), 8–21.

[14] N. N. Lebedev and R. A. Silverman, *Special Functions and Their Applications*, Prentice-Hall, Englewood Cliffs, New Jersey, 1965.

[15] R. K. Saxena and D. Kumar, Generalized fractional calculus of the Aleph-function involving a general class of polynomials, *Acta Math. Sinica* (Chin. Ser.) **35**(5) (2015), 1095-1110.

[16] R. K. Saxena and Y. Singh, A finite series for multivariate A-function, *Vijnana Parishad Anusandhan Patrika* **34** (3) (1991), 139–145.

[17] R.K. Saxena and Y. Singh, An expansion formula for multivariable A-function, *Vijnana Parishad Anusandhan Patrika* **34**(4) (1991), 197–205.

[18] H. M. Srivastava, A multilinear generating function for the Konhauser sets of biorthogonal polynomials suggested by the Laguerre polynomials, *Pacific J. Math.* **117**(1) (1985), 183-191.

[19] H. M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables, *Comment. Math. Univ. St. Paul.* **24** (1975), 119–137.

[20] H. M. Srivastava and R. Panda, Some bilateral generating function for a class of generalized hypergeometric polynomials, *J. Reine Angew Math.* **283/284** (1976), 265–274.

[21] J. L. Yadav and V. K. Srivastava, Application on product of multivariable H-function and generalized general class of polynomials, *Acta Ciencia Indica Math.* **34**(3) (2008), 1167–1174.

Dinesh Kumar

CoA Sumerpur-Pali, Agriculture University of Jodhpur, Jodhpur 342007, Rajasthan, India

E-mail address: dinesh_dino03@yahoo.com; dino.dinesh03@gmail.com

Frédéric Ayant

Collége Jean L'herminier, Allée des Nymphéas, 83500 La Seyne-sur-Mer, France; [2]Six-Fours-les-Plages 83140, Department of Var, France

E-mail address: fredericayant@gmail.com

Junesang Choi

Department of Mathematics, Dongguk University, Gyeongju 38066, Republic of Korea

E-mail address: junesang@mail.dongguk.ac.kr