

A Study on the Development of Mathematical Model of Three-stage Flow Control Valve

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Key Words : Hydraulic, flow control, proportional valve, mechanical feedback, three-stage valve, pilot spool.

Abstract: In this study, the theory of fluid mechanics and dynamics is used to build a mathematical model for a three-stage flow control valve. The significance of the study is that the mathematical model can easily be used to study the effect of different design parameters on the performance of the valve. The geometry of the valve and the properties of the fluid were used in this study to determine the variation in the performance of the valve when varying the magnetic force on the pilot spool. While a linearization technique is not used to solve the developed model, the solution of the mathematical model is found in the time domain by simulation of the equations using a software package. The results indicate that if the developed mathematical model is solved for the different values of magnetic force, the valve behaves linearly; the valve is thus called the proportional flow control valve.

Nomenclature

P : Main poppet (subscript)

SS : Sequence spool (subscript)

SP : Pilot spool (subscript)

io : A point shown in figure 6 (subscript)

$A_{ss,h}$: Area of sequence spool head side

$A_{ss,r}$: Area of sequence spool rod side

A_{00} : Area between chamber 'i' and point 'io'

A_0 : Flow area between 'io' and 'p,t'

A_{pe} : Poppet area in contact with chamber 'e'

A_{p0} : Outer cross-sectional area of poppet

A_{s0} : Area of outer diameter of the spool

A_{si} : Area of inner diameter of the spool

ρ : Density of the fluid

ν : Kinematic viscosity of the fluid

β : Bulk modulus of the fluid

$k_{spr,ss}$: Sequence spool spring constant

$k_{spr,sp}$: Spring constant of solenoid spring

P_s : Supply pressure

P_x : Pilot pressure

P_y : Pilot drain line pressure

$V_{chamber}$: Volume of the 'chamber'

m_p : Mass of the poppet

m_{ss} : Mass of the sequence spool

m_{sp} : Mass of pilot spool

Q_{12} : Flow rate from point '1' to point '2'

Q_e : The same as Q_{ie}

Other parameters are shown in figure 2.

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1. Introduction

Hydraulic power transmission units have high power to weight ratio [1] making them suitable for applications. In hydraulic units, valves control system dynamics. Purpose of valves is to allow required pressure or flow rate into actuator. In this study, a hydraulic valve is modeled using fluid mechanics and dynamics without linearization.

The valve chosen is a three-stage valve with a spring type mechanical feedback mechanism. It is a proportional flow control valve.

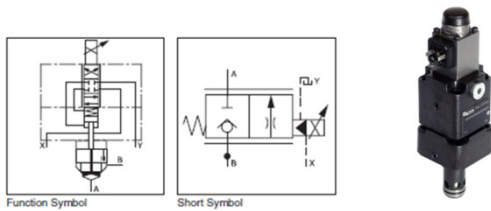


Fig. 1 Valve used in this study

Studies have been conducted on different combination of valves to find out which combination consumes the least amount of energy for a specific amount of power output. For example the studies conducted in [2], [3] and [4] contain discussions about independent metering valves. Another type of combination is called the main control valves as studied in [5] and [6]. All these studies involve modeling of valves in different arrangements.

Mathematical models are very helpful in discussing the stability of valves or the dependence of the system performance on different conditions. The studies conducted in [7] and [8] are a direct reflections of this fact. The parameters in a mathematical model can be varied for different geometry and its effects can be seen in the performance results of the valve model. The job of the simulation software is to solve the equations of the mathematical model simultaneously.

2. Working of the Valve

Valve schematic is shown in figure 2.

