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HYERS-ULAM STABILITY OF DERIVATIONS IN FUZZY BANACH SPACE: REVISITED

Gang Lu [a], Yuanfeng Jin [b], Gang Wu [c] and Sungsik Yun [d,*]

Abstract. Lu et al. [27] defined derivations on fuzzy Banach spaces and fuzzy Lie Banach spaces and proved the Hyers-Ulam stability of derivations on fuzzy Banach spaces and fuzzy Lie Banach spaces.

It is easy to show that the definitions of derivations on fuzzy Banach spaces and fuzzy Lie Banach spaces are wrong and so the results of [27] are wrong. Moreover, there are a lot of seroius problems in the statements and the proofs of the results in Sections 2 and 3.

In this paper, we correct the definitions of biderivations on fuzzy Banach algebras and fuzzy Lie Banach algebras and the statements of the results in [27], and prove the corrected theorems.

1. Introduction and Preliminaries

The stability problem of functional equations originated from a question of Ulam [36] concerning the stability of group homomorphisms. Hyers [20] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' theorem was generalized by Aoki [2] for additive mappings and by Rassias [33] for linear mappings by considering an unbounded Cauchy difference. Those results have been recently complemented in [7]. A generalization of the Aoki and Rassias theorem was obtained by Găvruta [19], who used a more general function controlling the possibly unbounded Cauchy difference in the spirit of Rassias' approach. The stability problems for several functional equations or inequalities have been extensively investigated by a number of authors and there are many interesting results concerning this problem (see [11, 12, 13], [21]–[26], [30]–[32]).

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*Corresponding author.

We recall a fundamental result in fixed point theory.

Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a *generalized metric* on X if d satisfies

(1) $d(x, y) = 0$ if and only if $x = y$;
(2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(3) $d(x, z) \le d(x, y) + d(y, z)$ for all $x, y, z \in X$.

Theorem 1.1 (see [10, 16]). *Let (X, d) be a complete generalized metric space and let $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then for each given element $x \in X$, either*

$$d(J^n x, J^{n+1} x) = \infty$$

for all nonnegative integers n or there exists a positive integer n_0 such that

(1) *$d(J^n x, J^{n+1} x) < \infty$, for all $n \ge n_0$;*
(2) *the sequence $\{J^n x\}$ converges to a fixed point y^* of J;*
(3) *y^* is the unique fixed point of J in the set $Y = \{y \in X | d(J^{n_0} x, y) < \infty\}$;*
(4) *$d(y, y^*) \le \frac{1}{1-L} d(y, Jy)$ for all $y \in Y$.*

By using the fixed point method, the stability problems of several functional equations have been extensively investigated by a number of authors (see [1, 8, 9, 10, 15, 17, 23, 29]).

In 1984, Katsaras [22] defined a fuzzy norm on a linear space and at the same year Wu and Fang [37] also introduced a notion of fuzzy normed space and gave the generalization of the Kolmogoroff normalized theorem for fuzzy topological linear space. In [6], Biswas defined and studied fuzzy inner product spaces in linear space. Since then some mathematicians have defined fuzzy metrics and norms on a linear space from various points of view [5, 18, 25, 35, 38]. In 1994, Cheng and Mordeson introduced a definition of fuzzy norm on a linear space in such a manner that the corresponding induced fuzzy metric is of Kramosil and Michalek type [24]. In 2003, Bag and Samanta [5] and Saadati and Vaezpour [34] modified the definition of Cheng and Mordeson [14] by removing a regular condition. They also established a decomposition theorem of a fuzzy norm into a family of crisp norms and investigated some properties of fuzzy norms (see [4]). Following [3], we give the employing notion of a fuzzy norm.

