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Coefficient Estimates for a Subclass of Bi-univalent Functions Defined by Sălăgean Type q-Calculus Operator

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Abstract. In this paper, we introduce and investigate a new subclass of bi-univalent functions defined by *Sălăgean q-calculus operator* in the open disk $\mathbb{U}$. For functions belonging to the subclass, we obtain estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$. Some consequences of the main results are also observed.

1. Introduction

Let $\mathcal{A}$ denote the family of functions *analytic* in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\},$$

which are normalized by the condition:

$$f(0) = f'(0) - 1 = 0$$

and given by the following Taylor-Maclaurin series:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k. \tag{1.1}$$

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Also let $\mathcal{S}$ be the class of functions $f \in \mathcal{A}$ of the form given by (1.1), which are univalent in $\mathbb{U}$. The Koebe one-quarter theorem [7] ensures that the image of $\mathbb{U}$ under every univalent function $f \in \mathcal{S}$ contains a disk of radius $\frac{1}{4}$. Hence every function $f \in \mathcal{S}$ has an inverse f^{-1}, defined by

$$f^{-1}(f(z)) = z, \quad (z \in \mathbb{U})$$

and

$$f^{-1}(f(w)) = w, \quad \left(|w| < r_0(f); r_0(f) \geq \frac{1}{4}\right),$$

where

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \dots .$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by the Taylor-Maclaurin series expansion (1.1). For a brief history and interesting examples of functions in the class Σ, see [28] (see also [4]). From the work of Srivastava *et al.* [28], we choose to recall the following examples of functions in the class Σ:

$$\frac{z}{1-z}, \qquad -log(1-z), \qquad \frac{1}{2}log\left(\frac{1+z}{1-z}\right).$$

However, familiar Koebe function is not a member of Σ.

The class of bi-univalent functions was investigated by Lewin [13], who proved that $|a_2| < 1.51$. In 1981, Styer and Wright [30] showed that $|a_2| > 4/3$. Subsequently, Brannan and Clunie [3] improved Lewin's result to $|a_2| \leq \sqrt{2}$. Netanyahu [14], showed that $\max_{f \in \Sigma} |a_2| = \frac{4}{3}$. In 1985, Branges [2] proved Bieberbach conjecture which showed that

$$|a_n| \leq n; \quad (n \in N - 1),$$

N being positive integer.

The problem of finding coefficient estimates for the bi-univalent functions has received much attention in recent years. In fact, the aforecited work of Srivastava *et al.* [28] essentially revived the investigation of various subclasses of bi-univalent function class Σ in recent years and that it leads to a flood of papers on the subject (see, for e.g., [6, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29]); it was followed by such works as those by Tang *et al.* [31], Xu *et al.* [32, 33] and Lashin [12], and others (see, for e.g., [1, 5, 8]). The coefficient estimate problem involving the bound of $|a_n| (n \in \mathbb{N} \setminus \{1, 2\})$ for each $f \in \Sigma$ is still an open problem.

In the field of geometric function theory, various subclasses of the normalized analytic function class $\mathcal{A}$ have been studied from different view points. The q-calculus as well as the fractional calculus provide important tools that have been used in order to investigate various subclasses of $\mathcal{A}$. Historically speaking, the firm footing of the usage of the q-calculus in the context of geometric function theory which was actually provided and q-hypergeometric functions were first used in geometric function theory in a book chapter by Srivastava (see, for details, [18, pp. 347 et seq.]). Ismail *et al.* [10] introduced the class of generalized complex functions via q-calculus on some subclasses of analytic functions. Recently, Purohit and Raina [16] investigated applications of fractional q-calculus operator to define new classes of functions which are analytic in unit disk $\mathbb{U}$ (see, for details, [9]).