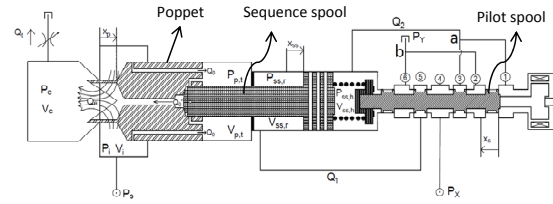


Fig. 2 Schematic of Valve used for Modeling

As shown in figure 2, the valve consists of three stages:

1. Solenoid actuator, made of solenoid and pilot spool
2. Sequence spool along with the sequence spool feedback spring
3. Main poppet or simply poppet

2.1 When Solenoid is not activated

When the solenoid is not activated, the main poppet and the sequence spool are to their extreme left but there is pre-compression in the sequence spool spring due to the valve geometry which keeps the pilot spool to the right side. Therefore, the pilot pressure, P_x , is connected to the chamber on the right side of the sequence spool while the left side which is the rod side of the sequence spool is connected to the tank line P_y . This pressure difference will keep the sequence spool at the left side hence achieving closure of the valve.

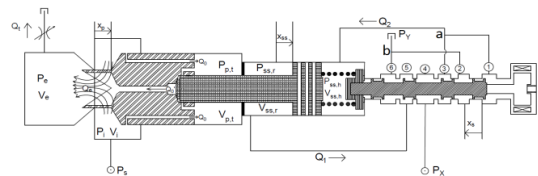


Fig. 3 Closed condition (solenoid not activated)

2.2 When Solenoid is activated

When the solenoid current is turned on, a magnetic force (F_{mag}) will push the pilot spool against the sequence spool spring leading the pilot spool to the left side allowing high pressure at the sequence spool rod side and low pressure at the head side. So, the sequence spool will move rightwards until the feedback spring force matches the magnetic force on the pilot spool. Now as the sequence spool moves rightward the main poppet will also move towards right thereby opening the opportunity for the fluid at the valve inlet to flow to the exit of the valve.

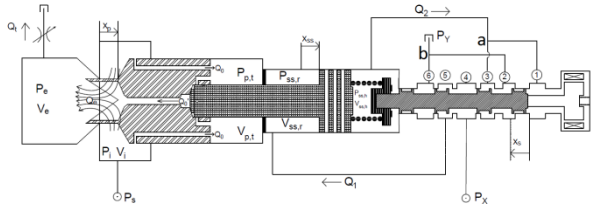


Fig. 4 Solenoid is activated (pilot spool towards left)

Now, if the solenoid current is turned off, the feedback spring force will shift the pilot spool to the right side hence shifting the valve back to the closed position.

3. Mathematical Modeling

3.1 Simplified Flow Configuration

In order to develop a mathematical model we need to breakdown the valve into a set of flow paths and control volumes as shown in figure 5.

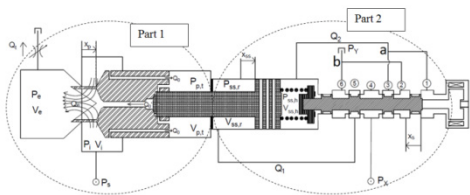


Fig. 5 Valve schematic divided into two parts

As above schematic shows, there are two portions of this valve on the basis of the fluid flow i.e. part 1 and part 2. Part 1 consists of the flow from the source of the pressure P_s to the poppet top side and to the exit of the valve. This flow is separate from the pilot flow which makes the second part of the valve. Part 2 is the part where flow occurs from the pilot source to the pilot drain line represented by encircled dotted line to the right side in figure 5. The flow configuration in this part depends on the position of the pilot spool. So we have one configuration for first portion and two configurations for the second portion:

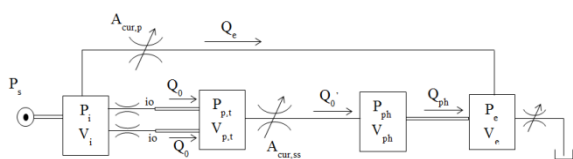


Fig. 6 Flow diagram of part 1

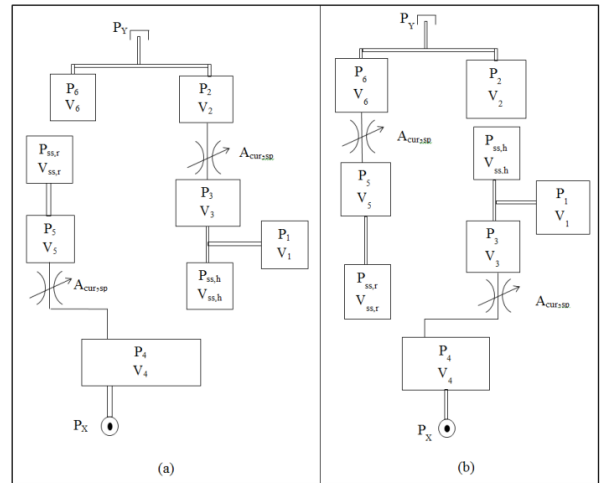


Fig. 7 Part 2 Flow diagram (a: solenoid activated, b: solenoid deactivated)

