Dynamical Analysis of the Mooring Vessel System Under Surge Excitations

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선박 계류시스템의 종방향 외력하의 비선형 동적거동 해석

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Abstract : *This paper deals with the dynamical analysis of a two-point mooring vessel under surge excitations. The characteristics of nonlinear behaviors are investigated completely including bifurcation and limit cycle according to particular input parameter changes. The strong nonlinearity of the mooring system is mainly caused by linear and cubic terms of restoring force. The numerical simulation is performed based on the fourth order Runge-Kutta algorithm. The bifurcation diagram and several instability phenomena are observed clearly by varying amplitudes as well as frequencies of surge excitations. Stable periodic solutions, called the periodic windows, can be obtained in succession between chaotic clouds of dots in case of frequency $\omega = 0.4$ rad/s. In addition, the chaotic region is unexpectedly increased when external forcing amplitude exceeds 1.0 with the angular frequency of $\omega = 0.7$ rad/s. Compared to the cases for $\omega = 0.4, 0.7$ rad/s, the region of chaotic behavior becomes more fragile than in the case of $\omega = 1.0$ rad/s. Finally, various types of steady states including sub-harmonic motion, limit cycle, and symmetry breaking phenomenon are observed in the two-point mooring system at each parameter value.*

Key Words : *Two-point mooring system, Nonlinear behavior, Bifurcation, Chaos, Limit cycle, Sub-harmonic*

요   약 : 본 연구는 두점식 선박 계류시스템의 종방향 외력에 대한 비선형 동적거동 해석을 수행하였다. 특정 입력 매개변수에 대한 카오스 운동과 한계주기궤도 등의 비선형 거동의 특성을 연구하였다. 주로 비선형복원력은 계류시스템의 강한 비선형성과 동적거동의 다양성을 제공한다. 계의 운동방정식 시뮬레이션에 사용된 수치 적분기는 4차 룽게쿠타법이다. 외력진폭과 주파수를 변화시킬 때 분기 그림과 동적불안정 현상들을 볼 수 있다. 외력의 주파수(진동수)가 0.4 rad/s인 경우 수많은 혼돈상태 점들 사이에 주기창이라 불리는 안정적인 주기해가 관측된다. 주파수가 0.7 rad/s인 경우는 외력진폭이 1.0을 초과할 때 혼돈 영역이 갑자기 증가한다. 주파수가 1.0 rad/s인 경우는 주파수가 0.4 rad/s 및 0.7 rad/s인 경우와 비교해 볼 때, 혼돈 운동이 약화된다. 아울러, 두점식 계류시스템은 각 매개변수에서 준주기 운동, 한계주기궤도, 대칭성의 깨짐과 같은 다양한 정상상태의 궤적이 관측된다.

핵심용어 : 두점식 계류시스템, 비선형 거동, 분기, 카오스(혼돈), 한계주기궤도, 준주기

1. Introduction

Recently, the complex dynamical behaviors of nonlinear system like periodic, aperiodic and chaotic motion have attracted much interests of many researchers in the fields of nonlinear aircraft and vessel motions. Ocean mooring systems including single and multi-point mooring systems exhibit complex dynamical behavior due to nonlinear characteristics of restoring force (Umar et al. 2010). Depending on the different initial conditions, these systems provide complex behaviors including co-existing periodic (harmonic as well as sub-harmonic) and aperiodic (quasi-periodic and chaotic) motions (Gottlieb and Yim, 1997). Therefore, the stability of mooring vessel is governed by strong sensitivity to initial conditions (Ellermann, 2005; Banik and Datta, 2010).

In addition, mechanism of route to chaos is of great interests since they define the ways the system loses stability near a bifurcation point (Belato et al., 2001). A bifurcation represents the

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sudden appearance of qualitatively different solutions for an nonlinear dynamical system as some parameter are varied. Bifurcation analysis is closely related to system stability and more complicated dynamical behaviors such as limit cycle and chaos (Zou and Nagarajaiah, 2015). The sensitivity depending on the initial condition is used to stabilize the chaotic behaviors in periodic orbits (Akhmet and Fen, 2012).

