J. Appl. Math. & Informatics Vol. **36***(2018), No. 1 - 2, pp. 39 - 50*
https://doi.org/10.14317/jami.2018.039

VALUE SHARING AND UNIQUENESS FOR THE POWER OF P-ADIC MEROMORPHIC FUNCTIONS†

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ABSTRACT. In this paper, we deal with the uniqueness problem for the power of p-adic meromorphic functions. The results obtained in this paper are the p-adic analogues and supplements of the theorems given by Yang and Zhang [Non-existence of meromorphic solution of a Fermat type functional equation, Aequationes Math. 76(2008), 140-150], Chen, Chen and Li [Uniqueness of difference operators of meromorphic functions, J. Ineq. Appl. 2012(2012), Art 48], Zhang [Value distribution and shared sets of differences of meromorphic functions, J. Math. Anal. Appl. 367(2010), 401-408].

AMS Mathematics Subject Classification : 30D35, 11S80.
Key words and phrases : value sharing, p-adic meromorphic function, uniqueness.

1. Introduction and main results

Mermorphic functions sharing values with their derivatives has become a subject of great interest in uniqueness theory recently. The paper by Rubel and Yang is the starting point of this topic, along with the following.

Theorem 1.1. [27] *Let f be a nonconstant entire function. If f and f' share two distinct finite values CM, then $f = f'$.*

In 1996, R. Brück [9] posed the following conjecture: Let f be a nonconstant entire function. Suppose that $\rho_1(f)$ is not a positive integer or infinite, if f and

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Received July 20, 2017. Revised October 13, 2017. Accepted October 26, 2017. *Corresponding author.
†The first author is supported by The Research Project of Education Department of Liaoning Province (L201612) and The Startup Foundation for Doctors of Shenyang Aerospace University (No. 16YB14). The third author is supported by The Research Project of Education Department of Liaoning Province (L201630).

f' share one finite value a CM, then

$$\frac{f'-a}{f-a}=c\,,$$

for some non-zero constant c, where $\rho_1(f)$ is the first iterated order of f which is defined by

$$\rho_1(f)=\limsup_{r\to\infty}\frac{\log\log T(r,f)}{\log r}\,.$$

In 1998, Gundersen and Yang [13] proved that the conjecture is true if f is of finite order, and in 1999, Yang [31] generalized their result to the k-th derivatives. In 2004, Chen and Shon [11] proved that the conjecture is true for entire functions of first iterated order $\rho_1(f)<1/2$.

In 2008, Yang and Zhang considered the uniqueness problems on meromorphic function f^n sharing value with its first derivative. One of their results can be stated as follows.

Theorem 1.2. [32] *Let $f(z)$ be a non-constant meromorphic function and $n\geq 12$ be an integer. Let $F=f^n$. If F and F' share 1 CM, then $F=F'$, and f assumes the form $f(z)=ce^{\frac{1}{n}z}$.*

In 2010, Zhang replaced F' by $F(z+c)$ and proved the following theorem.

Theorem 1.3. [33] *Set $S_1=\{1,\omega,...,\omega^{n-1}\}$ and $S_2=\{\infty\}$, where $\omega=cos\frac{2\pi}{n}+isin\frac{2\pi}{n}$. Suppose f is a nonconstant meromorphic function of finite order such that $E_{f(z)}(S_1)=E_{f(z+c)}(S_1)$ and $E_{f(z)}(S_2)=E_{f(z+c)}(S_2)$. If $n\geq 4$, then $f(z)=tf(z+c)$, where $t^n=1$.*

In 2012, Chen, Chen and Li replaced F' by Δ_cF where $\Delta_cF=F(z+c)-F(z)$ and proved the following theorem.

Theorem 1.4. [10] *Let $f(z)$ be a non-constant meromorphic function of finite order and $n\geq 9$ be an integer. Let $F(z)=f(z)^n$. If $F(z)$ and Δ_cF share 1, ∞ CM, then $F(z)=\Delta_cF$.*

In recent years, similar problems are investigated in non-Archimedean fields. Now let K be an algebraically closed field of characteristic zero, complete for a non-Archimedean absolute value. We denote by $A(K)$ the ring of entire functions in K and by $M(K)$ the field of meromorphic functions. The value shaing problems for meromorphic functions in K was investigated first in [1] and [20]. In recent years, numerous interesting results was obtained in the investigation of the value-sharing problem for meromorphic function in K (see, for example, [2],[4],[5], [7],[8], [12], [14]-[18], [21],[23]-[26], [28], [29]).