As can be seen in the figure 6 and figure 7, the flow configuration depends on the position of the pilot spool but also on the value of the flow area of the valve which we call the curtain area. As given in the above figure, the curtain areas are three in number i.e. $A_{cur,p}$, $A_{cur,ss}$ and $A_{cur,sp}$. As will be established in the next section, we will now derive the curtain areas as a function of the displacement of the poppet, sequence spool and pilot spool.

3.2 Curtain Area Calculations

Using the geometry of the valve the relationship of curtain areas with the respective displacements was calculated.

3.2.1 Poppet Curtain Area

When the main poppet is moved, the flow area from inlet to exit of the valve is shown by the white portion in the below figure.

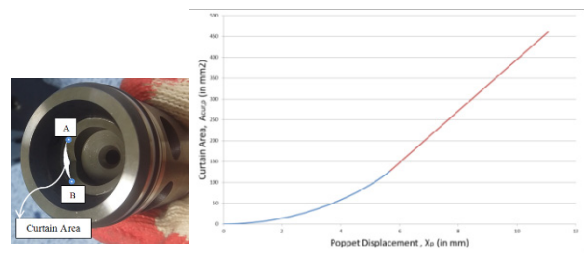


Fig. 8 Curtain area of the poppet and its relation with poppet displacement

$$A_{cur,p} = \begin{cases} 2x_p D_p \sin^{-1} \left(\frac{2\sqrt{3}x_p}{D_p} \right) + \frac{D_p^2}{\sqrt{3}} \left[\cos \left[\sin^{-1} \left(\frac{2\sqrt{3}x_p}{D_p} \right) \right] - 1 \right] & x_p \leq 5.68mm \\ 127.9 mm^2 + (x_p - 5.69mm)\pi D_p & x_p > 5.68mm \end{cases} \dots (i)$$

3.2.2 Sequence Spool Curtain Area

When the sequence spool is moved relative to the poppet, the flow area from poppet top side to poppet hole is given in this section.

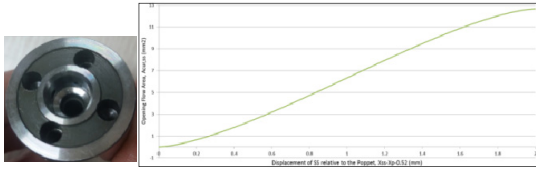


Fig. 9 Sequence spool curtain area of the poppet and its relation with SS displacement relative to poppet

$$A_{cur,ss} = -0.596(X_{ss} - X_p - 0.52)^5 + 3.0131(X_{ss} - X_p - 0.52)^4 - 7.1978(X_{ss} - X_p - 0.52)^3 + 9.4526(X_{ss} - X_p - 0.52)^2 + 1.6636(X_{ss} - X_p - 0.52) \dots (ii)$$

3.2.3 Pilot Spool Curtain Area

When the pilot spool is moved left or right from the middle position, pilot flow area is opened as given here.

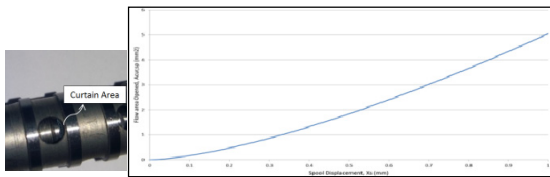


Fig. 10 Pilot spool curtain area created by spool motion

$$A_{cur,spool} = -2.7565X_s^5 + 8.4106X_s^4 - 10.685X_s^3 + 9.2088X_s^2 + 0.874X_s \dots (iii)$$

3.3 Derivation of Equations

3.3.1 Types of Equations and Modeling Approach

In order to develop the mathematical model, we can use three types of equations which are:

1. Continuity equations applied to each control volume
2. Relationship of pressure drop between two points

and the flow rate between them

3. Force balance equations or dynamic equations for poppet, sequence spool and the pilot spool

Our procedure starts with deriving continuity equations for each control volume. All the terms which include flow rate are eliminated by pressure drops in these equations.

