J. Appl. Math. & Informatics Vol. **36***(2018), No. 1 - 2, pp. 59 - 68*
https://doi.org/10.14317/jami.2018.059

ON SOFT REGULAR-OPEN(CLOSED) SETS IN SOFT TOPOLOGICAL SPACES

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Abstract. In this paper, We define and explore the characterizations and properties of soft regular open(closed) and soft semi-regular sets in soft topological spaces. The properties of soft extremally disconnected spaces are also introduced and discussed. The findings in this paper will help researcher to enhance and promote further study on soft topology to carry out a general framework for their applications in practical life.

AMS Mathematics Subject Classification : 06D72, 54A10, 54D10.
Key words and phrases : Soft topology, soft semi-open(closed) sets, soft semi-interior(closure), soft regular open(closed) sets, soft extremally disconnected space.

1. Introduction

To solve complicated problems having uncertainties, Molodstov[17] introduced soft set theory as a new mathematical tool. Recently, many researches have been done on the findings of different structures of soft set theory and its applications to many problems containing uncertainties. In [3-4], B. Chen defined and discussed the properties of soft semi-open and soft semi-closed sets in soft topological spaces. Later, S. Hussain[6] discussed the different characterizations and properties using soft semi-open(closed) sets and introduced and discussed the concepts of soft semi-interior(exterior), soft semi-closure, soft semi-boundary, soft semi-open neighborhood and soft semi-open neighborhood systems in soft topological spaces. Recently, S. Hussain and B. Ahmad[10] initiated and explored several properties of soft separation axioms including soft semi-T_i, $(i = 0, 1, 2)$, soft semi-regular, soft semi-T_3, soft semi-normal and soft semi-T_4 axioms using soft points. In [8], S. Hussain defined and discussed new form of continuity called soft pu-semi-continuity via soft semi-open set in soft topological spaces. Moreover the concepts of soft-pu-semi-open and soft pu-semi-closed

Received June 13, 2017. Revised August 20, 2017. Accepted August 24, 2017.

functions are introduced and discussed many of their characterizations and properties. It is interesting to mention that the soft functions defined and discussed in [8] are the generalization of soft functions explored in [5][21].

2. Preliminaries

To make our paper self contained, first we recall some definitions and results defined and discussed in [1],[3-4],[6-7],[9-10],[15-18], [20-21].

Definition 2.1. Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2.2. For two soft sets (F, A) and (G, B) over a common universe X, we say that (F, A) is a soft subset of (G, B), if
(1) $A \subseteq B$ and
(2) for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A)\tilde{\subseteq}(G, B)$.
(F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A)\tilde{\supseteq}(G, B)$.

Definition 2.3. Two soft sets (F, A) and (G, B) over a common universe X are said to be soft equal, if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4. The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A)\tilde{\cup}(G, B) = (H, C)$.

Definition 2.5. The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X , denoted $(F, A)\tilde{\cap}(G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e)\tilde{\cap}G(e)$, for all $e \in C$.

Definition 2.6. The difference (H, A) of two soft sets (F, A) and (G, A) over X, denoted by $(F, A)\tilde{\setminus}(G, A)$, is defined as $H(e) = F(e)\setminus G(e)$, for all $e \in A$

Definition 2.7. Let (F, A) be a soft set over X and Y be a non-empty subset of X. Then the sub soft set of (F, A) over Y denoted by (F_Y, A), is defined as follows: $F_Y(\alpha) = Y\tilde{\cap}F(\alpha)$, for all $\alpha \in A$. In other words $(F_Y, A) = \tilde{Y}\tilde{\cap}(F, A)$.

Definition 2.8. The relative complement of a soft set (F, A) is denoted by $(F, A)^{'}$ and is defined by $(F, A)^{'} = (F^{'}, A)$ where $F^{'} : A \to P(U)$ is a mapping given by $F^{'}(\alpha) = U \backslash F(\alpha)$, for all $\alpha \in A$.

Definition 2.9. Let $x \in X$, then (x, A) denotes the soft set over X for which $x(\alpha) = \{x\}$, for all $\alpha \in A$.

Definition 2.10. Let (F, A) be a soft set over X and $x \in X$. We say that $x \in (F, A)$ read as x belongs to the soft set (F, A), whenever $x \in F(\alpha)$, for all $\alpha \in A$. Note that $x \in X$, $x \notin (F, A)$, if $x \notin F(\alpha)$ for some $\alpha \in A$.