Let us recall some basic definitions. For $f \in M(K)$ and $S \subset \hat{K}$, we define

$$E_f(S) = \bigcup_{a \in S} \{(z,m) | f(z) = a \ \textit{multiplicity}\ m\},$$

and we denote by $E_f^k(a)$ the set of all a-points of f where an a-point with mutiplicity m is counted m times if $m \leq k$ and $k+1$ times if $m > k$. It's obvious that if $E_f^k(a) = E_g^k(a)$, then z_0 is a zero of $f - a$ with multiplicity $m(\leq k)$ if and only if it is a zero of $g - a$ with multiplicity $m(\leq k)$ and z_0 is a zero of $f - a$ with multiplicity $m(> k)$ if and only if it is a zero of $g - a$ with multiplicity $n(> k)$, where m is not necessarily equal to n.

Let F be a nonempty subset of $M(K)$. Two functions f, g of F are said to share S, counting multiplicity(share S CM), if $E_f(S) = E_g(S)$. Further, for $f \in M(K)$, we define the shift of f as $f(z+c)$, where $c \in K$ is a nonzero constant.

In the present paper, we discuss the uniqueness problem for the power of p-adic meromorphic functions and their shifts and prove the following theorems.

Theorem 1.5. *Let $f(z)$ be a p-adic meromorphic function and $n \geq 7$ be an integer. If $E_{f^n(z)}(1) = E_{f^n(z+c)}(1)$ and $E_{f(z)}(\infty) = E_{f(z+c)}(\infty)$, then $f(z) = tf(z+c)$, where $t^n = 1$.*

Corollary 1.6. *Let $f(z)$ be a p-adic entire function and $n \geq 5$ be an integer. If $E_{f^n(z)}(1) = E_{f^n(z+c)}(1)$, then $f(z) = tf(z+c)$, where $t^n = 1$.*

Theorem 1.7. *Let $f(z)$ be a p-adic meromorphic function and $n \geq 7$ be an integer. If $E^2_{f^n(z)}(1) = E^2_{f^n(z+c)}(1)$ and $E_{f(z)}(\infty) = E_{f(z+c)}(\infty)$, then $f(z) = tf(z+c)$, where $t^n = 1$.*

Theorem 1.8. *Let $f(z)$ be a p-adic meromorphic function and $n \geq 8$ be an integer. If $E^2_{f^n(z)}(1) = E^2_{f^n(z+c)}(1)$ and $E^0_{f(z)}(\infty) = E^0_{f(z+c)}(\infty)$, then $f(z) = tf(z+c)$, where $t^n = 1$.*

The main tool of the proof is the p-adic Nevanlinna theory (see, for example, [20], [22], [19]). So in the next section, we establish the basic properties of the characteristic functions of p-adic meromorphic functions.

2. Counting functions and Characteristic functions of p-adic meromorphic functions

Let f be a nonconstant entire function on K and $b \in K$. Then we can write f in the following form

$$f = \sum_{n=q}^{\infty} b_n (z-b)^n,$$

where $b_q \neq 0$ and we denote $\omega_f^0(b) = q$. For a point $a \in K$, we define the function $\omega_f^a : K \to N$ by $\omega_f^a(b) = \omega_{f-a}^0(b)$.

For a real number ρ with $0 < \rho \leq r$. Take $a \in K$ and we set

$$N_f(a,r) = \frac{1}{ln\rho}\int_{\rho}^{r}\frac{n_f(a,x)}{x}dx\,,$$

where $n_f(a,x)$ is the number of solutions of the equation $f(z) = a$(counting multiplicities) in the disk $D_x = \{z \in K : |z| \leq x\}$. If $a = 0$, the we set $N_f(r) = N_f(0,r)$.

If l is a positive integer, then we define

$$N_{l,f}(a,r) = \frac{1}{ln\rho}\int_{\rho}^{r}\frac{n_{l,f}(a,x)}{x}dx\,,$$

where $n_{l,f}(a,x) = \sum_{|z|\leq r}\min\{\omega_{f-a}(z), l\}$.