When the two-point mooring system is harmonically excited, it essentially represents the nonlinear Duffing oscillator (Mitra et al., 2017). Even when vessels are excited periodically by external waves, the system responses are ranging from harmonic or sub-harmonic to chaotic motions (Ellermann, 2005). Dynamical behaviors of a mooring vessel system under external excitations have been investigated by several researchers in the past years. With a time integration scheme, Umar and Datta (2003) studied the nonlinear behavior of multi-point moored buoy under the actions of first and second order of wave forces. By using a perturbation technique coupled with Hill's variational equation and Floquet's theory, they also analyzed the stability of a multi-point slack moored buoy under wave and wind forces (Umar et al., 2010). In addition, Ellermann (2005) investigated the dynamics of a moored barge under two different external excitations such as a periodic component and an addictive disturbance modelled as white noises with variable intensity. By employing incremental harmonic balance method with arc-length continuation, Banik and Datta (2010) analyzed the instability of a two-point mooring system in surge direction. Moreover, the stability analysis of surge oscillations of the two-point mooring system under state feedback control with time-delay is studied by Mitra et al. (2017). It is known that the choices of gains and delay values are shown to be highly effective to suppress primary responses.

In this paper, we consider the two-point mooring system (with damped Duffing oscillator) in wave excitations and provide rich nonlinear behaviors, including limit cycles and chaotic oscillations.

2. Mathematical Formulation

2.1 System Description

The mooring model is introduced by Gottlieb and Yim (1992) and is further modelled as a single degree of freedom (surge) nonlinear system under wave excitations by Banik and Datta (2010). As depicted in Fig. 1, we assume that the vessel is constrained to move in one dimension (surge).

Top
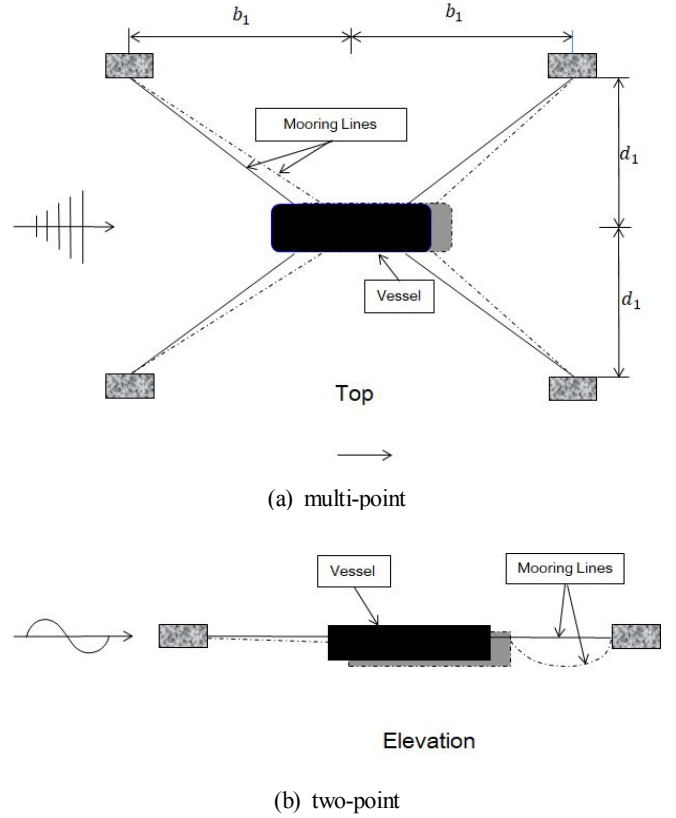
(a) multi-point

Elevation

(b) two-point

Fig. 1. Mooring system models: (a) for multi-point and (b) for two-point.

