

# Online Estimation of Rotational Inertia of an Excavator Based on Recursive Least Squares with Multiple Forgetting

Kwangseok Oh<sup>1</sup>, Kyong Su Yi<sup>2</sup>, Jaho Seo<sup>3\*</sup>, Yongrae Kim<sup>4</sup> and Geunho Lee<sup>4</sup>

Received: 09 Apr. 2017, Accepted: 24 Aug. 2017

**Key Words** : Inertial parameter estimation, Forgetting factor, Nominal parameter, Recursive least square, Updating rule

**Abstract:** This study presents an online estimation of an excavator’s rotational inertia by using recursive least square with forgetting. It is difficult to measure rotational inertia in real systems. Against this background, online estimation of rotational inertia is essential for improving safety and automation of construction equipment such as excavators because changes in inertial parameter impact dynamic characteristics. Regarding an excavator, rotational inertia for swing motion may change significantly according to working posture and digging conditions. Hence, rotational inertia estimation by predicting swing motion is critical for enhancing working safety and automation. Swing velocity and damping coefficient were used for rotational inertia estimation in this study. Updating rules are proposed for enhancing convergence performance by using the damping coefficient and forgetting factors. The proposed estimation algorithm uses three forgetting factors to estimate time-varying rotational inertia, damping coefficient, and torque with different variation rates. Rotational inertia in a typical working scenario was considered for reasonable performance evaluation. Three simulations were conducted by considering several digging conditions. Presented estimation results reveal the proposed estimation scheme is effective for estimating varying rotational inertia of the excavator.

## Nomenclature

$b$  : damping coefficient

$\bar{b}$  : nominal damping coefficient

$\hat{b}$  : estimated damping coefficient

$F$  : state transition matrix

$G$  : process noise matrix

$H$  : observation matrix

$I$  : identity matrix

$J_t$  : total swing inertia that include material inertia loaded in the bucket

$\hat{J}_t$  : estimated swing inertia

$K$  : Kalman gain matrix

$P_0$  : initial error covariance

$Q_k$  : covariance matrix of the process noise

$T$  : sampling period

$T_f$  : Coulomb friction torque

$T_{sw}$  : swing torque

$T_t$  : total torque

\* Corresponding author: seojaho@gmail.com

1 Department of Mechanical Engineering, Hankyong National University, Anseong-si, 17579, Korea

2 School of Mechanical and Aerospace Engineering, Seoul National University, Seoul 08826, Korea

3 Department of Automotive, Mechanical and Manufacturing Engineering, University of Ontario Institute of Technology, Oshawa, L1H 7K4, Canada

4 Department of System Reliability, Korea Institute of Machinery & Material, 156 Gajungbukno, Yuseong-gu, Daejeon 34103, Korea

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- $\hat{T}_t$ : combined torque  
 $v_k$ : measurement noise  
 $w_k$ : process noise  
 $x$ : state vector  
 $\hat{x}_0$ : initial state  
 $y$ : measured output vector  
 $\lambda_i$ : forgetting factors,  $i=1,2,3$   
 $\theta_{arm}$ : arm angle  
 $\theta_{boom}$ : boom angle  
 $\theta_{bucket}$ : bucket angle  
 $\dot{\theta}_{sw}$ : swing velocity  
 $\hat{\theta}_{sw}$ : estimated swing velocity  
 $\ddot{\theta}_{sw}$ : swing acceleration  
 $\hat{\ddot{\theta}}_{sw}$ : estimated swing acceleration

## 1. Introduction

Construction equipment such as excavators performs various tasks at construction sites. This is realized by using its working parts in coordination with nearby workers. Further, the excavator is usually operated in a stationary state, while its working parts can rotate 360°. Dynamic characteristics such as inertial parameters and friction of the excavator may change easily because the equipment is used with various digging materials and its working postures may vary. Various studies have been conducted on applications of construction equipment systems from the viewpoint of improving safety and automation.

