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Error Accumulation and Transfer Effects of the Retrieved Aerosol Backscattering Coefficient Caused by Lidar Ratios

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The errors in retrieved aerosol backscattering coefficients due to different lidar ratios are analyzed quantitatively in this paper. The actual calculation shows that the inversion error of the aerosol backscattering coefficients using the Fernald backward-integration method increases with increasing inversion distance. The greater the error in the lidar ratio, the faster the error in the aerosol backscattering coefficient increases. For the same error in lidar ratio, the smaller actual aerosol backscattering coefficient will get the larger relative error of the retrieved aerosol backscattering coefficient. The errors in the lidar ratios for dust or the cirrus layer have great impact on the retrievals of backscattering coefficients. The interval between the retrieved height and the reference range is one of the important factors for the derived error in the aerosol backscattering coefficient, which is revealed quantitatively for the first time in this paper. The conclusions of this article can provide a basis for error estimation in retrieved backscattering coefficients of background aerosols, dust and cirrus layer. The errors in the lidar ratio of an aerosol layer influence the retrievals of backscattering coefficients for the aerosol layer below it.

Keywords : Aerosol backscattering coefficient, Lidar ratio, Error analysis, Aerosol
OCIS codes : (010.0280) Remote sensing and sensors; (010.1110)Aerosols; (010.1350) Backscattering; (010.3640) Lidar

I. INTRODUCTION

Worldwide, nowadays Mie lidar is one of the most-used active remote-sensing tools for atmospheric sounding [1-8]. However, data inversion for Mie lidar still needs to be addressed, due to the unknown lidar ratio based on the Fernald backward-integration method (FBIM) [9]. As a result, the aerosol extinction coefficient and aerosol backscattering coefficient $\beta_a(z)$ retrieved from Mie lidar returns are treated as "data for reference only" in the field of atmospheric sounding.

The inversion error in $\beta_a(z)$ has been researched by Francesc Rocadenbosch *et al.* [10, 11], but the inversion results, calculation method, and research contents of this paper are different from those of previous studies. It can be said with certainty that the lidar ratio has influence on the retrieval of $\beta_a(z)$, but how great is this influence? If the interval from the calibration point to the inversion point continuously increases, what will be the errors in the retrievals of $\beta_a(z)$ obtained by the FBIM method? What is the impact on the vertical profile of $\beta_a(z)$ when the actual vertical profile of the lidar ratio is replaced by a constant value? These problems will be studied in this article.

The influences of lidar ratio on the vertical profile of $\beta_a(z)$ are analyzed quantitatively in this paper. It is necessary for us to understand the errors in the derived $\beta_a(z)$ and its vertical profiles. The contents of this article mainly discuss the influence of the error in lidar ratio on the aerosol

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backscattering coefficient. Based on the conclusions of this paper, when error exists in the lidar ratio, we can deduce the error in $\beta_a(z)$ at different heights from the retrieved $\beta_a(z)$ profile.

II. METHODS AND DISCUSSION

The equation for the backscattering signal of Mie lidar can be written as follows [2]:

$$P(z)=\frac{cE_0Y(z)A_r\beta(z)T^2(z)}{2z^2} \tag{1}$$

The Fernald backward-integral method is formulated in the backward backscattering coefficient form as below [2, 9]:

$$\beta_{\mathrm{a}}(z)=-\beta_{\mathrm{m}}(z)+\frac{X(z)\exp\{2(S_{\mathrm{a}}(z)-S_{\mathrm{m}})\int_z^{z_c}\beta_{\mathrm{m}}(z)dz\}}{\frac{X(z_c)}{\beta_{\mathrm{a}}(z_c)+\beta_{\mathrm{m}}(z_c)}+2S_{\mathrm{a}}(z)\int_z^{z_c}X(z)\exp\{2(S_{\mathrm{a}}(z)-S_{\mathrm{m}})\int_z^{z_c}\beta_{\mathrm{m}}(z')dz'\}dz} \tag{2}$$

where c refers to the speed of light, E_0 denotes laser emission energy, $Y(z)$ is the overlap factor, A_r is the area of the receiving telescope, $\beta_a(z)$ and $\beta_m(z)$ are the backscattering coefficients of aerosol particles and atmosphere molecules respectively at altitude z, and $\beta(z)=\beta_a(z)+\beta_m(z)$. $P(z)$ is the lidar backscattering signal at altitude z. $X(z)=P(z)z^2$, and z_c refers to the altitude of the lidar calibration point. $S_a(z)$ and S_m are the lidar ratios for aerosols and molecules respectively. $T(z)$ is the atmosphere transmittance from the lidar to the altitude z.