Let X be a real linear space. A function $N : X \times \mathbb{R} \to [0, 1]$ is said to be a fuzzy norm on X if for all $x, y \in X$ and all $a, b \in \mathbb{R}$:

(N_1) $N(x,a) = 0$ for $a \leq 0$;
(N_2) $x = 0$ if and only if $N(x,a) = 1$ for all $a > 0$;
(N_3) $N(ax,b) = N(x, \frac{b}{|a|})$ if $a \neq 0$;
(N_4) $N(x+y, a+b) \geq min\{N(x,a), N(y,b)\}$;
(N_5) $N(x,.)$ is a non-decreasing function on $\mathbb{R}$ and $\lim_{a\to\infty} N(x,a) = 1$;
(N_6) For $x \neq 0$, $N(x,.)$ is (upper semi) continuous on $\mathbb{R}$.
The pair (X,N) is called a fuzzy normed linear space. One may regard $N(x,a)$ as the truth value of the statement the norm of x is less than or equal to the real number a'.

Let $(X, \|.\|)$ be a normed space. Define

$$N(x,a) = \begin{cases} \frac{a}{a+\|x\|}, & a > 0, x \in X, \\ 0, & a \leq 0, x \in X. \end{cases}$$

Then (X,N) is called the induced fuzzy normed space.

Definition 1.2. Let (X,N) be a fuzzy normed linear space. Let x_n be a sequence in X. Then x_n is said to be convergent if there exists $x \in X$ such that $\lim_{n\to\infty} N(x_n - x, a) = 1$ for all $a > 0$. In that case, x is called the limit of the sequence x_n and we denote it by $N\text{-}lim_{n\to\infty}\, x_n = x$.

Definition 1.3. A sequence x_n in X is called *Cauchy* if for each $\epsilon > 0$ and each $a > 0$ there exists n_0 such that for all $n \geq n_0$ and all $p > 0$, we have $N(x_{n+p} - x_n, a) > 1 - \epsilon$.

It is known that every convergent sequence in fuzzy normed space is Cauchy. If each Cauchy sequence is convergent, then the fuzzy norm is said to be complete and the fuzzy normed space is called a *fuzzy Banach space.*

Definition 1.4 ([31])**.** Let A be an algebra and (A,N) a fuzzy normed space.

(1) The fuzzy normed space (A,N) is called a *fuzzy normed algebra* if

$$N(xy, st) \geq N(x,s) \cdot N(y,t)$$

holds for all $x, y \in A$ and all positive real numbers s and t.

(2) A complete fuzzy normed algebra is called a *fuzzy Banach algebra.*

Definition 1.5 ([31])**.** Let (A,N) be a fuzzy Banach algebra. Then an additive mapping $f : A \to A$ is called a *derivation* if

$$f(xy) = f(x)y + xf(y)$$

holds for all $x, y \in A$.

2. Stability of Derivations on Fuzzy Banach Algebras

Throughout this section, assume that (A, N) is a fuzzy Banach algebra. For any mapping $f : A \to A$, we define

$$Df(x, y, z) := f(2x - y - z) + f(x - z) + f(x + y + 2z) - f(4x)$$

for all $x, y, z \in A$.

Firstly, we prove that $Df(x, y, z) = 0$ implies the additivity of f.

Lemma 2.1. *Let (A, N) be a fuzzy normed vector space and $f : A \to A$ be a mapping such that*

$$N\left(f\left(2x - y - z\right) + f\left(x - z\right) + f\left(x + y + 2z\right), t\right) \geq N\left(f\left(4x\right), \frac{t}{2}\right) \tag{2.1}$$

for all $x, y, z \in A$ and all $t > 0$. Then the mapping $f : A \to A$ is additive.

Proof. Letting $x = y = z = 0$ in (2.1), we get

$$N(3f(0), t) = N\left(f(0), \frac{t}{3}\right) \geq N\left(f(0), \frac{t}{2}\right)$$

for all $t > 0$. By (N_5) and (N_6), $N(f(0), t) = 1$ for all $t > 0$. It follows from (N_2) that $f(0) = 0$.

Letting $x = z = 0$ in (2.1), we get

$$N(f(y) + f(-y) + f(0), t) \geq N\left(f(0), \frac{t}{2}\right) = 1$$

for all $t > 0$. It follows from (N_2) that $f(-y) + f(y) = 0$ for all $y \in A$. Thus

$$f(-y) = -f(y)$$

for all $y \in A$.