Tafazoli et al.<sup>1)</sup> developed a novel yet simple approach for experimentally determining link parameters and friction coefficients for a typical excavator arm. Tan et al.<sup>2)</sup> proposed a fast and robust technique to estimate the unknown parameters of soil mechanics equations by minimizing the error between measured failure forces and estimated forces for the experimental identification of soil. Lee et al.<sup>3)</sup> provided solutions to problems arising during the modeling of hydraulic excavators; these solutions were based on the bond graph method, the top-down and bottom-up methods,

and the developed modeling software.

This paper proposes an online inertial parameter estimation scheme for excavators based on recursive least squares with multiple forgetting. The working parts of an excavator comprise its body, boom, arm, and bucket with four degrees of freedom; the working parts can be moved to change the posture of the excavator.

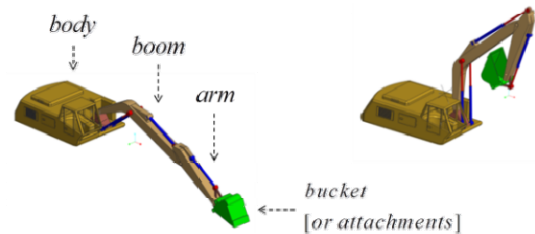


Fig. 1 Working parts (body, boom, arm, and bucket)

As the working posture, material load, and type of attachments such as bucket, ripper, and crusher influence the dynamic characteristics of the excavator, it is necessary to estimate the inertial parameter for swing motion in order to predict the behavior for improving safety and automation. Therefore, in this study, to estimate the rotational inertia, recursive least squares algorithm with multiple forgetting factors was used, and an updating rule was adopted for convergence performance. Multiple forgetting factors were defined considering the change rate of parameters that are required to be estimated. The damping coefficient in swing dynamics was also considered in the updating rule in this study.

The updating rule for the forgetting factor considered the convergence delay; this factor was applied for improving estimation performance. The performance evaluation of the estimation algorithm was conducted in a typical working scenario in Matlab/Simulink environment. The results show that the proposed estimation algorithm effectively estimates the actual moment of inertia during swing motion.

The rest of this paper is organized as follows: Section 2 describes the investigation of variations in the inertial parameter of the excavator. Section 3 proposes a swing inertia estimation method. Section 4 discusses the Matlab/Simulink-based evaluation of the estimation performance. Finally, the concluding remarks are provided in Section 5.

## 2. Investigation of Variations in Rotational Inertia

Various types of attachments such as crushers, grapples, and buckets, are used with an excavator. The excavator also works with several types of materials. The masses of different materials and attachments vary greatly and greatly affect the inertial property of the excavator. Furthermore, the rotational inertia for the swing motion may change because of changes in the working posture of the excavator. Therefore, the variation in the moment of inertia for swing motion was investigated to emphasize the need for estimation of inertia. Fig. 2 shows the materials and attachments considered in the investigation.



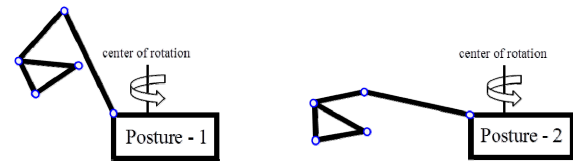
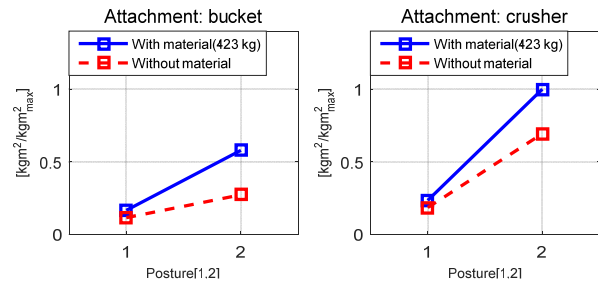
Division	Bucket		Crusher	
Attachment				
Material	0 kg	423 kg	0 kg	423 kg

Fig. 2 Materials and attachments considered in the investigation

In addition to buckets, crushers were also considered in the investigation because crushers are heavier than buckets, and hence, there is a large difference in the inertial parameters of excavators with buckets and those with crushers. Material and attachment specifications were determined for 5.5-ton class excavators. The moment of inertia was computed mathematically using the actual dimensions of the excavator. Fig. 3 describes the working postures for computation and the normalized moment of inertia with respect to the working posture, material, type, and attachment. All computed results shown in Fig. 3 were normalized for easy comparison. The rotational inertia for swing motion changed significantly with the attachment and material conditions. This observation provides motivation to study the methodology for estimating rotational inertia. The next section describes the rotational inertia estimation scheme.