Let k be a positive integer. Define the function ω_f^k from K into N by $\omega_f^k(z) = 0$ if $\omega_f^0(z) > k$ and $\omega_f^k(z) = \omega_f^0(z)$ if $\omega_f^0(z) \leq k$. And $n_f^{\leq k}(r) = \sum_{|z|\leq r}\omega_f^{\leq k}(z)$, $n_f^{\leq k}(a,r) = n_{f-a}^{\leq k}(r)$.

Define

$$N_f^{\leq k}(a,r) = \frac{1}{ln\rho}\int_{\rho}^{r}\frac{n_f^{\leq k}(a,x)}{x}dx\,,$$

If $a = 0$, then we set $N_f^{\leq k}(r) = N_f^{\leq k}(0,r)$. Set

$$N_{l,f}^{\leq k}(a,r) = \frac{1}{ln\rho}\int_{\rho}^{r}\frac{n_{l,f}^{\leq k}(a,x)}{x}dx\,,$$

where $n_{l,f}^{\leq k}(a,x) = \sum_{|z|\leq r}\min\{\omega_{f-a}^{\leq k}(z), l\}$. In a similar way, we can define $N_f^{<k}(a,r)$, $N_{l,f}^{<k}(a,r)$, $N_f^{>k}(a,r)$, $N_f^{\geq k}(a,r)$, $N_{l,f}^{\geq k}(a,r)$ and $N_{l,f}^{>k}(a,r)$ which are called truncated counting function. Such notations was firstly introduced by Han, Mori and Tohge [16], Han and Yi [17].

Recall that for a nonconstant entire function $f(z)$ on K, represented by the power series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

for each $r > 0$, we define $|f|_r = \max\{|a_n|r^n, 0 \leq n < \infty\}$.

Now let $f = \frac{f_1}{f_2}$ be a nonconstant meromorphic function on K, where f_1 and f_2 are entire functions on K having no common zeros. We set $|f|_r = \frac{|f_1|}{|f_2|}$. For a point $a \in K \cup \{\infty\}$, we define the function $\omega_f^a : K \to N$ by $\omega_f^a(b) = \omega_{f_1 - af_2}^0(b)$ with $a \neq \infty$ and $\omega_f^\infty(b) = \omega_{f_2}^0(b)$.

Taking $a \in K$, we denote the counting function of zeros of $f-a$, counting multiplicity, in the disk $D_r = \{z \in K : |z| \leq r\}$, i.e. we set $N_f(a,r) = N_{f_1-af_2}(r)$ and set $N_f(\infty, r) = N_{f_2}(r)$. In a similar way, for nonconstant meromorphic function on K, we can define $N_f^{<k}(a,r)$, $N_{l,f}^{<k}(a,r)$, $N_f^{>k}(a,r)$, $N_f^{\geq k}(a,r)$, $N_{l,f}^{\geq k}(a,r)$ and $N_{l,f}^{>k}(a,r)$.

We define

$$m_f(\infty, r) = \max\{0, log|f|_r\}, \quad m_f(a,r) = m_{\frac{1}{f-a}}(\infty, r)\,,$$

and then characteristic function of f by

$$T_f(r) = m_f(\infty, r) + N_f(\infty, r)\,.$$

Thus we get

$$N_f(a,r) + m_f(a,r) = T_f(r) + O(1)\,,$$

where $a \in K \cup \{\infty\}$ and

$$T_f(r) = T_{\frac{1}{f}}(r) + O(1), \quad m_{\frac{f^{(k)}}{f}}(\infty, r) = O(1)\,.$$

3. Some Lemmas

In this section, we present some lemmas which will be needed in the sequel.