2.1. Effects of Error Accumulation in the Retrieval of $\beta_a(z)$

Range-corrected lidar signals for the retrieval of $\beta_a(z)$ are shown in Fig. 1. Assuming that there is a homogeneous aerosol layer in the range of 2.5~5 km in altitude, which makes it easy to analyze the relationship between the error in $\beta_a(z)$ derivation and the interval from the calibration height to another height, where the aerosol backscattering coefficient need to be retrieved. In Fig. 2, signal trace A shows the $\beta_a(z)$ derived from the lidar signal denoted as A in Fig. 1, when 50 sr is assigned to $S_a(z)$. Within the interval of 2.5~5 km, $\beta_a(z)$ for signal B in Fig. 2 is 0.5 times as much as $\beta_a(z)$ for A. Trace C in Fig. 2 shows $\beta_m(z)$ from Eq. (2).

To simplify figure captions, "aerosol backscattering coefficient" is abbreviated as "ABSC" in this paper.

Based on the lidar signal represented as A in Fig. 1, $\beta_a(z)$ and its errors are retrieved with Eq. (2) when the value of $S_a(z)$ is varied from 30 sr to 70 sr in steps of 10 sr, and 10.02 km is chosen as the reference height [12].

Assuming that the true value of $S_a(z)$ is 50 sr, the corresponding retrievals of $\beta_a(z)$ in the range of 2.5~5 km are shown in Figs. 3 and 4.

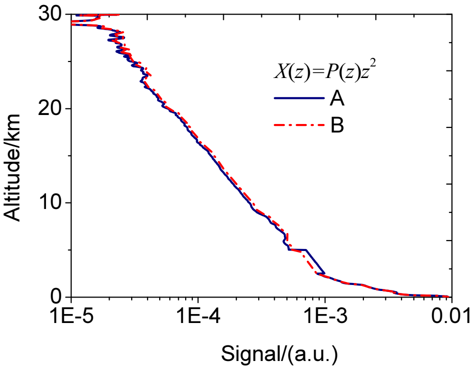


FIG. 1. Range-corrected lidar signals for comparative analysis. There is a uniform layer at altitudes of 2.5~5 km for the A and B signals, yet the lidar signals within the range of that uniform layer are different.

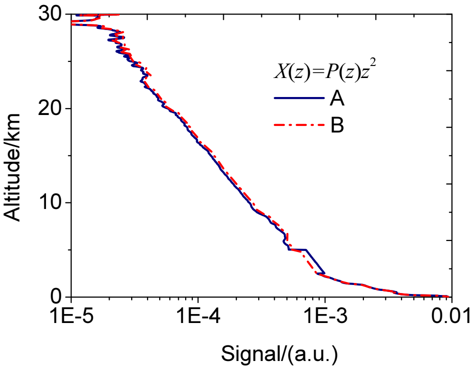


FIG. 2. The aerosol and molecule backscattering profiles. The aerosol profiles, plotted as a solid line and a dot-dash line, are derived from Eq. (2) using lidar signals A and B from Fig. 1, when $S_a(z)$ = 50 sr. The dashed line is the molecule backscattering profile using in the aerosol backscattering coefficient retrievals.

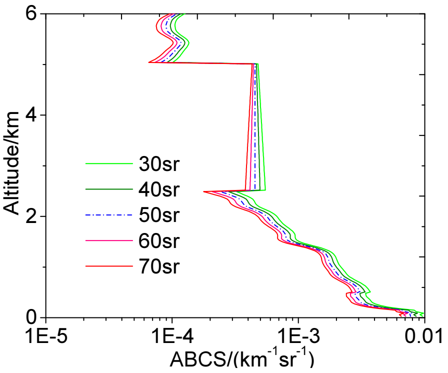


FIG. 3. The effect of $S_a(z)$ on the inversion results of ABSC.

2.1.1. Related factors in the accumulation of absolute error

(1) With the assumption that there is a certain error in $S_a(z)$, the absolute error of $\beta_a(z)$ continues to increase linearly with increasing interval between the height of calibration point and that of inverted atmosphere.