(a) Working postures 1 and 2



(b) Comparison of analytical results for rotational inertia Fig. 3 Working posture of the excavator and analytical rotational inertia

All computed results shown in Fig. 3 were normalized for easy comparison. The rotational inertia for swing motion of the excavator changed significantly with the attachment and material conditions. This observation provides motivation to study the methodology for estimating rotational inertia. The next section describes the rotational inertia estimation scheme.

## 3. Rotational Inertia Estimation

In order to estimate the swing inertia of excavator, this study proposes the recursive least square (RLS) estimation method with nominal parameter-based updating and multiple forgettings<sup>4, 5</sup>. A model schematic for estimating inertia is shown in Fig. 4.

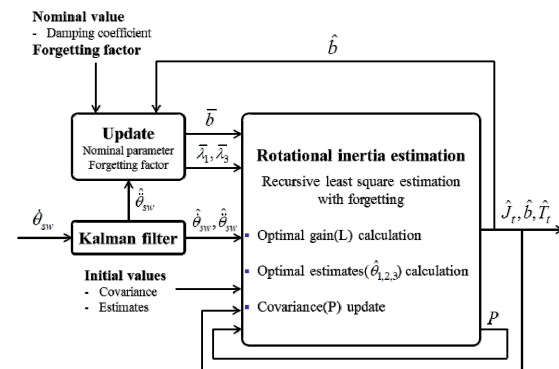


Fig. 4 Schematic of the model for estimating the rotational inertia of an excavator

$\hat{\theta}_{sw}$  and  $\hat{\theta}_{sw}$  are computed using the Kalman filter. In this study,  $\bar{b}$  is assumed to be known.  $\hat{J}_t$  and  $\hat{b}$  are computed by RLS estimation. The next section describes calculation of nominal swing inertia.

### 3.1 Kalman filter

A linear Kalman filter (LKF) was used for estimating the swing velocity and acceleration. A double integrator model was employed. The swing velocity was assumed to be measurable using a sensor. The discrete-time linear state space system used in the study is as follows.

$$\begin{aligned} x_k &= F_{k-1}x_{k-1} + G_{k-1}w_{k-1} \\ y_k &= H_k x_k + v_k \end{aligned} \quad (1)$$

where  $x_k = [\dot{\theta}_{sw,k} \quad \ddot{\theta}_{sw,k}]^T$  and  $w_k = [w_1 \quad w_2]^T$

$F$  and  $H$  are written as follows.

$$F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad H = [1 \quad 0] \quad (2)$$

where the process and measurement noise covariance terms are considered to be uncorrelated zero mean white Gaussian with covariance matrices  $Q_k$  and  $R_k$ , respectively.

$$w_k = N(0, Q_k) \quad (3)$$

$$v_k = N(0, R_k) \quad (4)$$

$$E[w_k v_k^T] = 0 \quad (5)$$

where  $Q_k = \text{diag}[0, q]$ . Here,  $q$  must be determined as a parameter to be adjusted because the actual accelerated motion is not characterized suitably by a stationary random process. It is assumed that  $\hat{x}_0$  is known with uncertainty given by  $P_0$ . In order to estimate  $x$ , the LKF was applied. The LKF is implemented in two steps: (1) prediction and (2) update. In the first step, the *a priori* estimates of the state and error covariance are given by

$$\hat{x}_{k|k-1} = F_{k-1}\hat{x}_{k-1} \quad (6)$$

$$P_{k|k-1} = F_{k-1}P_{k-1}F_{k-1}^T + Q_{k-1} \quad (7)$$

In the second step, the *a priori* values are updated using the computed  $K$ , yielding the following *a posteriori* estimates.

$$K_k = P_{k|k-1}H_k^T(H_kP_{k|k-1}H_k^T + R_k)^{-1} \quad (8)$$

$$\hat{x}_k = \hat{x}_{k|k-1} + K_k(y_k - H_k\hat{x}_{k|k-1}) \quad (9)$$

$$P_k = (I - K_kH_k)P_{k|k-1} \quad (10)$$

The next section describes the swing inertia estimation method based on the state estimated by the LKF.