For $0 < q < 1$, the q-derivative of a function f given by (1.1) is defined as

$$D_q f(z) = \begin{cases} \frac{f(qz)-f(z)}{(q-1)z} & \text{for } z \neq 0, \\ f'(0) & \text{for } z = 0. \end{cases} \tag{1.2}$$

We note that $\lim\limits_{q\to 1^-} D_q f(z) = f'(z)$. From (1.2), we deduce that

$$D_q f(z) = 1 + \sum_{k=2}^{\infty} [k]_q a_k\, z^{k-1}, \tag{1.3}$$

where as $q \to 1^-$

$$[k]_q = \frac{1-q^k}{1-q} = 1 + q + ... + q^k \longrightarrow k. \tag{1.4}$$

Making use of the q-differential operator for function $f \in \mathcal{A}$, we introduced the Sălăgean q-differential operator as given below

$$D_q^0 f(z) = f(z)$$
$$D_q^1 f(z) = zD_q f(z)$$
$$D_q^n f(z) = zD_q(D_q^{n-1} f(z))$$
$$D_q^n f(z) = z + \sum_{k=2}^{\infty} [k]_q^n a_k z^k \qquad (n \in \mathbb{N}_0, z \in \mathbb{U}). \tag{1.5}$$

We note that $\lim_q \longrightarrow 1^-$

$$D^n f(z) = z + \sum_{k=2}^{\infty} k^n a_k z^k \qquad (n \in \mathbb{N}_0, z \in \mathbb{U}), \tag{1.6}$$

the familiar Sălăgean derivative [17].

Recently, Kamble and Shrigan [11] introduce the following two subclasses of the bi-univalent function class Σ and obtained estimate on first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions in these subclasses as follows.

Definition 1.1.([11]) For $0 < \alpha \leq 1, 0 < q < 1, \lambda \geq 1, \mu \geq 0$ and $n \in \mathbb{N}_0$, a function $f(z)$ given by (1.1) is said to be *in the class* $\mathcal{H}_\Sigma^{q,\mu}(n, \alpha, \lambda)$ if the following conditions are satisfied

$$f \in \Sigma \text{ and } \left| arg\left((1-\lambda)\left(\frac{D_q^n f(z)}{z}\right)^{\mu} + \lambda\left(D_q^n f(z)\right)'\left(\frac{D_q^n f(z)}{z}\right)^{\mu-1} \right) \right| < \frac{\alpha\pi}{2} \tag{1.7}$$

and

$$\left| arg\left((1-\lambda)\left(\frac{D_q^n g(w)}{w}\right)^{\mu} + \lambda\left(D_q^n g(w)\right)'\left(\frac{D_q^n g(w)}{w}\right)^{\mu-1} \right) \right| < \frac{\alpha\pi}{2}, \tag{1.8}$$

where the function g is given by

$$g(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + ... \tag{1.9}$$

and D_q^n is the Sălăgean q-differential operator.

Theorem 1.2.([11]) *Let $f(z)$ given by (1.1) be in the function class $\mathcal{H}_{\Sigma}^{q,\mu}(n,\alpha,\lambda)$. Then*

$$|a_2| \le \frac{2\alpha}{\sqrt{\alpha(2(2\lambda+\mu)[3]_q^n - (\lambda^2+2\lambda+\mu)[2]_q^{2n}) + (\lambda+\mu)^2[2]_q^{2n}}} \tag{1.10}$$

and

$$|a_3| \le \frac{4\alpha^2}{(\lambda+\mu)^2[2]_q^{2n}} + \frac{2\alpha}{(2\lambda+\mu)[3]_q^n}, \tag{1.11}$$

where $0 < \alpha \le 1, 0 < q < 1, \lambda \ge 1, \mu \ge 0$ *and* $n \in \mathbb{N}_0$.

Definition 1.3.([11]) For $0 \le \beta < 1, 0 < q < 1, \lambda \ge 1, \mu \ge 0$ and $n \in \mathbb{N}_0$, a function $f(z)$ given by (1.1) is said to be *in the class* $\mathcal{H}_{\Sigma}^{q,\mu}(n,\beta,\lambda)$ if the following conditions are satisfied

$$f \in \Sigma \text{ and } Re\left\{ (1-\lambda)\left(\frac{D_q^n f(z)}{z}\right)^{\mu} + \lambda\left(D_q^n f(z)\right)'\left(\frac{D_q^n f(z)}{z}\right)^{\mu-1} \right\} > \beta \tag{1.12}$$

and

$$Re\left\{ (1-\lambda)\left(\frac{D_q^n g(w)}{w}\right)^{\mu} + \lambda\left(D_q^n g(w)\right)'\left(\frac{D_q^n g(w)}{w}\right)^{\mu-1} \right\} > \beta. \tag{1.13}$$