(2) The increasing rate of absolute error in $\beta_a(z)$ is related to the deviation of $S_a(z)$ from its actual value. The greater the deviation of $S_a(z)$, the faster the increase in rate of error in $\beta_a(z)$.

(3) By examining Fig. 3 carefully, we can see that, in spite of the same difference of 20 sr from the actual value (50 sr, for example), *i.e.* where $S_a(z)$ is 30 sr and 70 sr respectively, when the $S_a(z)$ is 20 sr *smaller* than the actual value, the absolute error in $\beta_a(z)$ obtained using FBIM is greater than the absolute error when $S_a(z)$ is 20 sr *larger* than the true value.

(4) There is a relationship between the absolute error in $\beta_a(z)$ and the error in $S_a(z)$ for the same inversion range; greater error in $S_a(z)$ results in greater absolute error in $\beta_a(z)$.

2.1.2. Relative error of the retrieved $\beta_a(z)$

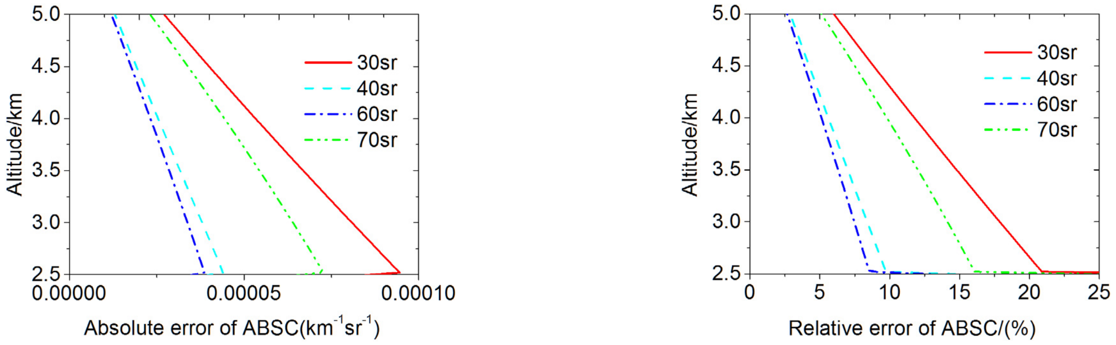
The relative error can be a true reflection of the credibility of the retrieved $\beta_a(z)$. Assuming that there are errors in $S_a(z)$, and the true value of $S_a(z)$ is 50 sr ,within a certain range it can be seen in Fig. 4 that the relative error in $\beta_a(z)$ obtained from Eq. (2) increases with range. For example, for $S_a(z) = 30$ sr, the relative error in $\beta_a(z)$ is 7% at 5 km, but 23% at 2.5 km. The relative error in $\beta_a(z)$ increased by 16% in the range from 5 to 2.5 km.

2.2. Comparative Analysis

The lidar data shown in the B traces of Figs. 1 and 2 are used for comparative analysis with the corresponding A data. The aerosol backscattering coefficients from 2.5 to 5 km shown in Fig. 2 as trace B are half as great as the corresponding ones in trace A. Trace B in Fig. 1 is the range-corrected lidar signal corresponding to the aerosol backscattering coefficient shown as curve B in Fig. 2, when $S_a(z) = 50$ sr.