### 3.2 Excavator swing dynamics

The estimation approach applied in this study is a model-based approach that employs excavator swing dynamics. It can be presented using the following equation.

$$J_t\ddot{\theta}_{sw} = -b\dot{\theta}_{sw} - T_f + T_{sw} \quad (11)$$

Eq. (11) can be rearranged such that the terms for inertia, damping, and torque are separated into

$$0 = J_t\ddot{\theta}_{sw} + b\dot{\theta}_{sw} + T_f \quad (12)$$

where  $T_t$  is equal to  $-T_f + T_{sw}$ . Eq. (12) can be rewritten in the following linear parametric form.

$$y_r = \phi^T\theta, \quad \phi = [\phi_1 \quad \phi_2 \quad \phi_3]^T, \quad \theta = [\theta_1 \quad \theta_2 \quad \theta_3]^T \quad (13)$$

where  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  are the unknown parameters to be estimated.

$$\theta = [\theta_1 \quad \theta_2 \quad \theta_3] = [J_t \quad b \quad T_t] \quad (14)$$

and  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$  are computed based on the estimated swing velocity and acceleration

$$\phi = [\phi_1 \quad \phi_2 \quad \phi_3] = [\ddot{\theta}_{sw} \quad \dot{\theta}_{sw} \quad -1] \quad (15)$$

The parameters  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  are generally vary with time.  $\theta_1$  depends on the working posture of the excavator.  $\theta_2$  and  $\theta_3$  depend on the condition of the hydraulic system for the swing motion. In order to estimate the time-varying parameters, recursive least

square estimation with multiple forgetting factors was adopted in this study. Section 3.3 describes RLS estimation with forgetting.

### 3.3 Recursive least square estimation with forgetting

The scheme of RLS estimation with forgetting was adopted to estimate  $\lambda_i (i=1,2,3)$ . These factors assign less weight to older data and more weight to recent data<sup>6)</sup>. In order to separate the error of each parameter (i.e., error for  $\theta_i (i=1,2,3)$ ) by applying suitable forgetting factors, the decoupled cost function is defined as follows.

$$\begin{aligned}
 J(\hat{\theta}_1(k), \hat{\theta}_2(k), \hat{\theta}_3(k), k) = & \\
 & \frac{1}{2} \sum_{i=1}^k \lambda_1^{k-i} (y_r(i) - \phi_1(i)\hat{\theta}_1(k) - \phi_2(i)\theta_2(i) - \phi_3(i)\theta_3(i))^2 + \\
 & \frac{1}{2} \sum_{i=1}^k \lambda_2^{k-i} (y_r(i) - \phi_1(i)\theta_1(i) - \phi_2(i)\hat{\theta}_2(k) - \phi_3(i)\theta_3(i))^2 + \\
 & \frac{1}{2} \sum_{i=1}^k \lambda_3^{k-i} (y_r(i) - \phi_1(i)\theta_1(i) - \phi_2(i)\theta_2(i) - \phi_3(i)\hat{\theta}_3(k))^2 \quad (16)
 \end{aligned}$$

Using this defined cost function, each term on the right-hand side of the defined cost function represents the error of step  $k$ . The optimal estimates that minimize the cost function in Eq. (16) can be computed as follows.

$$\begin{aligned}
 \frac{\partial J}{\partial \hat{\theta}_1(k)} = 0, \frac{\partial J}{\partial \hat{\theta}_2(k)} = 0, \frac{\partial J}{\partial \hat{\theta}_3(k)} = 0 \\
 \sum_{i=1}^k \lambda_1^{k-i} (-\phi_1(i))(y_r(i) - \phi_1(i)\hat{\theta}_1(k) - \phi_2(i)\theta_2(i) - \phi_3(i)\theta_3(i)) = 0 \\
 \sum_{i=1}^k \lambda_2^{k-i} (-\phi_2(i))(y_r(i) - \phi_1(i)\theta_1(i) - \phi_2(i)\hat{\theta}_2(k) - \phi_3(i)\theta_3(i)) = 0 \\
 \sum_{i=1}^k \lambda_3^{k-i} (-\phi_3(i))(y_r(i) - \phi_1(i)\theta_1(i) - \phi_2(i)\theta_2(i) - \phi_3(i)\hat{\theta}_3(k)) = 0 \quad (17)
 \end{aligned}$$