Letting $x = 0$ and $l = y + z$ in (2.1), we get

$$N(f(-l) + f(-z) + f(l + z), t) \geq N\left(f(0), \frac{t}{2}\right) = 1$$

for all $t > 0$. It follows from (N_2) that

$$f(-l) + f(-z) + f(l + z) = 0$$

for all $l, z \in A$. Thus

$$f(l + z) = f(l) + f(z)$$

for all $l, z \in A$, as desired. $\square$

Now we investigate the Hyers-Ulam stability of derivations on fuzzy Banach algebras for the functional equation

$$Df(x,y,z)=0$$

for all $x,y,z\in A$.

Theorem 2.2. *Let $\phi : A^3 \to [0,1]$ be a function such that there exists an $L<\frac{1}{2}$ with*

$$\phi\left(\frac{x}{2},\frac{y}{2},\frac{z}{2}\right)\le \frac{L}{2}\phi(x,y,z) \tag{2.2}$$

for all $x,y,z\in A$. Let $f : A\to A$ be a mapping such that

$$N\left(Df(x,y,z),t\right)\ge \frac{t}{t+\phi(x,y,z)}, \tag{2.3}$$

$$N(f(xy)-f(x)y-xf(y),t)\ge \frac{t}{t+\phi(x,y,0)} \tag{2.4}$$

for all $x,y,z\in A$ and all $t>0$. Then there exists a unique fuzzy derivation $\delta : A\to A$ such that

$$N(f(x)-\delta(x),t)\ge \frac{4(1-L)t}{4(1-L)t+L^2\phi\left(x,-x,x\right)} \tag{2.5}$$

for all $x\in A$ and all $t>0$.

Proof. Letting $y=-x$, $z=x$ in (2.4), we have

$$N\left(2f\left(2x\right)-f(4x),t\right)\ge \frac{t}{t+\phi\left(x,-x,x\right)} \tag{2.6}$$

and so

$$N\left(2f\left(\frac{x}{2}\right)-f(x),t\right)\ge \frac{t}{t+\phi\left(\frac{x}{4},-\frac{x}{4},\frac{x}{4}\right)}\ge \frac{t}{t+\frac{L^2}{4}\phi\left(x,-x,x\right)}$$

for all $x\in A$. Thus

$$N\left(2f\left(\frac{x}{2}\right)-f(x),\frac{L^2}{4}t\right)\ge \frac{\frac{L^2}{4}t}{\frac{L^2}{4}t+\frac{L^2}{4}\phi\left(x,-x,x\right)}=\frac{t}{t+\phi\left(x,-x,x\right)} \tag{2.7}$$

for all $x\in A$.

Consider the set

$$X:=\{g : A\to A\}$$

and introduce the generalized metric on X:

$$d(g,h):=\inf\{a\in\mathbb{R}^+ : N(g(x)-h(x),at)\ge \frac{t}{t+\phi\left(x,-x,x\right)}\}$$

for all $x \in A$ and all $t > 0$, where $a \in (0, \infty)$. It is easy to show that (X, d) is complete (see the proof of [28, Lemma 2.1]).

Now, we consider the linear mapping $Q : X \to X$ such that

$$Qg(x) := 2g\left(\frac{x}{2}\right)$$

for all $x \in A$.

Let $g, h \in X$ be given such that $d(g, h) = \epsilon$. Then

$$N(g(x) - h(x), \epsilon t) \geq \frac{t}{t + \phi(x, -x, x)}$$

for all $x \in A$ and all $t > 0$. Hence

$$\begin{aligned} N(Qg(x) - Qh(x), L\epsilon t) &= N\left(2g\left(\frac{x}{2}\right) - 2h\left(\frac{x}{2}\right), L\epsilon t\right) = N\left(g\left(\frac{x}{2}\right) - h\left(\frac{x}{2}\right), \frac{L}{2}\epsilon t\right) \\ &\geq \frac{\frac{Lt}{2}}{\frac{Lt}{2} + \phi\left(\frac{x}{2}, -\frac{x}{2}, \frac{x}{2}\right)} \geq \frac{\frac{Lt}{2}}{\frac{Lt}{2} + \frac{L}{2}\phi(x, -x, x)} \\ &= \frac{t}{t + \phi(x, -x, x)} \end{aligned}$$

for all $x \in A$ and all $t > 0$. Thus $d(g, h) = \epsilon$ implies that $d(Qg, Qh) \leq L\epsilon$. This means that

$$d(Qg, Qh) \leq Ld(g, h)$$

for all $g, h \in X$.