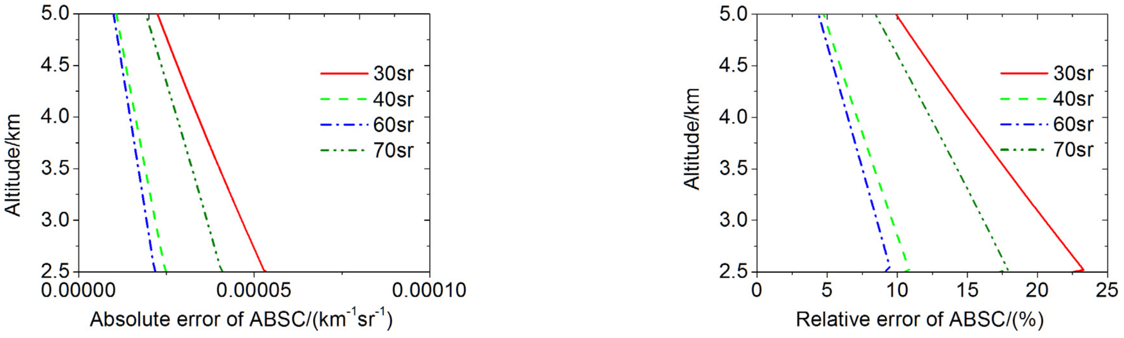
The retrievals of $\beta_a(z)$ in Fig. 5 are obtained with Eq. (2) from the lidar returns shown in Fig. 1 as curve B, when the value of $S_a(z)$ is varied from 30 to 70 sr in steps of 10 sr. The absolute and relative errors of $\beta_a(z)$ shown in Fig. 5 are calculated for quantitative analysis, as well as comparison to the corresponding errors of $\beta_a(z)$,which are acquired using the lidar data represented by the curves marked A in Figs. 1 and 2. We retrieve $\beta_a(z)$ at a height of 2.61 km as an example for quantitative analysis. Assigning 30 sr, 40 sr, 60 sr, and 70 sr to $S_a(z)$, one by one, the relative errors in $\beta_a(z)$ at a height of 2.61 km in Fig. 5 (retrieved from the lidar data shown in the B traces of Figs. 1 and 2) are 22.76%, 10.62%, 9.33%, and 17.57% respectively.

In contrast to the relative errors in $\beta_a(z)$ shown in Fig. 5(b), the relative errors in $\beta_a(z)$ in Fig. 4(b) (which are



(a) (b)

FIG. 4. The effect of $S_a(z)$ on the inversion results for errors in ABSC. The retrieval error in $\beta_a(z)$ within the uniform layer of 2.5~5 km, using signal A in Fig. 1, differs when $S_a(z)$ is the only variable and is set at 30, 40, 50, 60, and 70 sr respectively.



(a) (b)

FIG. 5. Absolute and relative errors in ABSC, based on the B signal in Fig. 1.

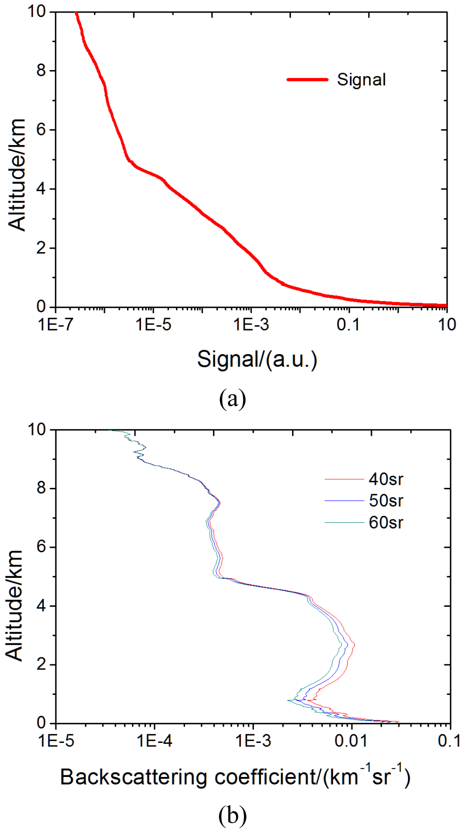
derived from the lidar signal represented as A in Figs. 1 and 2) are 20.32%, 9.48%, 8.34%, and 15.72% respectively.

It can be easily found from the comparative analysis of Figs. 4 and 5 that there exists a relation between the actual value of $\beta_a(z)$ and the relative error of the retrieved $\beta_a(z)$: When $S_a(z)$ and its error are the same, the relative error in the retrieved $\beta_a(z)$,acquired using Eq. (2), is larger when the actual value of $\beta_a(z)$ is smaller.

III. ERROR-ACCUMULATION EFFECT

3.1. The Effect of Errors in $S_d(z)$ on Retrieved $\beta_d(z)$

The lidar signal used for retrieval of the backscattering coefficient of dust $\beta_d(z)$ is shown in Fig. 6(a), while vertical profiles of the retrieved $\beta_d(z)$ are shown in Fig. 6(b), with lidar ratios for dust $S_d(z)$ of 40, 50, and 60 sr respectively. Assuming that the real value of $S_d(z)$ is 50 sr, and treating the $\beta_d(z)$ obtained using Eq. (2) as the true value of $\beta_d(z)$, it can be seen from Fig. 6(b) that the accurate value of $\beta_d(z)$ at 2.01 km is 0.00739 km^{-1} sr^{-1}. For $S_d(z) = 40$ and 60 sr [13, 14], the absolute error of the retrieved $\beta_d(z)$ at a height of 2.01 km is 0.00158 and 0.00112 km^{-1} sr^{-1} respectively.