Lemma 3.1. [19][6] *Let f be a nonconstant meromorphic function on K and let a_1, a_2,...,a_q be distinct points of K. Then*

$$(q-1)T_f(r) \leq N_{1,f}(\infty, r) + \sum_{i=1}^{q} N_{1,f}(a_i, r) - N_{0,f'}(r) - logr + O(1)$$

By similar discussions as in [3], we can obtain the analogous lemmas in p-adic case as follows:

Lemma 3.2. *Let f and g be nonconstant meromorphic functions on K. If $E_f(1) = E_g(1)$ and $E_f(\infty) = E_g(\infty)$, then one of the following three cases holds:*

$$(i) \quad T_f(r) \leq N_{1,f}(0,r) + N_{1,f}^{\geq 2}(0,r) + N_{1,g}(0,r) + N_{1,g}^{\geq 2}(0,r) + N_{1,f}(\infty, r) + N_{1,g}(\infty, r) - logr + O(1)\,,$$

$$(ii) \quad f = g\,, \quad (iii) \quad fg = 1\,.$$

Proof. Set

$$H = \left(\frac{f''}{f'} - \frac{2f'}{f-1}\right) - \left(\frac{g''}{g'} - \frac{2g'}{g-1}\right).$$

First we suppose that $H \not\equiv 0$. We consider the poles of H. It is clear that all poles of H are of order 1. We can deduce from the definition of H that the poles of H occur at the zeros of f' and g' since $E_f(1) = E_g(1)$ and $E_f(\infty) = E_g(\infty)$.

It's obvious that $m_H(\infty, r) = O(1)$, and

$$\begin{aligned} N_f^{\leq 1}(1,r) &\leq N_H(0,r) \leq T_H(r) + O(1) \leq N_H(\infty,r) + O(1) \\ &\leq N_{1,f}^{\geq 2}(0,r) + N_{1,g}^{\geq 2}(0,r) + N_{1,0,f'}(r) + N_{1,0,g'}(r) + O(1)\,, \end{aligned} \tag{1}$$

where $N_{1,0,f'}(r)$ is the counting function of those zeros of f' that are not zeros of $f(f-1)$, while each zero is counted with multiplicity 1.
On the other hand, by Lemma 3.1, we have

$$T_f(r) \leq N_{1,f}(\infty,r) + N_{1,f}(0,r) + N_{1,f}(1,r) - N_{0,f'}(r) - logr + O(1)\,, \tag{2}$$

Since $E_f(1) = E_g(1)$, we note that

$$N_{1,f}(1,r) = N_f^{\leq 1}(1,r) + N_{1,f}^{\geq 2}(1,r) = N_f^{\leq 1}(1,r) + N_{1,g}^{\geq 2}(1,r)\,, \tag{3}$$

Then

$$\begin{aligned} T_f(r) \leq\ & N_{1,f}(\infty,r) + N_{1,f}(0,r) + N_f^{\leq 1}(1,r) \\ & + N_{1,g}^{\geq 2}(1,r) - N_{0,f'}(r) - logr + O(1)\,. \end{aligned} \tag{4}$$

Next we consider $N_{1,g}^{\geq 2}(1,r)$.

$$\begin{aligned} N_{g'}(0,r) - N_g(0,r) + N_{1,g}(0,r) &= N_{\frac{g'}{g}}(0,r) \leq T_{\frac{g'}{g}}(r) + O(1) \\ = N_{\frac{g'}{g}}(\infty,r) + m_{\frac{g'}{g}}(\infty,r) + O(1) &= N_{1,g}(\infty,r) + N_{1,g}(0,r) + O(1)\,. \end{aligned} \tag{5}$$

So

$$N_{g'}(0,r) \leq N_{1,g}(\infty,r) + N_g(0,r) + O(1)\,. \tag{6}$$

Moreover

$$N_{0,g'}(r) + N_{1,g}^{\geq 2}(1,r) + N_g^{\geq 2}(0,r) - N_{1,g}^{\geq 2}(0,r) \leq N_{g'}(0,r)\,, \tag{7}$$

where $N_{0,g'}(r)$ is the counting function of those zeros of g' that are not zeros of $g(g-1)$. From (6) and (7) , we get

$$N_{0,g'}(r) + N_{1,g}^{\geq 2}(1,r) \leq N_{1,g}(\infty,r) + N_{1,g}(0,r) + O(1)\,. \tag{8}$$

Combining (1), (4) and (8), we obtain

$$\begin{aligned} T_f(r) \leq N_{1,f}(0,r) + N_{1,f}^{\geq 2}(0,r) + N_{1,g}(0,r) + N_{1,g}^{\geq 2}(0,r) + N_{1,f}(\infty,r) \\ + N_{1,g}(\infty,r) - logr + O(1)\,. \end{aligned}$$

Suppose $H \equiv 0$. Then by integration we get

$$f \equiv \frac{ag+b}{cg+d}\,, \tag{9}$$

where a, b, c and d are constants and $ad-bc \neq 0$. So $T_f(r) = T_g(r) + O(1)$.