Definition 2.11. Let τ be the collection of soft sets over X, then τ is said to be soft topology on X, if
(1) Φ , $\tilde{X}$ belong to τ.
(2) the union of any number of soft sets in τ belongs to τ.
(3) the intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, A) is called a soft topological space over X.

Definition 2.12. Let (X, τ, A) be a soft topological space over X then soft interior of soft set (F, A) over X is denoted by $(F, A)^{\circ}$ and is defined as the union of all soft open sets contained in (F, A). Thus $(F, A)^{\circ}$ is the largest soft open set contained in (F, A). A soft set (F, A) over X is said to be a soft closed set in X, if its relative complement $(F, A)^{'}$ belongs to τ.

Definition 2.13. Let (X, τ, A) be a soft topological space over X and (F, A) be a soft set over X. Then the soft closure of (F, A), denoted by $\overline{(F, A)}$ is the intersection of all soft closed super sets of (F, A). Clearly $\overline{(F, A)}$ is the smallest soft closed set over X which contains (F, A).

Definition 2.14. Let (X, τ, A) be a soft topological space over X and (F, A) be a soft set over X. Then (F, A) is called soft semi-open set if and only if there exists a soft open set (G, A) such that $(G, A) \tilde{\subseteq} (F, A) \tilde{\subseteq} \overline{(G, E)}$. The set of all soft semi-open sets is denoted by $S.S.O(X)$.

Note that every soft open set is soft semi-open set.
A soft set (F, A) is said to be soft semi-closed if its relative complement is soft semi-open . Equivalently there exists a soft closed set (G, A) such that $(G, A)^{\circ} \tilde{\subseteq} (F, A) \tilde{\subseteq} (G, A)$. Moreover, note that every soft closed set is soft semi-closed set.

Definition 2.15. Let (X, τ, A) be a soft topological space over X.
(i) soft semi-interior of soft set (F, A) over X is denoted by $int^{s}(F, A)$ and is defined as the union of all soft semi-open sets contained in (F, A).
(ii) soft closure of (F, A) over X is denoted by $cl^{s}(F, A)$ and is the intersection of all soft semi-closed super sets of (F, A).

Definition 2.16. Let (X, τ, A) be a soft topological space over X and (F, A) be a soft set over X. The soft set (F, A) is called a soft point in $\tilde{X}$, denoted by e_F, if for the element $e \in A$, $F(e) \neq \phi$ and $F(e^{'}) = \phi$, for all $e^{'} \in A - \{e\}$.

Definition 2.17. The soft point e_F is said to be in the soft set (G, A), denoted by $e_F \tilde{\in} (G, A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.

Proposition 2.18. *Let* $e_F \tilde{\in} \tilde{X}$ *and* (G, A) *be a soft set. If* $e_F \tilde{\in} (G, A)$, *then* $e_F \tilde{\notin} (G, A)^c$.

3. Soft Regular-Open(Closed) and Soft Semi-Regular Sets

Definition 3.1. Let (X, τ, A) be a soft topological space over X and (F, A) be a soft set in (X, τ, A). Then (F, A) is said to be a soft regular-open set, if $(F, A) \tilde{=} \{\overline{(F, A)}\}^\circ$. The set of soft regular open sets is denoted by $R^sO(X)$. Clearly, every soft regular-open set is soft open in soft topological space (X, τ, A).

Example 3.2. Let $X = \{x_1, x_2\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, \tilde{X}_A, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$, where $(F_1, A) = \{(e_1, \{x_2\}), (e_2, \{x_1\})\}$, $(F_2, A) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$, $(F_3, A) = \{(e_1, \{x_1\})\}$, $(F_4, A) = \{(e_1, X), (e_2, \{x_1\})\}$ Then (X, τ, A) is a soft topological space over X. Note that the soft closed sets are $\{(e_1, \{x_1\}), (e_2, \{x_2\})\}$, $\{(e_1, \{x_2\}), (e_2, \{x_1\})\}$, $\{(e_1, \{x_2\}), (e_2, X)\}$ and $\{(e_2, \{x_2\})\}$. Consider we take the soft set $(F, A) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$. Calculations shows that $(F, A) \tilde{=} \{\overline{(F, A)}\}^\circ$. This follows that (F, A) is soft regular-open set.