The differential equation of vessel motion is derived based on equilibrium of geometric restoring force and the body motion under wave and current excitations. The dynamical equation of motion is described by the following classical form:

$$x'' + \delta_1 x' + R(x) = \overline{F}(x'', x', t)/M \qquad (1)$$

where x represent the surge motion (m) with $x' = dx/dt$ (m/s) and $x'' = d^2x/dt^2$ (m/s^2). Specifically, the restoring force $R(x)$ has the form

$$R(x) = \Psi[x + \beta sgn(x)]\left\{\frac{1}{\sqrt{1+\beta^2}} - \frac{1}{\sqrt{1 + [x + \beta sgn(x)]^2}}\right\} \qquad (2)$$

with $\delta_1 = \dfrac{c}{M}$, $\Psi = \dfrac{X}{M + M_a}$, $M_a = \rho V C_a$,

$$\beta = \frac{b_1}{d_1}, \ sgn(x) = \begin{cases} +1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases} \qquad (3)$$

where c is the damping coefficient ($N.s/m$), M is the vessel mass (kg), M_a is the added mass, ρ is the water density, V is the displaced volume of fluid, C_a is the inertia coefficient, $\overline{F}$ represents external excitation force (N), and X is the stiffness (N/m) defined by $X = 2EA\sqrt{d_1^2 + b_1^2}$ (where EA is elastic cable force) and $sgn(x)$ is the signum function. The nonlinear relationship between the restoring force and displacement depends on the values of non-dimensional parameter $\beta = b_1/d_1$. For $\beta = 0\,(b_1 = 0)$, the mooring model represents a strongly nonlinear two-point system (Banik and Datta, 2010; King and Yim, 2007).

The restoring force include a strong geometric nonlinearity depending on the mooring angles. The restoring force assumes linear elastic behavior so that the nonlinearity is strictly due to the geometric configuration of the system (King and Yim, 2007). The nonlinear relationship given by equation (2) is sourced from Gottlieb and Yim (1992). The restoring force is approximated by a least square representation:

$$R(x) = \sum \Psi_n x^n, n = 1, 3, .., N \tag{4}$$

where the coefficients Ψ_n are functions of exciting frequency (Gottlieb, 1991). The governing system nonlinearity ($N = 3$) is mainly cubic (x^3) term ignoring quadratic term. Polynomials of various orders have been employed and optimum fit within experimental range. In practice, the restoring force using linear and cubic polynomial form can be expressed as:

$$R(x) = \mu_1 x + \mu_2 x^3 \tag{5}$$

where μ_1 represents the linear stiffness; μ_2 controls the amount of nonlinearity in the restoring force.

The two-point mooring system has strong nonlinearity due to the relatively large coefficient of the cubic term (Banik and Datta, 2010). As described in equation (1), any forces applied to the dynamic model are scaled with the vessel mass or $F = \dfrac{\overline{F}}{M}$ (N/kg). For simplicity, but without loss of generality, we assume that the vessel M has just unit mass. Then the equation of motion of the two-point mooring system can be written as follows:

$$x'' + \delta_1 x' + R(x) = F sin \omega t \tag{6}$$

where $x(t)$ is the displacement at time t, x' is the first derivative of x with respect to time t, i.e., velocity, and x'' is the second time derivative of x, i.e., acceleration, and the excitation forcing function of $F sin \omega t$ has the period of $T = 2\pi/\omega$ ($s/cycle$) and the frequency (Hz) of $f = 1/T$. By introducing the non-dimensional form $\tau = \omega t$ (rad), the equation of motion is now transformed to

$$\omega^2 \ddot{x} + \delta_1 \omega \dot{x} + \mu_1 x + \mu_2 x^3 = F sin(\tau) \tag{7}$$

where $\dot{x} = dx/d\tau$, and $\ddot{x} = d^2x/d\tau^2$ (Banik and Datta, 2010). The model parameters in the equation (7) are: δ_1 controls the amount of damping, F is the amplitude of the periodic excitation force, and ω is the angular velocity (or angular frequency) in (rad/s) of the driving force.

3. Numerical Analysis

Numerical simulations are conducted in Matlab^{TM}, adopting as a control parameter with initial condition. The parameter values for vessel model are listed in Table 1. Equation (7) is numerically integrated with the fourth-order Runge-Kutta method with time step $\triangle t = 0.005$(s) because the results are relatively stable over time if the step is less than this value. An unsuitable time step could lead to numerical instability. Thus the numerical analysis can be guaranteed if the time step is comparatively small enough.