We now consider the following cases.

Case 1. Let $ac \neq 0$. Since $E_f(\infty) = E_g(\infty)$, we can obtain that f and g have no pole from (9). Since

$$f - \frac{a}{c} = \frac{bc-ad}{c(cg+d)}\,, \tag{10}$$

it follows that $f - \frac{a}{c}$ has no zero. So By Lemma 3.1, we get

$$T_f(r) \leq N_{1,f}(\infty, r) + N_{1,f-\frac{a}{c}}(0, r) + N_{1,f}(0, r) + O(1) = N_{1,f}(0, r) + O(1)\,,$$

which implies (i).

Case 2. $a \neq 0$ and $c = 0$. Then $f = \frac{a}{d}g + \frac{b}{d}$. If $b \neq 0$, by Lemma 3.1,

$$\begin{aligned} T_f(r) &\leq N_{1,f}(\infty, r) + N_{1,f-\frac{b}{d}}(0, r) + N_{1,f}(0, r) + O(1) \\ &= N_{1,f}(\infty, r) + N_{1,g}(0, r) + N_{1,f}(0, r) + O(1)\,, \end{aligned}$$

which implies (i).

If $b = 0$, then $f = \frac{ag}{d}$. If $\frac{a}{d} = 1$, we obtain (ii). If $\frac{a}{d} \neq 1$, then by $E_f(1) = E_g(1)$ we get $f \neq 1$ and $f \neq \frac{a}{d}$. According to Lemma 3.1, we have

$$T_f(r) \leq N_{1,f}(\infty, r) + N_{1,f}(1, r) + N_{1,f}(\frac{a}{d}, r) + O(1) = N_{1,f}(\infty, r) + O(1)\,,$$

which implies (i).

Case 3. $a = 0$ and $c \neq 0$. Then $f = \frac{b}{cg+d}$. If $d \neq 0$, by Lemma 3.1,

$$\begin{aligned} T_f(r) &\leq N_{1,f}(\infty, r) + N_{1,f-\frac{b}{d}}(0, r) + N_{1,f}(0, r) + O(1) \\ &= N_{1,f}(\infty, r) + N_{1,g}(0, r) + N_{1,f}(0, r) + O(1)\,, \end{aligned}$$

which implies (i).

If $d = 0$, then $f = \frac{b}{cg}$. If $\frac{b}{c} = 1$, we obtain (iii). If $\frac{b}{c} \neq 1$, then by $E_f(1) = E_g(1)$ we get $f \neq 1$ and $f \neq \frac{b}{c}$. According to Lemma 3.1, we have

$$T_f(r) \leq N_{1,f}(\infty, r) + N_{1,f}(1, r) + N_{1,f}(\frac{b}{c}, r) + O(1) = N_{1,f}(\infty, r) + O(1)\,,$$

which implies (i). This completes the proof of the lemma. □

Lemma 3.3. *Let f and g be nonconstant meromorphic functions on K. If $E_f^2(1) = E_g^2(1)$ and $E_f(\infty) = E_g(\infty)$, then one of the following three cases holds:*

$$(i) \quad T_f(r) \le N_{1,f}(0,r) + N_{1,f}^{\ge 2}(0,r) + N_{1,g}(0,r) + N_{1,g}^{\ge 2}(0,r) + N_{1,f}(\infty,r) + N_{1,g}(\infty,r) - logr + O(1),$$

$$(ii) \quad f = g, \quad (iii) \quad fg = 1.$$

Lemma 3.4. *Let f and g be nonconstant meromorphic functions on K. If $E_f^2(1) = E_g^2(1)$ and $E_f^0(\infty) = E_g^0(\infty)$, then one of the following three cases holds:*

$$(i) \quad T_f(r) \le N_{1,f}(0,r) + N_{1,f}^{\ge 2}(0,r) + N_{1,g}(0,r) + N_{1,g}^{\ge 2}(0,r) + N_{1,f}(\infty,r) + N_{1,g}(\infty,r) + N_{1,*}(\infty,r) - logr + O(1),$$

$$(ii) \quad f = g, \quad (iii) \quad fg = 1.$$

where $N_{1,*}(\infty,r)$ denotes the reduced counting function of those poles of f whose multiplicities differ from the multiplicities of the corresponding poles of g.