Theorem 1.4.([11]) *Let $f(z)$ given by (1.1) be in the function class $\mathcal{H}_{\Sigma}^{q,\mu}(n,\beta,\lambda)$. Then*

$$|a_2| \le \min\left\{ \sqrt{\frac{4(1-\beta)}{|2[3]_q^n + (\mu-1)[2]_q^{2n}(2\lambda+\mu)|}}, \frac{2(1-\beta)}{(\lambda+\mu)[2]_q^n} \right\} \tag{1.14}$$

and

$$|a_3| \leq \min\left\{\frac{4(1-\beta)^2}{(\lambda+\mu)^2[2]_q^{2n}} + \frac{2(1-\beta)}{(2\lambda+\mu)[3]_q^n}, \frac{(1-\beta)\left\{|4[3]_q^n + [2]_q^{2n}(\mu-1)| - [2]_q^{2n}(|\mu-1|)\right\}}{|2[3]_q^n + (\mu-1)[2]_q^{2n}|(2\lambda+\mu)[3]_q^n}\right\}, \tag{1.15}$$

where $0 \leq \beta < 1, 0 < q < 1, \lambda \geq 1, \mu \geq 0$ *and* $n \in \mathbb{N}_0$.

Remark 1.5. By appropriately specializing the parameters in Definition 1.1 and 1.3, we can get several known subclasses of the bi-univalent function class Σ. For example:

(i) For $n = 0$ and $q \to 1^-$, we obtain the bi-univalent function classes

$$\mathcal{H}_\Sigma^{1,\mu}(0,\alpha,\lambda) = \mathcal{N}_\Sigma^\mu(\alpha,\lambda) \qquad \text{and} \qquad \mathcal{H}_\Sigma^{1,\mu}(0,\beta,\lambda) = \mathcal{N}_\Sigma^\mu(\beta,\lambda)(see[21]);$$

(ii) For $\mu = 1,\ n = 0$ and $q \to 1^-$, we obtain the bi-univalent function classes

$$\mathcal{H}_\Sigma^{1,1}(0,\alpha,\lambda) = \mathcal{B}_\Sigma(\alpha,\lambda) \qquad \text{and} \qquad \mathcal{H}_\Sigma^{1,1}(0,\beta,\lambda) = \mathcal{B}_\Sigma(\beta,\lambda)(see[8]);$$

(iii) For $\mu = 1$ and $q \to 1^-$ we obtain the bi-univalent function classes

$$\mathcal{H}_\Sigma^{1,1}(n,\alpha,\lambda) = \mathcal{B}_\Sigma(n,\alpha,\lambda) \qquad \text{and} \qquad \mathcal{H}_\Sigma^{1,1}(n,\beta,\lambda) = \mathcal{B}_\Sigma(n,\beta,\lambda)(see[15]);$$

(iv) For $\mu = 1,\ n = 0,\ \lambda = 1$ and $q \to 1^-$, we obtain the bi-univalent function classes

$$\mathcal{H}_\Sigma^{1,1}(0,\alpha,1) = \mathcal{H}_\Sigma^\alpha \qquad \text{and} \qquad \mathcal{H}_\Sigma^{1,1}(0,\beta,1) = \mathcal{H}_\Sigma(\beta)(see[28]);$$

(v) For $\mu = 0,\ n = 0,\ \lambda = 1$ and $q \to 1^-$, we obtain the bi-univalent function classes

$$\mathcal{H}_\Sigma^{1,0}(0,\alpha,1) = \mathcal{S}_\Sigma^*(\alpha) \qquad \text{and} \qquad \mathcal{H}_\Sigma^{1,0}(0,\beta,1) = \mathcal{S}_\Sigma^*(\beta)(see[4]).$$

This paper is a sequel to some of the aforecited works (especially see [11, 32, 33]). Here we introduce and investigate the general subclass $\mathcal{H}_\Sigma^{h,p}(\lambda,\mu,n,q)$ $(0 < q < 1, \lambda \geq 1, \mu \geq 0)$ of the analytic function class $\mathcal{A}$, which is given by Definition 1.6 below.