To illustrate the bifurcation diagram we have considered the state variables as x and $\dot{x}$. The numerical simulation shows that equation (7) can lead to complex dynamical behaviors such as multi-periodic and chaotic states. Bifurcation diagrams (variable x versus control parameter F) for the considered two-point mooring system are illustrated in Figs. 2, 4 and 6, where the values of x (recorded after each period $T = 2\pi/\omega$) is plotted versus the amplitude F of the excitation input. In addition to the bifurcation diagram, the phase portrait is displayed to describe various dynamical behaviors as shown in Figs. 3, 5 and 7.

Fig. 2 (a) is the bifurcation diagram obtained by varying F from 0.1 to 0.7 for $\omega = 0.4\,rad/s$. When the control parameter F is smoothly varied from 0.1, one can see the chaotic motions

persist for a range of F values. The magnification of a part of Fig. 2 (a) is shown in Fig. 2 (b). Here, stable periodic solutions, called the periodic windows, can be observed between chaotic clouds of dots. The phase portraits of Fig. 3 (a) and (b) show an example of chaotic orbit followed by periodic orbit at forcing amplitude $F = 0.341$ and 0.344, respectively.

Fig. 4 depicts the bifurcation phenomenon where the response of x at every period of external drive versus the driving amplitude F is plotted for $\omega = 0.7\, rad/s$. We may observe that equation (6) starts with periodic motion for smaller value of F. It is interesting to note from Fig. 4 that the chaotic region is unexpectedly increased when external forcing amplitude exceeds 1.0. In the damped, forced oscillatory system given by equation (5), various types of steady states are observed on the system parameters (ω, F) as well as on the initial condition (Ueda, 1991). Fig. 5 represents the steady states trajectories observed at each of parameter values. Fig. 5 (a) and (b) show the system responses to be sub-harmonic with a time period of 26 seconds. Banik and Datta (2010) introduced a variety of stable sub-harmonic responses such as $2\,T, 3\,T, 5\,T, 7\,T,$ and $9\,T$ in the range of $\omega = 1.0 \sim 0.5\, rad/s$. The phase portrait of Fig. 5 (c) and the time history curves of Fig. 5 (d) indicate symmetry breaking bifurcation. Also, Fig. 5 (e) and (f) show two coexisting period T partner orbits.

Fig. 6 illustrates the bifurcation phenomena when F is varied from 0.1 to 1.6 for $\omega = 1.0\, rad/s$. The mooring system exhibits periodic solutions in contrast to the previous cases ($\omega = 0.4, 0.7$ rad/s), even if the value of F increases. The region of chaotic behavior becomes more fragile in case of $\omega = 1.0\, rad/s$. When F is equal to 0.6, one stable symmetric oscillation occurs. Its limit cycle is presented in Fig. 7 (a).

Table 1. Model parameters for the mooring vessel

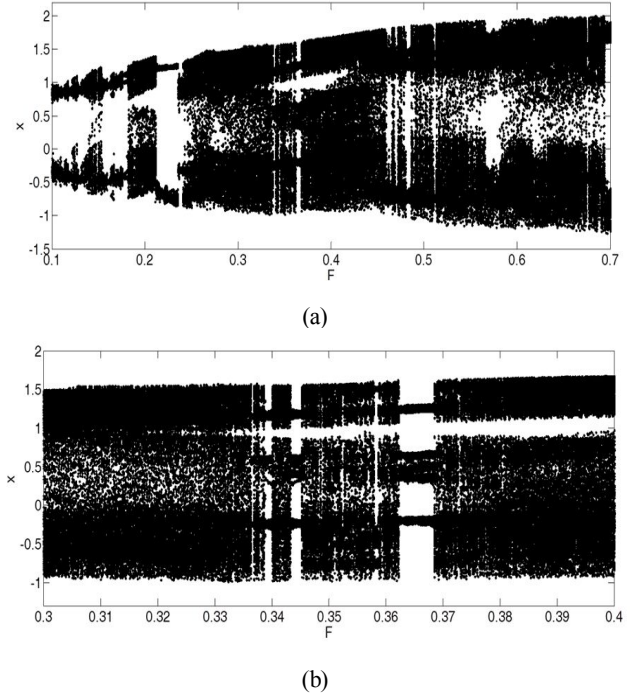| Parameters	Values
δ_1	$0.01\ s^{-1}$
μ_1	$0.0213\ s^{-2}$
μ_2	$0.319\ m^{-2}s^{-2}$

(a)

(b)

Fig. 2. (a) Bifurcation diagram obtained by varying F from 0.1 to 0.7 for ω=0.4 rad/s (b) Magnification of a part of bifurcation diagram of Fig. 2 (a).