(a)

(b)

FIG. 6. The effect of $S_a(z)$ on the backscattering coefficient of dust. (a) The lidar signal with a dust layer in the laser path. (b) A comparison of $\beta_d(z)$ derived for several values of $S_d(z)$, with lidar ratio varying from 40 to 60 sr in steps of 10 sr.

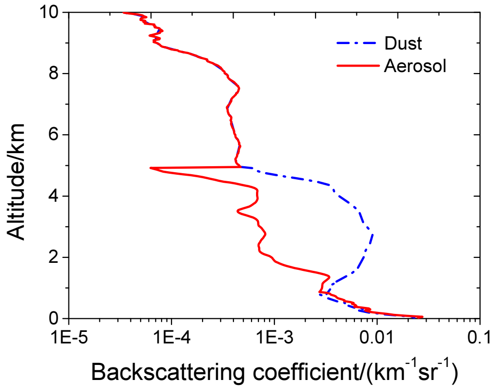


FIG. 7. Dust and aerosols for comparative analysis. The blue line shows the lidar signal with an elevated dust layer . The red line shows the lidar signal when there is no dust layer in the laser path.

In Fig. 7, if the dust layer (the dotted line) is replaced by the background aerosol (the solid line), which is treated as the actual profile of $\beta_a(z)$ with $S_a(z) = 50$ sr, we find that the actual value of $\beta_a(z)$ at 2.01 km is 0.000929 km^{-1} sr^{-1}. For $S_a(z) = 40$ and 60 sr, the absolute error of the retrieved $\beta_a(z)$ (which is derived from the simulated lidar signal obtained from the $\beta_a(z)$ profile in Fig. 7 using Eq. (2)) is 0.000102 km^{-1} sr^{-1} and 0.0000909 km^{-1} sr^{-1} respectively.

Assuming the lidar ratio and its error being the same, we come to the conclusion that the deviation in lidar ratio has larger impact on the absolute error of $\beta_d(z)$ than on that of $\beta_a(z)$ at the same height, because the true value of $\beta_d(z)$ is larger than that of $\beta_a(z)$.

3.2. The Effect of Error in $S_c(z)$ on the Retrieved $\beta_c(z)$

The retrieved backscattering coefficients of cirrus $\beta_c(z)$ and their absolute errors, which are obtained using the range-corrected backscattering signal for a cirrus layer in Fig. 8 with Eq. (2), are shown in Figs. 9 and 10, with 20, 30, 40, 50, and 60 sr as the value of $S_c(z)$ respectively, while 20 sr is treated as the actual value of $S_c(z)$ [15].

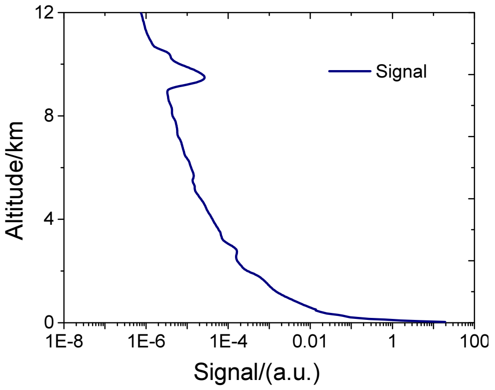


FIG. 8. Range-corrected lidar signal with a cirrus layer in the laser path, which is used to analyze the influence of error in $S_c(z)$ on the retrieval of $\beta_c(z)$.

In Fig. 10 we can see that if some error exists in $S_c(z)$, a larger true value of $\beta_c(z)$ will result in a larger absolute error in $\beta_c(z)$. In many retrievals of $\beta_c(z)$, the absolute error is so great that it cannot be ignored. For the same error in lidar ratio, the absolute error in the obtained $\beta_c(z)$ is much greater than that in $\beta_a(z)$. This conclusion is well comparable to the relationship between the absolute error of the derived $\beta_d(z)$ and $S_d(z)$, as was analyzed quantitatively in section 3.1 above.