Definition 3.3. Let (X, τ, A) be a soft topological space over X and (F, A) be a soft set in (X, τ, A). Then (F, A) is said to be a soft regular closed set, if $(F, A)^c$ is soft regular open set.

Remark 3.1. Clearly, (F, A) is soft regular closed set, if $(F, A) \tilde{=} \overline{(F, A)^\circ}$. The set of all soft regular closed sets is denoted by $R^sC(X)$.

Definition 3.4. Let (X, τ, A) be a soft topological space over X and (F, A) be a soft set in (X, τ, A). Then (F, A) is said to be a soft semi-regular , if it is soft semi-open and soft semi-closed. The set of all soft semi-regular sets is denoted by $SR^s(X)$.

Keeping in view the definition of soft semi-closure[11], the soft semi-closure point is defined as:

Definition 3.5. Let (X, τ, A) be a soft topological space over X and (F, A) be a soft set in (X, τ, A). The soft point e_G is said to be soft closure point of (F, A), if for every soft open set (H, A) with $e_G \tilde{\in} (H, A)$ implies $(F, A) \tilde{\cap} (H, A) \tilde{\neq} \tilde{\Phi}$.

Example 3.6. Let $X = \{x_1, x_2\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, \tilde{X}_A, (F_1, A), (F_2, A), (F_3, A)\}$, where $(F_1, A) = \{(e_1, \{x_1\})\}$, $(F_2, A) = \{(e_2, \{x_2\})\}$, $(F_3, A) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$. Then (X, τ, A) is a soft topological space over X. Consider the soft set $(F, A) = \{(e_1, \{x_1\}), (e_2, X)\}$. Then clearly the soft point $e_G = (e_1, \{x_1\})$, is the soft closure point of the soft set (F, A).

Lemma 3.7. *Let* (F, A) *and* (G, A) *are soft sets in soft topological space* (X, τ, A). *Then*

(1) $\overline{(F,A)}\tilde{-}\overline{(G,A)}\tilde{\subseteq}\overline{\{(F,A)\tilde{-}(G,A)\}}$.
(2)$\{(F,A)\tilde{-}(G,A)\}^\circ\tilde{\subseteq}(F,A)^\circ\tilde{-}(G,A)^\circ$.
(3) If (F,A) *is soft open, then* $(F,A)\tilde{\cap}\overline{(G,A)}\tilde{\subseteq}\overline{(F,A)\tilde{\cap}(G,A)}$.

Proof. (1) Let e_H be a soft point such that $e_H\tilde{\in}\{\overline{(F,A)}\tilde{-}\overline{(G,A)}\}$. Then $e_H\tilde{\in}\overline{(F,A)}$ and $e_H\tilde{\notin}\overline{(G,A)}$. Thus there exists a soft open set (K,A) soft contains e_H such that $(F,A)\tilde{\cap}(K,A)\tilde{\neq}\tilde{\Phi}$ and $(G,A)\tilde{\cap}(K,A)\tilde{=}\tilde{\Phi}$. This follows that $(K,A)\tilde{\cap}\{(F,A)\tilde{-}(G,A)\}\tilde{\neq}\tilde{\Phi}$. Therefore $e_H\tilde{\in}\overline{\{(F,A)\tilde{-}(G,A)\}}$. This proves (1).
(2) The proof is same as of (1).
(3) Suppose that (F,A) is soft open. Then $(F,A)\tilde{=}(F,A)^\circ$. Using (1) and properties of soft interior and soft closure[11], we get

$$\begin{aligned}(F,A)\tilde{\cap}\overline{(G,A)} &\tilde{=} (F,A)^\circ\tilde{\cap}\overline{(G,A)}\\ &\tilde{=} \overline{(G,A)}\tilde{-}((F,A)^\circ)^c\\ &\tilde{=} \overline{(G,A)}\tilde{-}\overline{(F,A)^c}\\ &\tilde{\subseteq} \overline{(G,A)\tilde{-}(F,A)^c}\ (By(1))\\ &\tilde{=} \overline{(G,A)\tilde{\cap}(F,A)}\\ &\tilde{=} \overline{(F,A)\tilde{\cap}(G,A)}.\end{aligned}$$

This completes the proof. □

Lemma 3.8. *Let* (X,τ,A) *be a soft topological space over* X *and* (F,A) *be a soft set in* (X,τ,A)*. Then* $(\overline{(F,A)})^\circ\tilde{\subseteq}\ cl^s(F,A)$.