(a) ω=0.4 rad/s and F=0.341
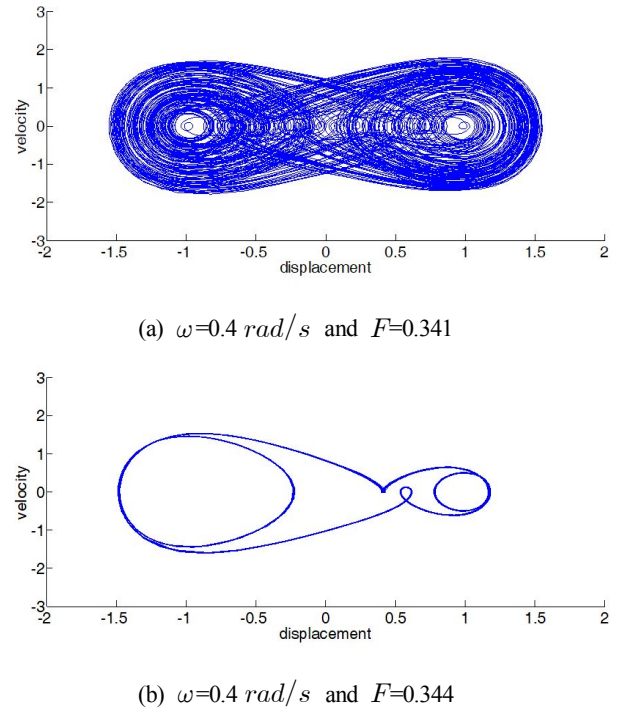

(b) ω=0.4 rad/s and F=0.344

Fig. 3. Phase portraits (a) for F=0.341 and (b) for F=0.344.

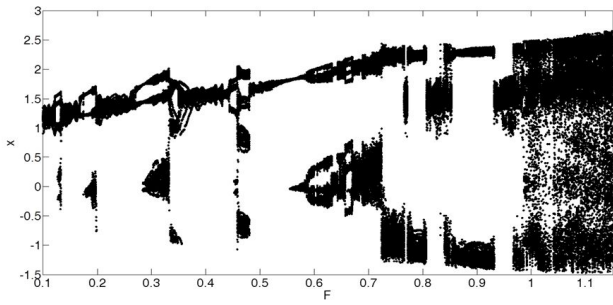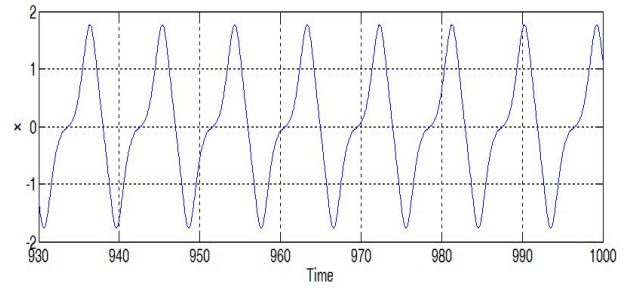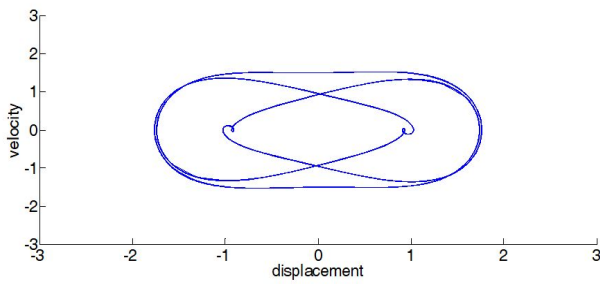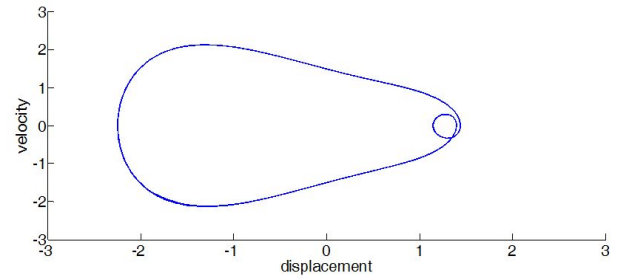Fig. 4. Bifurcation diagram obtained by varying F from 0.1 to 1.15 for ω=0.7 rad/s.