$\hat{\theta}_1(k)$ ,  $\hat{\theta}_2(k)$ , and  $\hat{\theta}_3(k)$  can be determined by rearranging Eq. (17) as follows.

$$\begin{aligned}
 \hat{\theta}_1(k) &= \left( \sum_{i=1}^k \lambda_1^{k-i} \phi_1(i)^2 \right)^{-1} \left( \sum_{i=1}^k \lambda_1^{k-i} (y_r(i) - \phi_2(i)\theta_2(i) - \phi_3(i)\theta_3(i)) \right) \\
 \hat{\theta}_2(k) &= \left( \sum_{i=1}^k \lambda_2^{k-i} \phi_2(i)^2 \right)^{-1} \left( \sum_{i=1}^k \lambda_2^{k-i} (y_r(i) - \phi_1(i)\theta_1(i) - \phi_3(i)\theta_3(i)) \right) \\
 \hat{\theta}_3(k) &= \left( \sum_{i=1}^k \lambda_3^{k-i} \phi_3(i)^2 \right)^{-1} \left( \sum_{i=1}^k \lambda_3^{k-i} (y_r(i) - \phi_1(i)\theta_1(i) - \phi_2(i)\theta_2(i)) \right) \quad (18)
 \end{aligned}$$

However, a recursive form is required for real-time parameter estimation. The recursive form can be deduced by using the analogy between Eqs. (17) and (18) as follows.

$$\begin{aligned}
 \hat{\theta}_1(k) &= \hat{\theta}_1(k-1) + L_1(y_r(k) - \phi_1(k)\hat{\theta}_1(k-1) - \phi_2(k)\theta_2(k) - \phi_3(k)\theta_3(k)) \\
 \hat{\theta}_2(k) &= \hat{\theta}_2(k-1) + L_2(y_r(k) - \phi_1(k)\theta_1(k) - \phi_2(k)\hat{\theta}_2(k-1) - \phi_3(k)\theta_3(k)) \\
 \hat{\theta}_3(k) &= \hat{\theta}_3(k-1) + L_3(y_r(k) - \phi_1(k)\theta_1(k) - \phi_2(k)\theta_2(k) - \phi_3(k)\hat{\theta}_3(k-1)) \quad (19)
 \end{aligned}$$

where

$$\begin{aligned}
 L_1(k) &= P_{r,1}(k-1)\phi_1(k)(\lambda_1 + \phi_1^T(k)P_{r,1}(k-1)\phi_1(k))^{-1} \\
 P_{r,1}(k) &= (I - L_1(k)\phi_1^T(k))P_{r,1}(k-1)\frac{1}{\lambda_1} \quad (20)
 \end{aligned}$$

$$\begin{aligned}
 L_2(k) &= P_{r,2}(k-1)\phi_2(k)(\lambda_2 + \phi_2^T(k)P_{r,2}(k-1)\phi_2(k))^{-1} \\
 P_{r,2}(k) &= (I - L_2(k)\phi_2^T(k))P_{r,2}(k-1)\frac{1}{\lambda_2} \quad (21)
 \end{aligned}$$

$$\begin{aligned}
 L_3(k) &= P_{r,3}(k-1)\phi_3(k)(\lambda_3 + \phi_3^T(k)P_{r,3}(k-1)\phi_3(k))^{-1} \\
 P_{r,3}(k) &= (I - L_3(k)\phi_3^T(k))P_{r,3}(k-1)\frac{1}{\lambda_3} \quad (22)
 \end{aligned}$$