3.3.2 Modeling of Part 1 by Employing Type 1 and Type 2 Equations

As shown in figure 5, part 1 refers to the portion shown below:

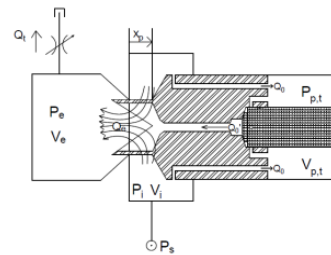


Fig. 11 Part 1 of valve (fluid flow diagram is in figure 6)

Applying continuity equation along with stress-strain relationship to chamber i in figure 11 above, we have:

$$Q_{si} = \frac{V_i}{\beta} \frac{dP_i}{dt} + 2Q_0 + Q_e + (A_{p0} - A_{pe}) \dot{x}_p$$

In this equation the three terms of flow rate are:

$$Q_{si} = K_1(P_s - P_i) \text{ (Darcy equation)}$$

$$Q_0 = C_{d1} A_{00} \sqrt{\frac{2(P_i - P_0)}{\rho}}$$

$$Q_e = C_{d2} A_{cur,p} \sqrt{\frac{2(P_i - P_e)}{\rho}}$$

Replacing the flow rate terms, we get:

$$K_1(P_s - P_i) = \frac{V_i}{\beta} \frac{dP_i}{dt} + 2C_{d1} A_{00} \sqrt{\frac{2(P_i - P_0)}{\rho}} + C_{d2} A_{cur,p} \sqrt{\frac{2(P_i - P_e)}{\rho}} + (A_{p0} - A_{pe}) \dot{x}_p \dots (1)$$

Similarly, equations for other control volumes follow:

$$K_2(P_{i0} - P_{p,t}) = C_{d1} A_{00} \sqrt{\frac{2(P_i - P_{i0})}{\rho}} \dots (2)$$

$$2K_2(P_{i0} - P_{p,t}) = \frac{V_{p,t}}{\beta} \frac{dP_{p,t}}{dt} + C_{d3}A_{cur,ss} \sqrt{\frac{2(P_{p,t} - P_{ph})}{\rho}} - (A_{p0} - A_{ss,r})\dot{x}_p \dots\dots\dots(3)$$

$$C_{d3}A_{cur,ss} \sqrt{\frac{2(P_{p,t} - P_{ph})}{\rho}} = \frac{V_{ph}}{\beta} \frac{dP_{ph}}{dt} + K_3(P_{ph} - P_e) \dots\dots\dots(4)$$

$$C_{d2}A_{cur,p} \sqrt{\frac{2(P_i - P_e)}{\rho}} + K_3(P_{ph} - P_e) = \frac{V_e}{\beta} \frac{dP_e}{dt} + C_{d4}A_{et} \sqrt{\frac{2(P_e - P_t)}{\rho}} \dots\dots\dots(5)$$

3.3.3 Modeling of Part 2 by Employing Type 1 and Type 2 Equations

Now we deal with part 2 of the valve which includes equations which change with the spool position. So, we have two cases i.e. pilot spool moved leftward and rightward.