It follows from (2.7) that $d(f, Qf) \leq \frac{L^2}{4}$.

By Theorem 1.1, there exists a mapping $\delta : A \to A$ satisfying the following:

(1) δ is a fixed point of Q, *i.e.*,

$$\delta\left(\frac{x}{2}\right) = \frac{1}{2}\delta(x) \tag{2.8}$$

for all $x \in A$. The mapping δ is a unique fixed point of Q in the set

$$M = \{g \in G : d(f, g) < \infty\}.$$

This implies that δ is a unique mapping satisfying (2.8) such that there exists an $a \in (0, \infty)$ satisfying

$$N(f(x) - \delta(x), at) \geq \frac{t}{t + \phi(x, -x, x)}$$

for all $x \in A$ and $t > 0$.

(2) $d(Q^k f, \delta) \to 0$ as $k \to \infty$. This implies the equality

$$N - \lim_{k \to \infty} 2^k f\left(\frac{x}{2^k}\right) = \delta(x)$$

for all $x \in A$;

(3) $d(f,\delta) \le \frac{1}{1-L} d(f, Qf)$, which implies the inequality

$$d(f, A) \le \frac{L^2}{4(1-L)}.$$

This implies that the inequality (2.5) holds.

Next we show that δ is additive. It follows from (2.2) that

$$\begin{aligned}\sum_{k=0}^{\infty} 2^k \phi\left(\frac{x}{2^k}, \frac{y}{2^k}, \frac{z}{2^k}\right) &= \phi(x,y,z) + 2\phi\left(\frac{x}{2}, \frac{y}{2}, \frac{z}{2}\right) + 2^2\phi\left(\frac{x}{2^2}, \frac{y}{2^2}, \frac{z}{2^2}\right) + \cdots \\ &\le \phi(x,y,z) + L\phi(x,y,z) + L^2\phi(x,y,z) + \cdots \\ &= \frac{1}{1-L}\phi(x,y,z) < \infty\end{aligned}$$

for all $x, y, z \in A$.

By (2.3),

$$\begin{aligned}&N\left(2^k f\left(\frac{2x-y-z}{2^k}\right) + 2^k f\left(\frac{x-z}{2^k}\right) + f\left(\frac{x+y+2z}{2^k}\right) - 2^k f\left(\frac{4}{2^k}x\right), 2^k t\right) \\ &\ge \frac{t}{t + \phi\left(\frac{x}{2^k}, \frac{y}{2^k}, \frac{z}{2^k}\right)}\end{aligned}$$

and so

$$\begin{aligned}&N\left(2^k f\left(\frac{2x-y-z}{2^k}\right) + 2^k f\left(\frac{x-z}{2^k}\right) + 2^k f\left(\frac{x+y-2z}{2^k}\right) - 2^k f\left(\frac{4}{2^k}x\right), t\right) \\ &\ge \frac{\frac{t}{2^k}}{\frac{t}{2^k} + \phi\left(\frac{x}{2^k}, \frac{y}{2^k}, \frac{z}{2^k}\right)} = \frac{t}{t + 2^k\phi\left(\frac{x}{2^k} \cdot \frac{y}{2^k}, \frac{z}{2^k}\right)}\end{aligned}$$

for all $x, y, z \in A$ and all $t > 0$. Since $\lim_{k\to\infty} \frac{t}{t+2^k\phi\left(\frac{x}{2^k}, \frac{y}{2^k}, \frac{z}{2^k}\right)} = 1$ for all $x, y, z \in A$ and all $t > 0$,