Definition 1.6. Let $h, p : \mathbb{U} \to \mathbb{C}$ be analytic functions and

$$min\{Re(h(z)),\ Re(p(z))\} > 0 \qquad (z \in \mathbb{U}) \qquad and \qquad h(0) = p(0) = 1.$$

Also let the function f given by (1.1), be in the analytic function class $\mathcal{A}$. We say that

$$f \in \mathcal{H}_\Sigma^{h,p}(\lambda,\mu,n,q) \qquad (0 < q < 1, \lambda \geq 1, \mu \geq 0 \text{ and } n \in \mathbb{N}_0)$$

if the following conditions satisfied:
(1.16)
$$f \in \Sigma \text{ and } (1-\lambda)\left(\frac{D_q^n f(z)}{z}\right)^{\mu} + \lambda\left(D_q^n f(z)\right)'\left(\frac{D_q^n f(z)}{z}\right)^{\mu-1} \in h(\mathbb{U}) \ (z \in \mathbb{U})$$

and

$$(1.17) \qquad (1-\lambda)\left(\frac{D_q^n g(w)}{w}\right)^{\mu} + \lambda\left(D_q^n g(w)\right)'\left(\frac{D_q^n g(w)}{w}\right)^{\mu-1} \in p(\mathbb{U}) \ (w \in \mathbb{U}),$$

where the function g is given by (1.9).

If $f \in \mathcal{H}_{\Sigma}^{h,p}(\lambda, \mu, n, q)$, then

(1.18)
$$f \in \Sigma \text{ and } \left| arg\left((1-\lambda)\left(\frac{D_q^n f(z)}{z}\right)^{\mu} + \lambda\left(D_q^n f(z)\right)'\left(\frac{D_q^n f(z)}{z}\right)^{\mu-1}\right)\right| < \frac{\alpha\pi}{2}$$

and

$$(1.19) \qquad \left| arg\left((1-\lambda)\left(\frac{D_q^n g(w)}{w}\right)^{\mu} + \lambda\left(D_q^n g(w)\right)'\left(\frac{D_q^n g(w)}{w}\right)^{\mu-1}\right)\right| < \frac{\alpha\pi}{2}$$

or

$$(1.20) \quad f \in \Sigma \text{ and } Re\left\{(1-\lambda)\left(\frac{D_q^n f(z)}{z}\right)^{\mu} + \lambda\left(D_q^n f(z)\right)'\left(\frac{D_q^n f(z)}{z}\right)^{\mu-1}\right\} > \beta$$

and

$$(1.21) \qquad Re\left\{(1-\lambda)\left(\frac{D_q^n g(w)}{w}\right)^{\mu} + \lambda\left(D_q^n g(w)\right)'\left(\frac{D_q^n g(w)}{w}\right)^{\mu-1}\right\} > \beta.$$

where the function g is given by (1.9).

Our paper is motivated and stimulated especially by the work of Srivastava *et al.* [21, 28]. Here we propose to investigate the bi-univalent function subclass $\mathcal{H}_{\Sigma}^{h,p}(\lambda, \mu, n, q)$ of the function class Σ and find estimates on the initial coefficients $|a_2|$ and $|a_3|$ for functions in the new subclass of the function class Σ using Sălăgean q-differential operator.

2. A Set of General Coefficient Estimates

In this section, we derive estimates on the initial coefficients $|a_2|$ and $|a_3|$ for functions in subclass $\mathcal{H}_{\Sigma}^{h,p}(\lambda, \mu, n, q)$ given by Definition 1.6.

Theorem 2.1. *Let the function $f(z)$ given by Taylor-Maclaurin series expansion (1.1) be in the function class $\mathcal{H}_{\Sigma}^{h,p}(\lambda,\mu,n,q)$. Then*

$$|a_2| \leq \min\left\{\sqrt{\frac{|h'(0)|^2+|p'(0)|^2}{2(\lambda+\mu)^2[2]_q^{2n}}},\ \sqrt{\frac{|h''(0)|+|p''(0)|}{2(2\lambda+\mu)\left|(\mu-1)[2]_q^{2n}+2[3]_q^n\right|}}\right\} \tag{2.1}$$

and

$$|a_3| \leq \min\left\{\frac{|h'(0)|^2+|p'(0)|^2}{2(\lambda+\mu)^2[2]_q^{2n}}+\frac{|h''(0)|+|p''(0)|}{4(2\lambda+\mu)[3]_q^n},\right.$$
$$\left.\frac{\left|(\mu-1)[2]_q^{2n}+4[3]_q^n\right||h''(0)|+|\mu-1|[2]_q^{2n}|p''(0)|}{4(2\lambda+\mu)[3]_q^n\left|(\mu-1)[2]_q^{2n}+2[3]_q^n\right|}\right\}, \tag{2.2}$$

where $0<q<1, \lambda\geq 1, \mu\geq 0$ *and* $n\in\mathbb{N}_0$.