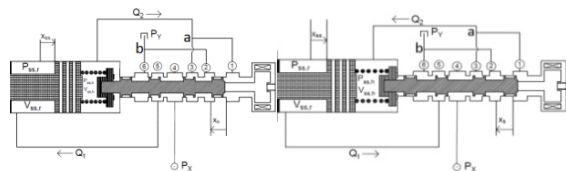


Fig. 12 Part 2 when spool moves left and right

For chamber 4:

When spool is on the left:

$$Q_{x4} = \frac{V_4}{\beta} \frac{dP_4}{dt} + Q_{43}$$

$$Q_{x4} = K_4(P_x - P_4)$$

$$Q_{43} = C_{d5}A_{cur,sp} \sqrt{\frac{2(P_4 - P_3)}{\rho}}$$

$$K_4(P_x - P_4) = \frac{V_4}{\beta} \frac{dP_4}{dt} + C_{d5}A_{cur,sp} \sqrt{\frac{2(P_4 - P_3)}{\rho}} \dots\dots\dots(6L)$$

When spool is on the right:

$$Q_{x4} = \frac{V_4}{\beta} \frac{dP_4}{dt} + Q_{43}$$

$$Q_{43} = C_{d5}A_{cur,sp} \sqrt{\frac{2(P_4 - P_3)}{\rho}}$$

$$K_4(P_x - P_4) = \frac{V_4}{\beta} \frac{dP_4}{dt} + C_{d5}A_{cur,sp} \sqrt{\frac{2(P_4 - P_3)}{\rho}} \dots\dots\dots(6R)$$

Applying the same procedure for other control volumes of part 2, we get below set of equations:

$$C_{d5}A_{cur,sp} \sqrt{\frac{2(P_4 - P_5)}{\rho}} = \frac{V_5}{\beta} \frac{dP_5}{dt} + K_5(P_5 - P_{ss,r}) \dots\dots\dots(7L)$$

$$K_5(P_{ss,r} - P_5) = \frac{V_5}{\beta} \frac{dP_5}{dt} + C_{d5}A_{cur,sp} \sqrt{\frac{2(P_5 - P_6)}{\rho}} \dots\dots\dots(7R)$$

$$K_5(P_5 - P_{ss,r}) = \frac{V_{ss,r}}{\beta} \frac{dP_{ss,r}}{dt} + (A_{ss,h} - A_{ss,r})\dot{x}_{ss} \dots\dots\dots(8L)$$

$$-(A_{ss,h} - A_{ss,r})\dot{x}_{ss} = \frac{V_{ss,r}}{\beta} \frac{dP_{ss,r}}{dt} + K_5(P_{ss,r} - P_5) \dots\dots\dots(8R)$$

$$(A_{ss,h})\dot{x}_{ss} = \frac{V_{ss,h}}{\beta} \frac{dP_{ss,h}}{dt} + K_6(P_{ss,h} - P_a) \dots\dots\dots(9L)$$

$$K_6(P_a - P_{ss,h}) = \frac{V_{ss,h}}{\beta} \frac{dP_{ss,h}}{dt} + (A_{ss,h})\dot{x}_{ss} \dots\dots\dots(9R)$$

$$K_6(P_{ss,h} - P_a) = K_7(P_a - P_1) + K_8(P_a - P_3) \dots\dots\dots(10L)$$

$$K_8(P_3 - P_a) = K_6(P_a - P_{ss,h}) + K_7(P_a - P_1) \dots\dots\dots(10R)$$

$$K_7(P_a - P_1) = \frac{V_1}{\beta} \frac{dP_1}{dt} + \dot{x}_s(A_{so} - A_{si})$$

.....(11L)

$$K_7(P_a - P_1) = \frac{V_1}{\beta} \frac{dP_1}{dt} + \dot{x}_s(A_{so} - A_{si})$$

.....(11R)

$$C_{d5}A_{cur,sp} \sqrt{\frac{2(P_3 - P_2)}{\rho}} = \frac{V_2}{\beta} \frac{dP_2}{dt} + K_9(P_2 - P_b)$$

.....(12L)

$$K_9(P_b - P_2) = \frac{V_2}{\beta} \frac{dP_2}{dt}$$

.....(12R)

$$K_8(P_a - P_3) = \frac{V_3}{\beta} \frac{dP_3}{dt} + C_{d5}A_{cur,sp} \sqrt{\frac{2(P_3 - P_2)}{\rho}}$$

.....(13L)

$$C_{d5}A_{cur,sp} \sqrt{\frac{2(P_4 - P_3)}{\rho}} = \frac{V_3}{\beta} \frac{dP_3}{dt} + K_8(P_3 - P_a)$$