The unknown parameters  $\theta_1(k)$ ,  $\theta_2(k)$ , and  $\theta_3(k)$  in the aforementioned equations can be replaced with their estimates,  $\hat{\theta}_1(k)$ ,  $\hat{\theta}_2(k)$ , and  $\hat{\theta}_3(k)$  because in this study, it is assumed that the actual and estimated values are very close to each other or are within the region of convergence. By substituting for  $\theta_1(k)$ ,  $\theta_2(k)$ , and  $\theta_3(k)$ , the following equations can be obtained.

$$\begin{aligned}
 \hat{\theta}_1(k) + L_1(k)\phi_2(k)\hat{\theta}_2(k) + L_1(k)\phi_3(k)\hat{\theta}_3(k) \\
 = \hat{\theta}_1(k-1) + L_1(k)(y_r(k) - \phi_1(k)\hat{\theta}_1(k-1)) \quad (23)
 \end{aligned}$$

$$\begin{aligned}
 L_2(k)\phi_1(k)\hat{\theta}_1(k) + \hat{\theta}_2(k) + L_2(k)\phi_3(k)\hat{\theta}_3(k) \\
 = \hat{\theta}_2(k-1) + L_2(k)(y_r(k) - \phi_2(k)\hat{\theta}_2(k-1)) \quad (24)
 \end{aligned}$$

$$\begin{aligned}
 L_3(k)\phi_1(k)\hat{\theta}_1(k) + L_3(k)\phi_2(k)\hat{\theta}_2(k) + \hat{\theta}_3(k) \\
 = \hat{\theta}_3(k-1) + L_3(k)(y_r(k) - \phi_3(k)\hat{\theta}_3(k-1)) \quad (25)
 \end{aligned}$$

The solutions for the estimates such as  $\hat{\theta}_1(k)$ ,  $\hat{\theta}_2(k)$ , and  $\hat{\theta}_3(k)$  can be obtained as shown in Eq. (26).

$$\begin{aligned}
 \begin{bmatrix} \hat{\theta}_1(k) \\ \hat{\theta}_2(k) \\ \hat{\theta}_3(k) \end{bmatrix} &= \begin{bmatrix} 1 & L_1(k)\phi_2(k) & L_1(k)\phi_3(k) \\ L_2(k)\phi_1(k) & 1 & L_2(k)\phi_3(k) \\ L_3(k)\phi_1(k) & L_3(k)\phi_2(k) & 1 \end{bmatrix}^{-1} \times \\
 & \begin{bmatrix} \hat{\theta}_1(k-1) + L_1(k)(y_r(k) - \phi_1(k)\hat{\theta}_1(k-1)) \\ \hat{\theta}_2(k-1) + L_2(k)(y_r(k) - \phi_2(k)\hat{\theta}_2(k-1)) \\ \hat{\theta}_3(k-1) + L_3(k)(y_r(k) - \phi_3(k)\hat{\theta}_3(k-1)) \end{bmatrix} \quad (26)
 \end{aligned}$$

It can be proved that the determinant of the matrix in the following equation is always nonzero as  $P_1$ ,  $P_2$ , and  $P_3$  are always positive<sup>7)</sup>.

$$\begin{bmatrix} 1 & L_1(k)\phi_2(k) & L_1(k)\phi_3(k) \\ L_2(k)\phi_1(k) & 1 & L_2(k)\phi_3(k) \\ L_3(k)\phi_1(k) & L_3(k)\phi_2(k) & 1 \end{bmatrix} \quad (27)$$

Therefore, the inverse of Eq. (27) always exists. Section 3.4 describes the nominal-parameter-based updating rule.

### 3.4 Updating rule: nominal parameter

Using the value of the damping coefficient in Eq. (11), which is obtained from a previous study<sup>8)</sup>, the updating rule has been defined such that the estimated damping coefficient is updated depending on the given damping coefficient. The updating rule was defined by modifying Eq. (19). Fig. 5 describes the defined updating rule for the nominal parameter.