Proof. Suppose e_G be a soft point in $(\overline{(F,A)})^\circ$ and (H,A) be a soft semi-open set in (X,τ,A) such that $e_G\tilde{\in}(H,A)$. Then for some soft open set (K,A) in (X,τ,A), $(K,A)\tilde{\subseteq}(H,A)\tilde{\subseteq}\overline{(K,A)}$. Now $e_G\tilde{\in}(H,A)\tilde{\subseteq}\overline{(K,A)}$ and $e_G\tilde{\in}(\overline{(F,A)})^\circ$ and above Lemma 3.7 implies that $\tilde{\Phi}\tilde{\neq}(\overline{(F,A)})^\circ\tilde{\cap}(K,A)\tilde{\subseteq}\overline{(F,A)}\tilde{\cap}(K,A)\tilde{\subseteq}\overline{(F,A)\tilde{\cap}(K,A)}$. This follows $(F,A)\tilde{\cap}(K,A)\tilde{\neq}\tilde{\Phi}$. Which implies that $(F,A)\tilde{\cap}(H,A)\tilde{\neq}\tilde{\Phi}$. Hence $e_G\tilde{\in}cl^s(F,A)$. This completes the proof. □

Using above Lemma 3.8, we have the following theorem.

Theorem 3.9. *Let* (X,τ,A) *be a soft topological space over* X *and* (F,A) *be a soft set in* (X,τ,A)*. Then*
(1) $(F,A)\tilde{\in}SR^s(X)$.
(2) $(F,A)\tilde{=}sint^s(scl^s(F,A))$.
(3) There exists $(G,A)\tilde{\in}R^sO(X)$ *with the property that* $(G,A)\tilde{\subseteq}(F,A)\tilde{\subseteq}\overline{(G,A)}$.

Proof. (1) ⇒ (2). Suppose $(F,A)\tilde{\in}SR^s(X)$, then $sint^s(scl^s(F,A))\tilde{=}sint^s(F,A)$ $\tilde{=}(F,A)$.
(2) ⇒ (3). Using (2) and Lemma 3.8, we have
$(\overline{(F,A)})^\circ\tilde{\subseteq}int^s(cl^s(F,A))\tilde{\subseteq}\ cl^s(F,A)\tilde{=}(F,A)$. Also (F,A) is soft semi-open implies that $(F,A)\tilde{\subseteq}\overline{(F,A)^\circ}$. Moreover (F,A) is soft semi-closed implies soft closed [6], which follows that $(\overline{(F,A)})^\circ\tilde{\subseteq}(F,A)\tilde{\subseteq}\overline{(F,A)^\circ}\tilde{\subseteq}\overline{(\overline{(F,A)})^\circ}$. Take $(G,A)\tilde{=}$ $(\overline{(F,A)})^\circ$. Then $(G,A)\tilde{=}(\overline{(G,A)})^\circ$ implies that (G,A) is soft regular-open set.

Thus (3) is satisfied.
(3) $\Rightarrow$ (1). (3) follows that (F,A) is soft semi-open set. This implies that $(\overline{(F,A)})^\circ \tilde{=} (\overline{(G,A)})^\circ \tilde{=} (G,A) \tilde{\subseteq} (F,A)$. Therefore, (F,A) is soft semi-closed. Hence $(F,A) \tilde{\in} SR^s(X)$. Hence the proof. ☐

Theorem 3.10. *Let (F,A) be a soft semi-open set in soft topological space (X,τ,A). Then $cl^s(F,A)$ is both soft semi-closed and soft semi-open set.*

Proof. This follows directly that $cl^s(F,A)$ is soft semi-closed. It remains to prove that $cl^s(F,A)$ is soft semi-open. (F,A) is soft semi-open implies that there exist soft open set (G,A) such that $(G,A) \tilde{\subseteq} (F,A) \tilde{\subseteq} \overline{(G,A)}$. Thus $(G,A) \tilde{\subseteq} cl^s(G,A) \tilde{\subseteq} cl^s(F,A) \tilde{\subseteq} cl^s(\overline{(G,A)}) \tilde{=} \overline{(G,A)}$. Therefore, $cl^s(F,A)$ is soft semi-open. This completes the proof. ☐

Remark 3.2. From above Theorem 3.10, it follows that
$(F,A) \tilde{\in} S.S.O(X) \Rightarrow cl^s(F,A) \tilde{\in} SR^s(X)$.