.....(13R)

$$K_9(P_2 - P_b) = K_{10}(P_b - P_y) + K_{11}(P_b - P_6)$$

.....(14L)

$$K_{11}(P_6 - P_b) = K_9(P_b - P_2) + K_{10}(P_b - P_y)$$

.....(14R)

$$K_{11}(P_b - P_6) = \frac{V_6}{\beta} \frac{dP_6}{dt}$$

.....(15L)

$$C_{d5}A_{cur,sp} \sqrt{\frac{2(P_5 - P_6)}{\rho}} = \frac{V_6}{\beta} \frac{dP_6}{dt} + K_{11}(P_6 - P_b)$$

.....(15R)

3.3.4 Type 3 Equations for Poppet, Sequence Spool and Pilot Spool

There are three moving masses in the valve,

1. Main poppet or poppet (m_p)

2. Sequence spool (m_{ss})

3. Pilot spool or spool (m_{sp})

Now we apply dynamic equations as below:

For the poppet:

$$P_e(A_{pe} - A_{ph}) + P_i(A_{po} - A_{pe}) - P_{p,t}(A_{po} - A_{ss,r}) - P_{ph}(A_{ss,r} - A_{ph}) - 2 \frac{\rho[K_2(P_{io} - P_{p,t})]^2}{A_0} - \frac{\rho[K_3(P_{ph} - P_e)]^2}{A_{ph}} + \frac{2(P_i - P_e)[C_{d2}A_{cur,p}]^2}{A_{pe}} - C_{d1}\dot{x}_p = m_p\ddot{x}_p$$

.....(16)

For the Sequence Spool:

$$P_{ss,r}(A_{ss,h} - A_{ss,r}) + P_{ph}A_{ss,r} - P_{ss,h}A_{ss,h} - F_{spr,ss} - C_{d2}\dot{x}_{ss} = m_{ss}\ddot{x}_{ss}$$

The spring force was derived from experiments and geometry of the valve as below:

$$F_{spr,ss} = 4070(x_s + x_{ss}) + 13.3694N$$

$$P_{ss,r}(A_{ss,h} - A_{ss,r}) + P_{ph}A_{ss,r} - P_{ss,h}A_{ss,h} - [4070(x_s + x_{ss}) + 13.3694N] - C_{d2}\dot{x}_{ss} = m_{ss}\ddot{x}_{ss}$$

.....(17)

For the pilot spool:

$$-F_{spr,ss} - P_{ss,h}A_{so} + F_{sol,mag} + F_{spr,sol} - C_{D3}\dot{x}_s + P_1A_{so} = m_s\ddot{x}_s$$

After testing the solenoid spring for spring constant, we get:

$$F_{spr,sol} = -4920x_s + 3.444N$$

So, we get:

$$-[4070(x_s + x_{ss}) + 13.3694N] - P_{ss,h}A_{so} + F_{sol,mag} + [-4920x_s + 3.444N] - C_{D3}\dot{x}_s + P_1A_{so} = m_s\ddot{x}_s$$

.....(18)

4. Results and Discussion

We use a software package to solve the model for specific supply pressure, P_s , the pilot pressure, P_x and the tank pressure, P_t . The tank pressure, P_t , can be replaced with a load pressure. Parameters related to valve geometry were obtained by taking measurements of the valve.

Laminar and steady state flow conditions were assumed.

Changing magnetic force, the output displacement of poppet, x_p , the displacement of sequence spool, X_{ss} and displacement of pilot spool, X_{sp} was obtained.

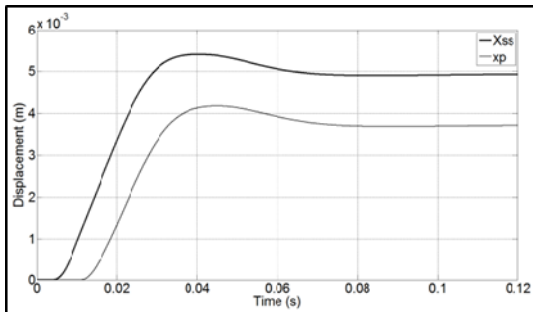


Fig. 13 When $P_x = 10\text{bar}$ ($F_{mag} = 30\text{ N}$ and $P_s = 10\text{ bar}$)

In figure 13, we can see that the poppet displacement lags behind the sequence spool. This is because the sequence spool movement does not allow the sequence spool curtain area to be opened instantaneously. Instead, the flow area starts to open after the sequence spool is moved by 0.52mm which is attributed to the geometry of the valve. Even after 0.52 mm, the poppet is not moved abruptly because it takes more sequence spool displacement for the pressure difference across the poppet to be sufficient to move the poppet to the right side (referring to figure 2).