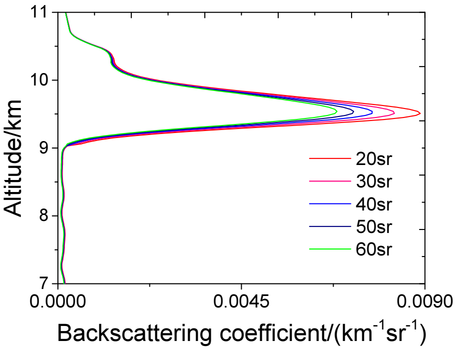


FIG. 9. The retrievals of $\beta_c(z)$. The profile of $\beta_c(z)$ obtained according to the lidar signal in Fig. 8, when the assumed constant lidar ratio of the cirrus layer is $S_c(z)$ = 20, 30, 40, 50, and 60 sr respectively.

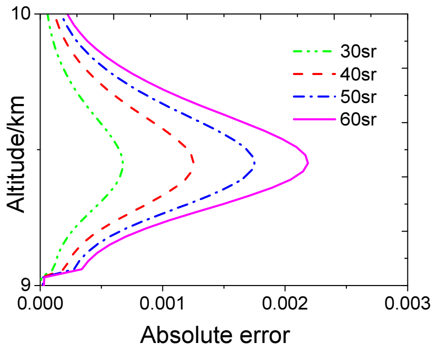


FIG. 10. Error analysis for $\beta_c(z)$. The absolute error in $\beta_c(z)$ is shown for $S_c(z)$ = 30, 40, 50, and 60 sr respectively.

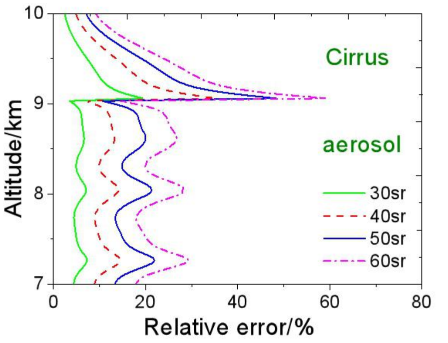


FIG. 11. Accumulation and transfer effects for relative error in the retrieval of $\beta_c(z)$. The relationship between the relative error in the retrieval of $\beta_c(z)$ and the lidar ratio, as well as the retrieval distance. The bottom of the layer of cirrus clouds is at an altitude of 9.0 km.

The maximum relative error of $\beta_c(z)$, which occurs at the lowest edge of the cirrus structure (at an altitude of 9 km), is observed in Fig. 11. It shows that the relative error in the received $\beta_c(z)$ is not only concerned with the actual value of $\beta_c(z)$, but also relates to the interval from the calibration point to the point where $\beta_c(z)$ will be retrieved. Greater relative error in $\beta_c(z)$ is observed when the greater actual value of $\beta_c(z)$ and greater reversal distance exist simultaneously. This conclusion is consistent with the corresponding retrievals for the backscattering coefficients of aerosols and dust, which were presented earlier in the article.

IV. ERROR TRANSFER EFFECT BETWEEN LAYERS

In the earlier inversion of section 3.2, there are errors in the lidar ratio of the cirrus layer, but not in the lidar ratio of the background aerosol, which kept a constant value of 50 sr. From Fig. 10, it can be seen that the lidar ratio of the cirrus layer is not only the source of the error in retrievals for the cirrus, but also the source of the error in the calculated value of the background aerosol layer below the cirrus. Assuming that the error of the cirrus lidar ratio is in the range 10~40 sr, it can be seen from Fig. 10 that the relative error in the retrieval of the backscattering coefficients for aerosol layers below the cirrus are in the range 5~30%.

V. CONCLUSION

In conclusion, if there is a certain error in the lidar ratio and the atmosphere is uniform in the altitude range of interest, the absolute and relative errors in $\beta_a(z)$ continue to accumulate with increasing inversion range along the lidar operating path.

The greater the error in aerosol lidar ratio, the faster the accumulation of absolute and relative errors in the received $\beta_a(z)$.

The error in lidar ratio has great influence on the retrieved backscattering coefficient for a layer of dust or cirrus clouds, the absolute error in the derived $\beta_c(z)$ or $\beta_d(z)$ profile being so large that it cannot be neglected in most cases.

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