Definition 3.11. Let (F,A) be a soft set in soft topological space (X,τ,A). Then the soft point e_G is said to be a soft semi-θ-adherent point of (F,A), if for any soft semi-open set (K,A), $cl^s(K,A) \tilde{\cap} (F,A) \tilde{\neq} \tilde{\Phi}$. The set of all soft semi-θ-adherent points of (F,A) denoted as $cl^s_\theta(F,A)$ and is called the soft semi-θ-closure of (F,A). The soft set (F,A) is called soft semi-θ-closed, if $cl^s_\theta(F,A) \tilde{=} (F,A)$.

Theorem 3.12. *Let (F,A) be a soft set in soft topological space (X,τ,A). Then*
(1) $cl^s(F,A) \tilde{=} cl^s_\theta(F,A)$, for soft semi-open set (F,A) in (X,τ,A).
(2) (F,A) is soft semi-θ-closed , for soft semi-regular set (F,A) in (X,τ,A).

Proof. (1). First we prove $cl^s_\theta(F,A) \tilde{\subseteq} cl^s(F,A)$ by using contrapositive method. Let $e_G \tilde{\notin} cl^s(F,A)$. Then for soft semi-open set (K,A), $(F,A) \tilde{\cap} (K,A) \tilde{=} \tilde{\Phi}$. Also (F,A) is soft semi-open implies that $(F,A) \tilde{\cap} cl^s(K,A) \tilde{=} \tilde{\phi}$. This follows that $e_G \tilde{\notin} cl^s_\theta(F,A)$. The other inclusion in obvious. Hence $cl^s(F,A) \tilde{=} cl^s_\theta(F,A)$.
(2). This follows directly from (1). Hence the proof. ☐

Proposition 3.13. *Let (F,A) be a soft open set in soft topological space (X,τ,A). Then $cl^s(F,A) \tilde{=} (\overline{(F,A)})^\circ$.*

Proof. $(\overline{(F,A)})^\circ \tilde{\subseteq} cl^s(F,A)$ follows from Lemma 3.8. For the reverse inclusion, we use contrapositive method. For this, let $e_G \tilde{\notin} (\overline{(F,A)})^\circ$. Then $e_G \tilde{\in} \overline{((F,A)^c)^\circ}$, where $\overline{((F,A)^c)^\circ}$ is soft semi-open set. (F,A) is soft open follows that $(F,A) \tilde{\subseteq} (\overline{(F,A)})^\circ$. Also $(F,A) \tilde{\cap} \overline{((F,A)^c)^\circ} \tilde{=} \tilde{\Phi}$. Therefore $e_G \tilde{\notin} cl^s(F,A)$. Therefore $cl^s(F,A) \tilde{=} (\overline{(F,A)})^\circ$. This completes the proof. ☐

4. Properties of Soft Extremally Disconnected Spaces

Definition 4.1. A soft topological space (X,τ,A) over X is said to be soft extremally disconnected , if for any soft open set (F,A) in (X,τ,A), $\overline{(F,A)}$ is soft open.

Lemma 4.2. *Let (F, A) be a soft set in soft topological space (X, τ, A). Then (F, A) is a soft semi-open set if and only if $\overline{(F, A)^\circ} \tilde{=} \overline{(F, A)}$.*

Proof. $(\Rightarrow)$ (F, A) is soft semi-open implies that $(F, A) \tilde{\subseteq} \overline{(F, A)^\circ}$. Therefore $\overline{(F, A)} \tilde{\subseteq} \overline{\overline{(F, A)^\circ}} \tilde{=} \overline{(F, A)^\circ}$. Also $(F, A)^\circ \tilde{\subseteq} (F, A)$ implies that $\overline{(F, A)^\circ} \tilde{\subseteq} \overline{(F, A)}$. Hence $\overline{(F, A)^\circ} \tilde{=} \overline{(F, A)}$.
$(\Leftarrow)$ By hypothesis, we have $(F, A) \tilde{\subseteq} \overline{(F, A)} \tilde{=} \overline{(F, A)^\circ}$. This follows that (F, A) is soft semi-open. This completes the proof. □