$$N\left(\delta\left(2x-y-z\right) + \delta\left(x-z\right) + \delta\left(x+y+2z\right) - \delta\left(4x\right), t\right) = 1$$

for all $x, y, z \in A$ and all $t > 0$. So

$$\delta\left(2x-y-z\right) + \delta\left(x-z\right) + \delta\left(x+y+2z\right) = \delta\left(4x\right) \tag{2.9}$$

for all $x, y, z \in A$ and all $t > 0$. By Lemma 2.1, $\delta : A \to A$ is an additive mapping.

It follows from (2.4) that

$$\begin{aligned}
&N\left(2^{2k}f\left(\frac{xy}{2^{2k}}\right)-y\cdot 2^{k}f\left(\frac{x}{2^{k}}\right)-x\cdot 2^{k}f\left(\frac{y}{2^{k}}\right),t\right)\\
&=N\left(f\left(\frac{xy}{2^{2k}}\right)-\frac{y}{2^{k}}f\left(\frac{x}{2^{k}}\right)-\frac{x}{2^{k}}f\left(\frac{y}{2^{k}}\right),\frac{t}{2^{2k}}\right)\\
&\geq \frac{\frac{t}{2^{2k}}}{\frac{t}{2^{2k}}+\phi\left(\frac{x}{2^{k}},\frac{y}{2^{k}},0\right)}=\frac{t}{t+2^{2k}\phi\left(\frac{x}{2^{k}},\frac{y}{2k},0\right)}\geq\frac{t}{t+(2L)^{k}\phi\left(x,y,0\right)}
\end{aligned}$$

for all $x, y \in A$ and all $t > 0$. Since $\lim_{k\to\infty} \frac{t}{t+(2L)^k\phi(x,y,0)} = 1$ for all $x, y \in A$ and all $t > 0$, we get

$$\delta(xy) = y\delta(x) + x\delta(y)$$

for all $x, y \in A$. □

Corollary 2.3. *Let p be a real number with $p > 2$, $\theta \geq 0$, and (A, N) be a fuzzy Banach algebra with norm $\|\cdot\|$. Let $f : A \to A$ be a mapping satisfying*

$$N\left(Df(x,y,z),t\right) \geq \frac{t}{t+\theta(\|x\|^p+\|y\|^p+\|z\|^p)}, \tag{2.10}$$

$$N(f(xy)-f(x)y-xf(y),t) \geq \frac{t}{t+\theta(\|x\|^p+\|y\|^p)} \tag{2.11}$$

(2.12)

for all $x, y \in A$ and all $t > 0$. Then there exists a unique fuzzy derivation $\delta : A \to A$ satisfying

$$N(f(x)-\delta(x),t) \geq \frac{(2^p-2)t}{(2^p-2)t+3\theta\|x\|^p}$$

for all $x \in A$ and all $t > 0$.

Proof. The proof follows from Theorem 2.2 by taking

$$\phi(x,y,z) = \theta(\|x\|^p+\|y\|^p+\|z\|^p)$$

and $L = 2^{1-p}$. □

3. STABILITY OF DERIVATIONS ON FUZZY LIE BANACH ALGEBRAS

A fuzzy Banach algebra, endowed with the Lie product

$$[x,y] := \frac{xy-yx}{2}$$

on $\mathbb{R}$, is called a fuzzy Lie Banach algebra.

Definition 3.1. Let (A, N) be a fuzzy Lie Banach algebra. An additive mapping $\delta : A \to A$ is called a *fuzzy Lie derivation* if

$$\delta([x, y]) = [\delta(x), y] + [x, \delta(y)]$$

holds for all $x, y \in A$.