Lemma 3.5. [2] *Let f be a nonconstant p-adic meromorphic function. Then*

$$m_{\frac{f(z+c)}{f}}(\infty,r) = O(1); \; T_{f(z+c)}(r) = T_{f(z)}(r) + O(1).$$

4. Proof of Theorem 1.5

Let

$$F = f^n(z), \; G = F(z+c) = f^n(z+c). \tag{11}$$

Then it is easy to verify $E_F(1) = E_G(1)$ and $E_F(\infty) = E_G(\infty)$. Suppose the Case (i) in Lemma 3.2 holds

$$T_F(r) \le N_{1,F}(0,r) + N_{1,F}^{\ge 2}(0,r) + N_{1,G}(0,r) + N_{1,G}^{\ge 2}(0,r) + N_{1,F}(\infty,r) + N_{1,G}(\infty,r) - logr + O(1), \tag{12}$$

It's obvious that

$$N_{1,F}(0,r) + N_{1,F}^{\ge 2}(0,r) \le 2N_{1,F}(0,r) = 2N_{1,f}(0,r) \le 2T_f(r). \tag{13}$$

According to Lemma 3.5, we obtain

$$N_{1,G}(0,r) + N_{1,G}^{\ge 2}(0,r) \le 2N_{1,G}(0,r) = 2N_{1,F(z+c)}(0,r) = 2N_{1,f(z+c)}(0,r) \le 2T_{f(z+c)}(r) = 2T_f(r) + O(1), \tag{14}$$

$$N_{1,F}(\infty,r) = N_{1,f}(\infty,r) \le T_f(r), \tag{15}$$

$$N_{1,G}(\infty,r) = N_{1,F(z+c)}(\infty,r) = N_{1,f(z+c)}(\infty,r) \le T_{f(z+c)}(r) = T_f(r) + O(1). \tag{16}$$

Combining (12), (13), (14), (15) and (16), we deduce

$$T_F(r) = nT_f(r) \le 6T_f(r) + O(1)\,, \tag{17}$$

that is,

$$(n-6)T_f(r) \le O(1)\,, \tag{18}$$

which contradicts with $n \ge 7$. Therefore $F = G$ or $FG = 1$.

If $F = G$, that is $f^n(z) = f^n(z+c)$. So we deduce $f(z) = tf(z+c)$, where t is a constant and $t^n = 1$.

If $FG = 1$, that is

$$f(z)f(z+c) = 1\,. \tag{19}$$

From (19) we obtain $f(z) \ne 0, \infty$ and $\frac{f(z+c)}{f(z)} = \frac{1}{f^2(z)}$. According to Lemma 3.5, we can deduce

$$\begin{aligned} T_{f^2}(r) &= T_{\frac{1}{f^2}}(r) + O(1) = T_{\frac{f(z+c)}{f(z)}}(r) + O(1) \\ &= m_{\frac{f(z+c)}{f(z)}}(\infty, r) + N_{\frac{f(z+c)}{f(z)}}(\infty, r) + O(1) = O(1)\,, \end{aligned} \tag{20}$$

which is a contradiction. This completes the proof of Theorem 1.5.

5. Proof of Theorem 1.7

Let

$$F = f^n(z),\ G = F(z+c) = f^n(z+c)\,. \tag{21}$$

Then it is easy to verify $E_F^2(1) = E_G^2(1)$ and $E_F(\infty) = E_G(\infty)$. Suppose the Case (i) in Lemma 3.3 holds

$$\begin{aligned} T_F(r) \le N_{1,F}(0,r) &+ N_{1,F}^{\ge 2}(0,r) + N_{1,G}(0,r) + N_{1,G}^{\ge 2}(0,r) + N_{1,f}(\infty, r) \\ &+ N_{1,g}(\infty, r) - logr + O(1)\,, \end{aligned} \tag{22}$$

Similar to the proof of Theorem 1.5, we can get the conclusion of Theorem 1.7.