Theorem 4.3. *A soft topological space (X, τ, A) is soft extremally disconnected if and only if for any soft semi-open set (F, A), $cl^s(F, A) \tilde{=} \overline{(F, A)}$.*

Proof. $(\Rightarrow)$ This follows form the definition that for any soft set (F, A), $cl^s(F, A) \tilde{\subseteq} \overline{(F, A)}$. Now we prove the reverse inclusion for any soft semi-open set (F, A). For this, let $e_G \tilde{\notin} cl^s(F, A)$, then there exists a soft semi-open set (K, A) such that $e_G \tilde{\in} (K, A)$ and $(F, A) \tilde{\cap} (K, A) \tilde{=} \tilde{\Phi}$. This follows that $(F, A)^\circ \tilde{\cap} (K, A)^\circ \tilde{=} \tilde{\Phi}$. Our hypothesis implies that $\overline{((F, A)^\circ)} \tilde{\cap} \overline{((K, A)^\circ)} \tilde{=} \tilde{\Phi}$. Therefore, by Lemma 4.2, $e_G \tilde{\notin} \overline{(F, A)^\circ} \tilde{=} \overline{(F, A)}$.
$(\Leftarrow)$ The fact that every soft open set is soft semi-open and Proposition 3.13, implies that $\overline{(F, A)} \tilde{=} cl^s(F, A) \tilde{=} (\overline{(F, A)})^\circ$. This follows that $\overline{(F, A)}$ is soft open, for any soft open set (F, A). Hence the proof. □

Remark 4.1. Let (X, τ, A) be soft topological space and (F, A) be a soft set in (X, τ, A). Clearly, if (F, A) is soft open and soft closed set then it is a soft regular-open set. Moreover the converse is not true in general. But if soft topological space (X, τ, A) is soft extremally disconnected then the converse holds.

Theorem 4.4. *Let (X, τ, A) be soft extremally disconnected soft topological space and (F, A) be a soft set in (X, τ, A). If (F, A) is soft regular-open then (F, A) is a soft open and soft closed.*

Proof. (F, A) is soft regular-open implies (F, A) is a soft open set. Moreover soft extremally disconnected space follows that $\overline{(F, A)}$ is soft open. Therefore, $(F, A) \tilde{=} (\overline{(F, A)})^\circ \tilde{=} \overline{(F, A)}$. Hence (F, A) is a soft closed set. This completes the proof. □

Theorem 4.5. *Let (X, τ, A) be soft extremally disconnected soft topological space and (F, A) be soft set in (X, τ, A). Then the following statements are equivalent:*
(1) $(F, A)^c$ is a soft regular-open set.
(2) (F, A) is a soft regular-open set.
(3) (F, A) is soft open and soft closed.
(4) $(F, A) \tilde{=} (\overline{(F, A)})^\circ$.

Proof. (1) $\Rightarrow$ (2). Using Theorem 4.4 and since (X, τ, A) is soft extremally disconnected soft topological space then $(F, A)^c$ is soft open and soft closed. Therefore (F, A) is soft open and soft closed. Hence $(F, A) \tilde{=} (\overline{(F, A)})^\circ$. This

follows that (F,A) is a soft regular-open set.
(2) $\Rightarrow$ (3). This follows directly by using Theorem 4.4.
(3) $\Rightarrow$ (4). This is trivial.
(4) $\Rightarrow$ (1). Suppose that $(F,A)\tilde{=}(\overline{(F,A)})^{\circ}$. This implies that
$(F,A)^c\tilde{=}((\overline{(F,A)})^{\circ})^c\tilde{=}(((F,A)^{\circ})^c)^{\circ}\tilde{=}(\overline{(F,A)^c})^{\circ}$. Therefore, $(F,A)^c$ is soft regular-open. Hence the proof. □

Theorem 4.6. *Let (X,τ,A) be soft extremally disconnected soft topological space and (F,A) be a soft set in (X,τ,A). Then (F,A) is soft open if and only if $\overline{(F,A)}$ is soft regular-open.*