In this section, suppose that (A, N) is a fuzzy Lie Banach algebra. We prove the Hyers-Ulam stability of fuzzy Lie derivations on fuzzy Lie Banach algebras for the functional equation

$$Df(x, y, z) = 0.$$

Theorem 3.2. *Let $\phi : A^3 \to [0, \infty)$ be a function such that there exists an $L < \frac{1}{2}$ with*

$$\phi\left(\frac{x}{2}, \frac{y}{2}, \frac{z}{2}\right) \leq \frac{L}{2}\phi(x, y, z)$$

for all $x, y, z \in A$. Let $f : A \to A$ be a mapping satisfying

$$N(Df(x, y, z), t) \geq \frac{t}{t + \phi(x, y, z)}$$

and

$$N(f([x, y]) - [f(x), y] - [x, f(y)], t) \geq \frac{t}{t + \phi(x, y, 0)} \tag{3.1}$$

for all $x, y, z \in A$ and $t > 0$. Then there exists a unique fuzzy Lie derivation $\delta : A \to A$ satisfying

$$N(f(x) - \delta(x), t) \geq \frac{4(1 - L)t}{4(1 - L)t + L^2\phi(x, -x, x)} \tag{3.2}$$

for all $x \in A$ and all $t > 0$.

Proof. By the same reasoning as in the proof of Theorem 2.2, we can define an additive mapping $\delta : A \to A$ given by

$$\delta(x) = \lim_{n\to\infty} 2^n f(\frac{x}{2^n})$$

for all $x \in A$. It follows from (3.1) that

$$\begin{aligned}
&N(\delta([x, y]) - [\delta(x), y] - [x, \delta(y)], t)\\
&= \lim_{n\to\infty} N(f(\frac{1}{2^{2n}}[x, y]) - [f(\frac{x}{2^n}), \frac{y}{2^n}] - [\frac{x}{2^n}, f(\frac{y}{2^n})], \frac{1}{2^{2n}}t)\\
&\geq \lim_{n\to\infty} \frac{t}{t + (2L)^n\phi(x, y, 0)} = 1
\end{aligned}$$

for all $x, y \in A$ and $t > 0$. So

$$\delta([x, y]) = [\delta(x), y] + [x, \delta(y)]$$

for all $x, y \in A$. Thus $\delta : A \to A$ is a fuzzy Lie Banach derivation satisfying (3.2). This completes the proof. $\square$

Corollary 3.3. *Let $\theta \geq 0$ and let p be a positive real number with $p > 2$. Let (A, N) be a fuzzy Lie Banach algebra with norm $\|\cdot\|$. Let $f : A \to A$ be a mapping satisfying*

$$N(Df(x, y, z), t) \geq \frac{t}{t + \theta(\|x\|^p + \|y\|^p + \|z\|^p)}$$

and

$$N(f([x, y]) - [f(x), y] - [x, f(y)], t) \geq \frac{t}{t + \theta(\|x\|^p + \|y\|^p)}$$

for all $x, y, z \in A$ and $t > 0$. Then there exists a unique fuzzy Lie derivation $\delta : A \to A$ such that

$$N(f(x) - \delta(x), t) \geq \frac{(2^p - 2)t}{(2^p - 2)t + 3\theta\|x\|^p}$$

for all $x \in A$ and all $t > 0$.

Proof. The proof follows from Theorem 3.2 by taking

$$\phi(x, y, z) = \theta(\|x\|^p + \|y\|^p + \|z\|^p)$$

and $L = 2^{1-p}$. $\square$

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[a]DEPARTMENT OF MATHEMATICS, ZHEJIANG UNIVERSITY, HANGZHOU 310027, P. R. CHINA; DEPARTMENT OF MATHEMATICS, SCHOOL OF SCIENCE, SHENYANG UNIVERSITY OF TECHNOLOGY, SHENYANG 110178, P. R. CHINA
Email address: lvgang1234@hanmail.net

[b]Department of Mathematics, Yanbian University, Yanji 133001, P. R. China
Email address: yfkim@ybu.edu.cn

[c]Department of Mathematics and Applied Mathematics, Harbin University of Commerce, Harbin 150076, P. R. China
Email address: w3393@163.com

[d]Department of Financial Mathematics, Hanshin University, Gyeonggi-do 18101, Korea
Email address: ssyun@hs.ac.kr