Proof. It follows from (1.16) and (1.17) that

$$(1-\lambda)\left(\frac{D_q^n f(z)}{z}\right)^{\mu}+\lambda\left(D_q^n f(z)\right)'\left(\frac{D_q^n f(z)}{z}\right)^{\mu-1}=h(\mathbb{U}) \tag{2.3}$$

and

$$(1-\lambda)\left(\frac{D_q^n g(w)}{w}\right)^{\mu}+\lambda\left(D_q^n g(w)\right)'\left(\frac{D_q^n g(w)}{w}\right)^{\mu-1}=p(\mathbb{U}) \tag{2.4}$$

Comparing the coefficients of z and z^2 in (2.3) and (2.4), we have

$$(\lambda+\mu)[2]_q^n a_2 = h_1, \tag{2.5}$$

$$(\mu-1)\left(\lambda+\frac{\mu}{2}\right)[2]_q^{2n}a_2^2+(2\lambda+\mu)[3]_q^n a_3 = h_2, \tag{2.6}$$

$$-(\lambda+\mu)[2]_q^n a_2 = p_1 \tag{2.7}$$

and

$$-(2\lambda+\mu)[3]_q^n a_3+\left(4[3]_q^n+(\mu-1)[2]_q^{2n}\right)\left(\lambda+\frac{\mu}{2}\right)a_2^2 = p_2. \tag{2.8}$$

From (2.5) and (2.7), we obtain

$$h_1 = -p_1 \tag{2.9}$$

and

$$2(\lambda+\mu)^2[2]_q^{2n}a_2^2 = h_1^2+p_1^2. \tag{2.10}$$

Also, from (2.6) and (2.8), we find that

$$\left\{(\mu-1)[2]_q^{2n}+2[3]_q^n\right\}(2\lambda+\mu)^2a_2^2=h_2+p_2. \tag{2.11}$$

Therefore, we find from the equations (2.10) and (2.11) that

$$|a_2|\leq\sqrt{\frac{|h'(0)|^2+|p'(0)|^2}{2(\lambda+\mu)^2[2]_q^{2n}}}$$

and

$$|a_2|\leq\sqrt{\frac{|h''(0)|+|p''(0)|}{2(2\lambda+\mu)\left|(\mu-1)[2]_q^{2n}+2[3]_q^n\right|}}\,,$$

respectively. So we get the desired estimate on the coefficients $|a_2|$ as asserted in (2.1).

Next, in order to find the bound on the coefficient $|a_3|$, we subtract (2.8) from (2.6), we get

$$2(2\lambda+\mu)[3]_q^na_3-2[3]_q^n(2\lambda+\mu)a_2^2=h_2-p_2. \tag{2.12}$$

Upon substituting the value of a_2^2 from (2.10) into (2.12), we arrive at

$$a_3=\frac{h_1^2+p_1^2}{2(\lambda+\mu)^2[2]_q^{2n}}+\frac{h_2-p_2}{2(2\lambda+\mu)[3]_q^n}.$$

We thus find that

$$|a_3|\leq\frac{|h'(0)|^2+|p'(0)|^2}{2(\lambda+\mu)^2[2]_q^{2n}}+\frac{|h''(0)|+|p''(0)|}{4(2\lambda+\mu)[3]_q^n}. \tag{2.13}$$

On the other hand, upon substituting the value of a_2^2 from (2.11) into (2.12), we arrive at

$$a_3=\frac{\left\{(\mu-1)[2]_q^{2n}+4[3]_q^n\right\}h_2+(\mu-1)[2]_q^{2n}p_2}{2(2\lambda+\mu)[3]_q^n\left\{(\mu-1)[2]_q^{2n}+2[3]_q^n\right\}}.$$

Consequently, we have

$$|a_3|\leq\frac{\left|(\mu-1)[2]_q^{2n}+4[3]_q^n\right||h''(0)|+|\mu-1|[2]_q^{2n}|p''(0)|}{4(2\lambda+\mu)[3]_q^n\left|(\mu-1)[2]_q^{2n}+2[3]_q^n\right|}. \tag{2.14}$$

This evidently completes the proof of Theorem 2.1. □

3. Corollaries and Consequences

By Setting $\mu=1,\ q\to1^-$ and $n=0$ in Theorem 2.1, we deduce the following consequence of Theorem 2.1.