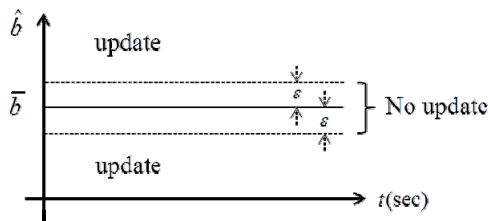


Fig. 5 Updating rule for the nominal parameter

$\varepsilon$  in Fig. 5 is the difference between the actual and estimated values. The estimation error is defined as follows.

$$e_2 = \theta_2 - \hat{\theta}_1 = b - \hat{b} \quad (28)$$

Then, the estimated value in the recursive form is updated by applying the following updating rule.

$$\begin{aligned} & \text{If } |e_2| \leq \varepsilon \\ & \hat{\theta}_2(k) = \hat{\theta}_2(k-1) + L_2(y_r(k) - \phi_1(k)\hat{\theta}_1(k) - \phi_2(k)\hat{\theta}_2(k-1) - \phi_3(k)\theta_3(k)) \\ & \text{else} \\ & \hat{\theta}_2(k) = \bar{b} + L_2(y_r(k) - \phi_1(k)\hat{\theta}_1(k) - \phi_2(k)\hat{\theta}_2(k-1) - \phi_3(k)\theta_3(k)) \\ & \text{end} \end{aligned} \quad (29)$$

The updating rule can enhance the convergence performance of estimation because the  $\hat{J}_i$  and  $\hat{T}_i$  can approach the actual values when the estimated damping

coefficient is updated when the absolute value of the estimation error is larger than the defined error threshold. As the actual damping coefficient cannot be determined owing to its nonlinearity,  $\varepsilon$  must be determined as a parameter to be adjusted. The next section describes the updating rule for the forgetting factor.

### 3.5 Updating rule: forgetting factor

Forgetting factors ( $\lambda_1, \lambda_2$ ) defined in the previous section for the RLS algorithm are designed to be updated. The estimator does not perform well when the swing acceleration changes significantly because of the convergence delay in the LKF used for estimating the swing acceleration. Therefore, it is necessary to maintain the value of the rotational inertia until the swing acceleration converges to an actual value upon updating the defined forgetting factor close to unity. The estimator is designed such that the defined forgetting factor close to zero is updated for  $\hat{\theta}_3$  to improve the estimation performance of  $T_i$ . Fig. 6 shows the designed updating rule for the forgetting factor.

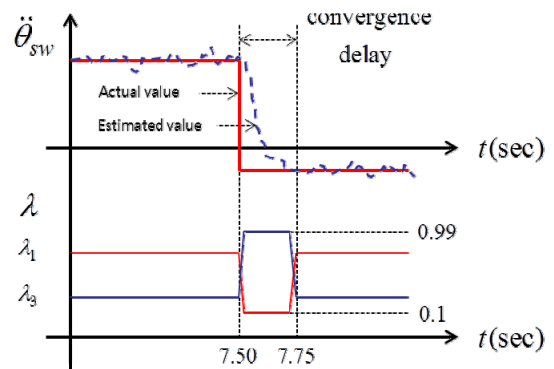


Fig. 6 Updating rule for the forgetting factor

An update value of 0.99 was chosen for the forgetting factor to effectively track the slowly varying parameter,  $\theta_1(k)$ . 0.1 was the value chosen for the rapidly changing parameter,  $\theta_3(k)$ . The convergence delay was derived by comparing the estimated acceleration with the actual acceleration offline. The convergence delay was defined as 0.25 s in the study. The designed updating rule was used to secure the estimation performance for rotational inertia when the excavator begins to decelerate for unloading. Section 4

describes the performance evaluation of the proposed estimation algorithm based on RLS with updating rules and multiple forgetting.

### 4. Performance Evaluation

In order to evaluate the estimation performance of the proposed algorithm, Matlab/Simulink-based simulations were conducted. The swing dynamic model for estimation was constructed in Matlab/Simulink environment. The values of  $b$  and  $T_f$  used in the swing dynamic model were obtained from a previous study<sup>8)</sup>. The rotational inertia of the excavator varying with the working posture was derived analytically (mathematically) in a typical working scenario, and was used as a reference value tracked by the proposed estimation algorithm.

#### 4.1 Working scenario and analytical rotational inertia

In this study, the working scenario (dumping) was divided into three stages: loading, transporting, and unloading. Figure 7 describes the divided working scenario.