Proof. ($\Rightarrow$) Suppose that (X,τ,A) be soft extremally disconnected soft topological space and (F,A) be a soft open set. Then $\overline{(F,A)}$ is soft open implies that (F,A) is soft open and soft closed. Using Theorem 4.5, $\overline{(F,A)}$ is soft regular-open.
($\Leftarrow$) Suppose that $\overline{(F,A)}$ is soft regular-open. This follows that
$(F,A)\tilde{\subseteq}\overline{(F,A)}\tilde{\subseteq}\{\overline{\overline{(F,A)}}\}^{\circ}\tilde{=}(\overline{(F,A)})^{\circ}\tilde{=}(F,A)^{\circ}$. Hence (F,A) is soft open. This completes the proof. □

Corollary 4.7. *Let (X,τ,A) be soft extremally disconnected soft topological space and (F,A) be any soft set in (X,τ,A). Then $\overline{(F,A)^{\circ}}$ is soft regular-open.*

Definition 4.8. Let (F,A) be soft set in soft topological space (X,τ,A). Then the soft point e_G is said to be a soft θ-cluster point of (F,A), if for any soft open set (K,A) with $e_G\tilde{\in}(K,A)$, $\overline{(K,A)}\tilde{\cap}(F,A)\tilde{\neq}\tilde{\Phi}$. The set of all soft θ-cluster points of (F,A) is called the soft θ-closure of (F,A) and is denoted by $\overline{(F,A)_{\theta}}$.

Definition 4.9. Let (F,A) be a soft set in soft topological space (X,τ,A). Then the soft set (F,A) is called soft θ-closed, if $\overline{(F,A)_{\theta}}\tilde{=}(F,A)$. The complement of soft θ-closed set is soft θ-open.
Clearly soft θ-open(soft θ-closed) set) is soft open(soft closed) set.

Theorem 4.10. *Let (F,A) and (K,A) be soft sets in soft topological space (X,τ,A). Then*
(1) $\overline{(F,A)_{\theta}}\tilde{\subseteq}\overline{(K,A)_{\theta}}$, for $(F,A)\tilde{\subseteq}(K,A)$.
(2) If for each $i\in I$, (F,A_i) is soft θ-closed, then $\tilde{\cap}_{i\in I}(F,A_i)$ is soft θ-closed.

Proof. (1) This is trivial.
(2) Let us assume that for each $i\in I$, (F,A_i) is soft θ-closed. Then for each $i\in I$, $\overline{(F,A_i)_{\theta}}\tilde{=}(F,A_i)$. Therefore,
$\overline{(\tilde{\cap}_{i\in I}(F,A_i))_{\theta}}\tilde{\subseteq}\tilde{\cap}_{i\in I}\overline{(F,A_i)_{\theta}}\tilde{=}\tilde{\cap}_{i\in I}(F,A_i)\tilde{\subseteq}\overline{(\tilde{\cap}_{i\in I}(F,A_i))_{\theta}}$. This implies that $\overline{(\tilde{\cap}_{i\in I}(F,A_i))_{\theta}}\tilde{=}\tilde{\cap}_{i\in I}(F,A_i)$. This follows that $\tilde{\cap}_{i\in I}(F,A_i)$ is soft θ-closed. Hence the proof. □

Theorem 4.11. *Let (F,A) be a soft set in soft extremally disconnected soft topological space (X,τ,A). Then*

$\overline{(F,A)_\theta}\tilde{=}\tilde{\cap}\{(K,A)$:$(F,A)\tilde{\subseteq}(K,A)$ *and* (K,A) *is soft* θ*-closed* $\}$
$\tilde{=}\tilde{\cap}\{(K,A)$:$(F,A)\tilde{\subseteq}(K,A)$ *and* (K,A) *is soft regular-open* $\}$.