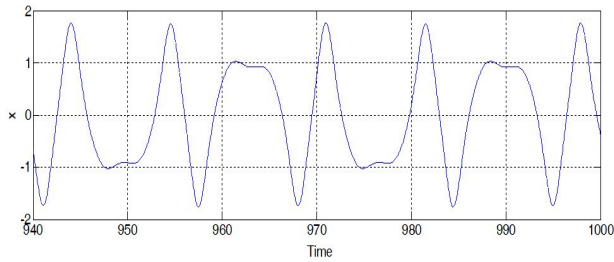(a) ω=0.7 rad/s and F=0.34

(b) time series of ω=0.7 rad/s and F=0.34

(c) ω=0.7 rad/s and F=0.53

(d) time series of ω=0.7 rad/s and F=0.53
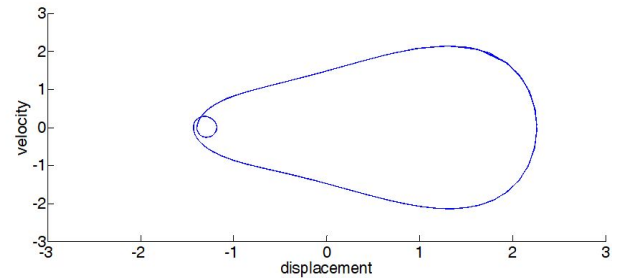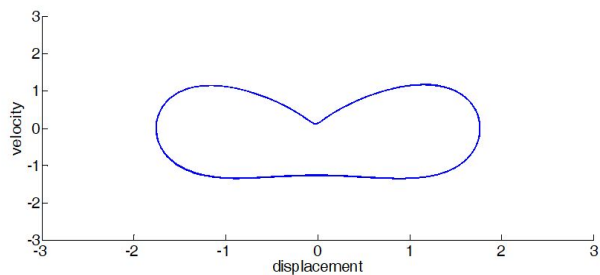
(e) ω=0.7 rad/s and F=0.82
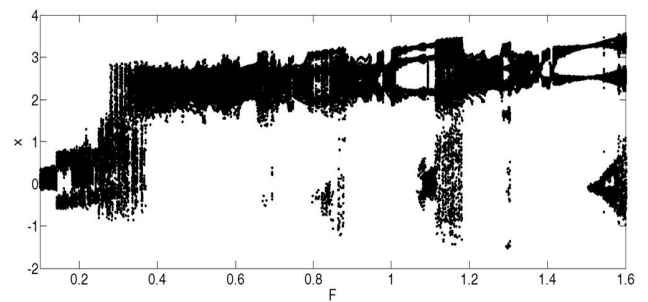

(f) ω=0.7 rad/s and F=0.84

Fig. 5. Phase portraits (a) for F=0.34, (c) for F=0.53, (e) for F=0.82, and (f) for F=0.84. Time history curves (b) for F=0.34 and (d) for F=0.53.

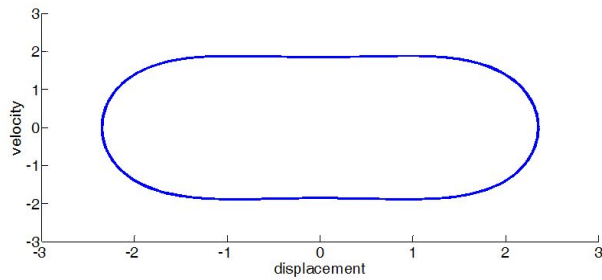Fig. 6. Bifurcation diagram obtained by varying F from 0.1 to 1.6 for ω=1.0 rad/s.