Corollary 3.1. *Let the function $f(z)$ given by Taylor-Maclaurin series expansion (1.1) be in the function class $\mathcal{B}_{\Sigma}^{h,p}(\lambda)(\lambda \geq 1)$. Then*

$$|a_2| \leq \min\left\{\sqrt{\frac{|h'(0)|^2+|p'(0)|^2}{2(1+\lambda)^2}},\ \sqrt{\frac{|h''(0)|+|p''(0)|}{4(1+2\lambda)}}\right\} \tag{3.1}$$

and

$$|a_3| \leq \min\left\{\frac{|h'(0)|^2+|p'(0)|^2}{2(1+\lambda)^2}+\frac{|h''(0)|+|p''(0)|}{4(1+2\lambda)}, \frac{|h''(0)|}{2(1+2\lambda)}\right\}. \tag{3.2}$$

By Setting $\mu = 0,\ \lambda = 1,\ q \to 1^-$ and $n = 0$ in Theorem 2.1, we deduce the following.

Corollary 3.2.([5]) *Let the function $f(z)$ given by Taylor-Maclaurin series expansion (1.1) be in the function class $\mathcal{B}_{\Sigma}^{h,p}$. Then*

$$|a_2| \leq \min\left\{\sqrt{\frac{|h'(0)|^2+|p'(0)|^2}{2}},\ \sqrt{\frac{|h''(0)|+|p''(0)|}{4}}\right\} \tag{3.3}$$

and

$$|a_3| \leq \min\left\{\frac{|h'(0)|^2+|p'(0)|^2}{8}+\frac{|h''(0)|+|p''(0)|}{8}, \frac{3|h''(0)|+|p''(0)|}{8}\right\}. \tag{3.4}$$

Remark 3.3. Corollary 3.2 is an improvement of the following estimates obtained by Xu *et al.* [33].

Corollary 3.4.([33]) *Let the function $f(z)$ given by Taylor-Maclaurin series expansion (1.1) be in the function class $\mathcal{B}_{\Sigma}^{h,p}(\lambda)$ $(\lambda \geq 1)$. Then*

$$|a_2| \leq \sqrt{\frac{|h''(0)|+|p''(0)|}{4(1+2\lambda)}} \tag{3.5}$$

and

$$|a_3| \leq \frac{|h''(0)|}{2(1+2\lambda)}. \tag{3.6}$$

By Setting $\lambda = 1,\ \mu = 1,\ q \to 1^-$ and $n = 0$ in Theorem 2.1, we deduce the following Corollary 3.5.

Corollary 3.5.([32]) *Let the function $f(z)$ given by Taylor-Maclaurin series expansion (1.1) be in the function class $\mathcal{H}_{\Sigma}^{h,p}$. Then*

$$|a_2| \leq \sqrt{\frac{|h''(0)| + |p''(0)|}{12}} \tag{3.7}$$

and

$$|a_3| \leq \frac{|h''(0)|}{6}. \tag{3.8}$$

4. Concluding Remarks and Observations

The main objective in this paper has been to derive first two Taylor-Maclaurin coefficient estimates for functions belonging to a new subclass $\mathcal{H}_{\Sigma}^{h,p}(\lambda, \mu, n, q)$ of analytic and bi-univalent function in the open unit disk $\mathbb{U}$. Indeed, by using *Sălăgean q-calculus operator,* we have successfully determined the first two Taylor-Maclaurin coefficient estimates for functions belonging to a new subclass $\mathcal{H}_{\Sigma}^{h,p}(\lambda, \mu, n, q)$.

By means of corollaries and consequences which we discuss in the preceding section by suitable specializing the parameters λ and μ, we have also shown already that the results presented in this paper would generalize and improve some recent works of Xu *et al*. [32, 33] and other authors.

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