Proof. Suppose that $e_G\tilde{\notin}\overline{(F,A)_\theta}$. Then there exists a soft open set (K,A) such that $e_G\tilde{\in}(K,A)$ and $\overline{(K,A)}\tilde{\cap}(F,A)\tilde{=}\tilde{\Phi}$. Now Theorem 4.6 implies that $(\overline{(K,A)})^c$ is soft regular-open and therefore $(\overline{(K,A)})^c$ is soft θ-closed with $(F,A)\tilde{\subseteq}(\overline{(K,A)})^c$ and $e_G\tilde{\notin}(\overline{(K,A)}_\theta)^c$. This follows that $e_G\tilde{\notin}\tilde{\cap}\{(K,A)$:$(F,A)\tilde{\subseteq}(K,A)$ and (K,A) is soft θ-closed $\}$. Thus $\overline{(F,A)_\theta}\tilde{\subseteq}\tilde{\cap}\{(K,A)$:$(F,A)\tilde{\subseteq}(K,A)$ and (K,A) is soft θ-closed $\}$. Now consider $e_G\tilde{\notin}\tilde{\cap}\{(K,A)$:$(F,A)\tilde{\subseteq}(K,A)$ and (K,A) is soft θ-closed $\}$. This implies that there exists a soft θ-closed set (K,A) with $(F,A)\tilde{\subseteq}(K,A)$ and $e_G\tilde{\notin}(K,A)$. This follows that there exists a soft open set (H,A) such that $e_G\tilde{\in}(H,A)$ and $(H,A)\tilde{\subseteq}\overline{(H,A)}\tilde{\subseteq}(K,A)^c$. Therefore,
$\overline{(H,A)}\tilde{\cap}(F,A)\tilde{\subseteq}\overline{(H,A)}\tilde{\cap}(K,A)\tilde{=}\tilde{\Phi}$. Hence $e_G\tilde{\notin}\overline{(F,A)_\theta}$. Thus
$\tilde{\cap}\{(K,A)$:$(F,A)\tilde{\subseteq}(K,A)$ and (K,A) is soft θ-closed $\}\tilde{\subseteq}\overline{(F,A)_\theta}$.
The second proof follows similarly. This completes the proof. □

Theorem 4.12. *Let* (F,A) *be a soft set in soft extremally disconnected soft topological space* (X,τ,A)*. Then*
(1) $e_G\tilde{\in}\overline{(F,A)_\theta}$ $\Leftrightarrow$ *for any soft regular-open set* (K,A) *with* $e_G\tilde{\in}(K,A)$
implies $(K,A)\tilde{\cap}(F,A)\tilde{\neq}\tilde{\Phi}$.
(2) (F,A) *is soft* θ*-open* $\Leftrightarrow$ *for any soft point* $e_G\tilde{\in}(F,A)$*, there exists a soft regular-open set* (K,A) *with* $e_G\tilde{\in}(K,A)$ *such that* $(K,A)\tilde{\subseteq}(F,A)$.
(3) (F,A) *is soft regular-open* $\Leftrightarrow$ (F,A) *is soft* θ*-open and soft* θ*-closed.*

Proof. (1) and (2) follows directly by using Theorems 4.5 and 4.6.
(3) ($\Rightarrow$) (F,A) is soft regular-open implies that (F,A) is soft open and soft closed with $(F,A)\tilde{=}\overline{(F,A)}\tilde{=}\overline{(F,A)}_\theta$. Therefore, (F,A) is soft θ-closed. Using above argument and since $(F,A)^c$ is soft regular-open, $(F,A)^c$ is soft θ-closed. This follows that (F,A) is soft θ-open.
($\Leftarrow$) This is trivial. Hence the proof. □

Remark 4.2. Note that soft regular-open $\Rightarrow$ soft θ-open $\Rightarrow$ soft open. The following example shows that he converse is not true.

Example 4.13. Let $X=\{x_1,x_2\}$, $A=\{e_1,e_2\}$ and $\tau=\{\Phi_A,\tilde{X}_A,(F_1,A),$ $(F_2,A),(F_3,A)\}$, where $(F_1,A)=\{(e_1,\{x_1\})\}$, $(F_2,A)=\{(e_2,\{x_2\})\}$, $(F_3,A)=\{(e_1,\{x_1\}),(e_2,\{x_2\})\}$. Then (X,τ,A) is a soft topological space over X. Clearly, $(F,A)=\{(e_1,\{x_1\}),(e_2,\{x_2\})\}$ is soft open but not soft regular-open set. As, $(\overline{(F,A)})^\circ\tilde{=}\tilde{X}_A\tilde{\neq}(F,A)$.

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