Fig. 7. Phase portraits (a) for ω=1.0 rad/s and F=0.6.

4. Conclusions

In this study, the nonlinear dynamical behaviors of a two-point mooring system have been extensively investigated using the numerical analysis. Particularly, the bifurcation techniques are applied to analyze the dynamical vessel motions for varying excitation amplitude as well as angular frequency, modelled as a single degree of freedom (surge).

1) The vessel system exhibits complex dynamical behaviors such as periodic and chaotic responses. Stable periodic solutions, called the periodic windows, can be observed in succession between chaotic motion clouds of dots in case of $\omega = 0.4\ rad/s$.

2) Chaotic region is unexpectedly increased when external forcing amplitude exceeds 1.0 in case of $\omega = 0.7\ rad/s$. Compared to the angular velocity for $\omega = 0.4\ rad/s$ and $0.7\ rad/s$, the region of chaotic behavior becomes more fragile when $\omega = 1.0\ rad/s$.

3) Various types of steady states including sub-harmonic motion and symmetry breaking phenomenon are obtained at each of parameter values.

The test results are extremely helpful for understanding of nonlinear stability and bifurcation mechanism in mooring system. Finally, further research will be made to consider the mooring control synthesis for safety vessel operations.

References

[1] Akhmet, M. U. and M. O. Fen(2012), Chaotic Period-Doubling and OGY Control for the Forced Duffing Equation, Journal of Communications in Nonlinear Science and Numerical Simulation, Vol. 17, pp. 1929-1946.

[2] Banik, A. K. and T. K. Datta(2010), Stability Analysis of Two-Point Mooring System in Surge Oscillation, Journal of Computational and Nonlinear Dynamics, Vol. 5, pp. 1-8 (021005).

[3] Belato, D., H. I. Weber, J. M. Balthazar and D. T. Mook (2001), Chaotic Vibrations of a Nonideal Electro-Mechanical System, Journal of Solids and Structures, Vol. 38, pp. 1699-1706.

[4] Ellermann, K.(2005), Dynamics of a Moored Barge under Periodic and Randomly Disturbed Excitation, Ocean Engineering, Vol. 32, pp. 1420-1430.

[5] Gottlieb, O.(1991), Nonlinear Oscillations, Bifurcations and Chaos in Ocean Mooring Systems, Doctoral Dissertation, Oregon State University, p. 28.

[6] Gottlieb, O. and S. C. S. Yim(1992), Nonlinear Oscillations, Bifurcations and Chaos in a Multi-Point Mooring System with a Geometric Nonlinearity, Applied Ocean Research, Vol. 14, pp. 241-257.

[7] Gottlieb, O. and S. C. S. Yim(1997), Nonlinear Dynamics of a Coupled Surge-Heave Small-Body Ocean Mooring System, Ocean Engineering, Vol. 24. No. 5, pp. 479-495.

[8] King, P. E. and S. C. Yim(2007), Stochastic Control of Sensitive Nonlinear Motions of an Ocean Mooring System, Journal of Offshore Mechanics and Arctic Engineering, Vol. 129, pp. 29-38.

[9] Mitra, R. K., A. K. Banik and S. Chatterjee(2017), State Feedback Control of Surge Oscillations of Two-Point Mooring System, Journal of Sound and Vibration, Vol. 386, pp. 1-20.

[10] Ueda, Y.(1991), Survey of Regular and Chaotic Phenomena in the Forced Duffing Oscillator, Journal of Chaos, Solitons & Fractals, Vol. 1, No. 3, pp. 199-231.

[11] Umar, A. and T. K. Datta(2003), Nonlinear Response of a Moored Buoy, Ocean Engineering, Vol. 30, No. 13, pp. 1625-1646.

[12] Umar, A., T. K. Datta and S. Ahmad(2010), Complex Dynamics of Slack Mooring System Under Wave and Wind Excitations, The Open Oceanography Journal, Vol. 4, pp. 9-31.

[13] Zou, K. and S. Nagarajaiah(2015), An Analytical Method for Analyzing Symmetry-Breaking Bifurcation and Period-Doubling Bifurcation, Journal of Communications in Nonlinear Science and Numerical Simulation, Vol. 22, pp. 780-792.