6. Proof of Theorem 1.8

Let

$$F = f^n(z),\ G = F(z+c) = f^n(z+c)\,. \tag{23}$$

Then it is easy to verify $E_F^2(1) = E_G^2(1)$ and $E_F^0(\infty) = E_G^0(\infty)$. Suppose the Case (i) in Lemma 3.4 holds

$$\begin{aligned} T_F(r) &\le N_{1,F}(0,r) + N_{1,F}^{\ge 2}(0,r) + N_{1,G}(0,r) + N_{1,G}^{\ge 2}(0,r) \\ &+ N_{1,F}(\infty, r) + N_{1,G}(\infty, r) + N_{1,*}(\infty, r) - logr + O(1)\,, \end{aligned} \tag{24}$$

It's obvious that

$$N_{1,F}(0,r) + N_{1,F}^{\ge 2}(0,r) \le 2N_{1,F}(0,r) = 2N_{1,f}(0,r) \le 2T_f(r)\,, \tag{25}$$

And

$$N_{1,F}(\infty,r) = N_{1,f}(\infty,r) \leq T_f(r)\,, \tag{26}$$

$$N_{1,*}(\infty,r) \leq N_{1,F}(\infty,r) \leq T_f(r)\,. \tag{27}$$

According to Lemma 3.5, we obtain

$$\begin{aligned} N_{1,G}(0,r) + N_{1,G}^{\geq 2}(0,r) &\leq 2N_{1,G}(0,r) = 2N_{1,F(z+c)}(0,r) \\ &= 2N_{1,f(z+c)}(0,r) \leq 2T_{f(z+c)}(r) = 2T_f(r) + O(1)\,, \end{aligned} \tag{28}$$

$$\begin{aligned} N_{1,G}(\infty,r) = N_{1,F(z+c)}(\infty,r) &= N_{1,f(z+c)}(\infty,r) \\ &\leq T_{f(z+c)}(r) = T_f(r) + O(1)\,. \end{aligned} \tag{29}$$

Combining (24), (25), (26), (27), (28) and (29), we deduce

$$T_F(r) = nT_f(r) \leq 7T_f(r) + O(1)\,, \tag{30}$$

that is,

$$(n-7)T_f(r) \leq O(1)\,, \tag{31}$$

which contradicts with $n \geq 8$. Therefore $F = G$ or $FG = 1$. Similar to the proof of Theorem 1.5, we can get the conclusion of Theorem 1.8.

7. Remarks

With $S_1 = \{1, \omega, ..., \omega^{n-1}\}$ and $S_2 = \{\infty\}$, where $\omega = cos\frac{2\pi}{n} + isin\frac{2\pi}{n}$, we can get the following equivalent forms of Theorem 1.5, Corollary 1.6, Theorem 1.7 and Theorem 1.8 respectively.

Theorem 7.1. *Let $f(z)$ be a p-adic meromorphic function and $n \geq 7$ be an integer. If $E_{f(z)}(S_1) = E_{f(z+c)}(S_1)$ and $E_{f(z)}(S_2) = E_{f(z+c)}(S_2)$, then $f(z) = tf(z+c)$, where $t^n = 1$.*

Corollary 7.2. *Let $f(z)$ be a p-adic entire function and $n \geq 5$ be an integer. If $E_{f(z)}(S_1) = E_{f(z+c)}(S_1)$, then $f(z) = tf(z+c)$, where $t^n = 1$.*

Theorem 7.3. *Let $f(z)$ be a p-adic meromorphic function and $n \geq 7$ be an integer. If $E^2_{f(z)}(S_1) = E^2_{f(z+c)}(S_1)$ and $E_{f(z)}(S_2) = E_{f(z+c)}(S_2)$, then $f(z) = tf(z+c)$, where $t^n = 1$.*

Theorem 7.4. *Let $f(z)$ be a p-adic meromorphic function and $n \geq 8$ be an integer. If $E^2_{f(z)}(S_1) = E^2_{f(z+c)}(S_1)$ and $E^0_{f(z)}(S_2) = E^0_{f(z+c)}(S_2)$, then $f(z) = tf(z+c)$, where $t^n = 1$.*

Acknowledgments The authors would like to thank the referee for reading the manuscript very carefully and making a number of valuable and kind comments which improved the presentation.

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