The pilot spool response attains the middle position because of sequence spool spring feedback mechanism. This phenomenon can be observed in figure 14 below:

The negative values in figure 14 mean that the pilot spool is to the right side of the middle position and positive valve means the left side. As can be seen, the displacement reaches the middle position after some time which goes to show the effect of the sequence spool spring as a mechanical feedback mechanism.

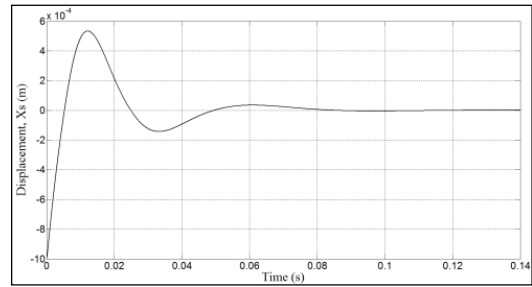


Fig. 14 Pilot spool response when magnetic force of 25 N is applied with $C_D = 100\text{ Ns/m}$, $P_s = 10\text{ bar}$ and $P_x = 50\text{ bar}$

Now let's find out how the steady values are affected by the change in magnetic force. This phenomenon is shown in figure 15 below:

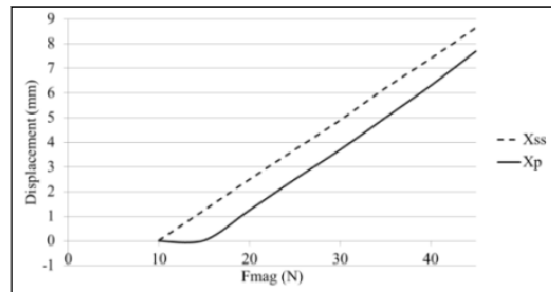


Fig. 15 Displacement of poppet and sequence spool with magnetic force

The poppet and sequence spool behave linearly with the change in magnetic force which means that the valve can be called a proportional valve. This relationship is not affected by changing the supply pressure, the pilot pressure or the friction coefficient.

The steady state value of flow rate, however, depends on the value of supply pressure as well as the magnetic force as shown in figure 16.

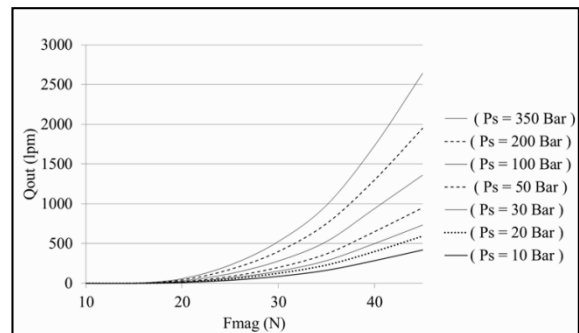


Fig. 16 Steady state flow rate response for different values of supply pressure

5. Conclusion

As shown in the previous section, the valve properties are determined by the conditions of the valve operation and the geometrical details of different parts of the valve.

The steady state poppet displacement increases linearly with increasing the magnetic force as shown in figure 15. It is important because flow rate is affected by this behavior.

Pressure or flow rate shocks can be reduced by designing the curtain area in a particular fashion. One example of this phenomenon can be found in this study. As shown in figure 16, the flow area changes slowly at the start, however, it starts to increase quickly as the poppet is moved further. Due to this behavior we see the flow rate increasing in the same fashion as shown in figure 16.

The work done in this study can be extended to use the developed mathematical model to find other properties of the valve like response time and its dependence on the geometry or operating parameters.

Acknowledgement

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