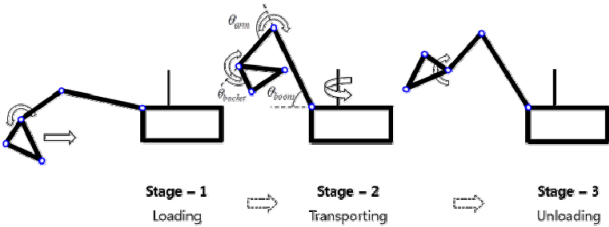
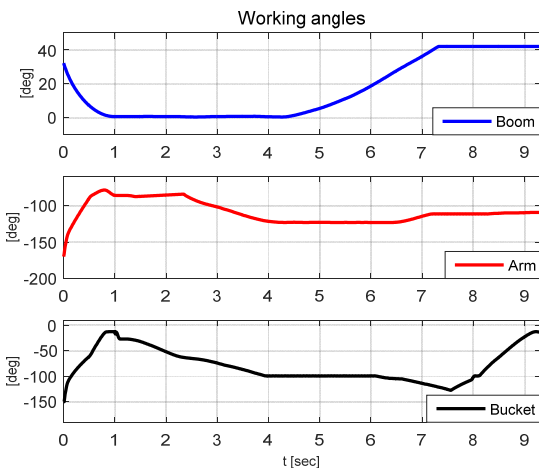
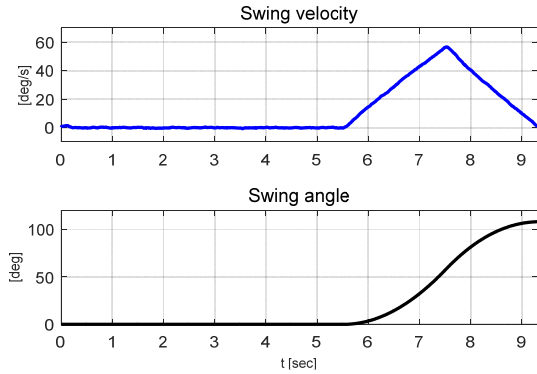


Fig. 7 Three stages of the working scenario



(a) Working angles: boom, arm, and bucket



(b) Working angles and velocity: swing

Fig. 8 Working angles and velocity: boom, arm, bucket, and swing

The working part angles (boom, arm, bucket, and swing) and velocity (swing) for the working scenario are shown in Fig. 8.

Based on the typical working scenario, the analytical rotational inertia of the excavator was derived. Fig. 9 shows the derived analytical rotational inertia used for performance evaluation.

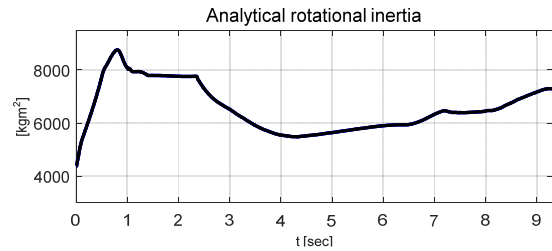


Fig. 9 Analytic rotational inertia for the working scenario

The maximum and minimum values of the derived analytical rotational inertia are 9,761 kgm<sup>2</sup> and 4,380 kgm<sup>2</sup> during working.

#### 4.2 Rotational inertia estimation

In order to conduct a reasonable performance evaluation, three simulation conditions with different material types were considered. Fig. 10 shows the three simulation conditions.

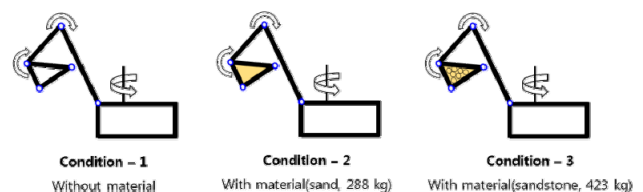


Fig. 10 Three simulation conditions



Three simulations were conducted for rotational inertia estimation in the region where the swing motion existed because the proposed estimation algorithm requires the swing velocity and acceleration to estimate the rotational inertia of the excavator. Furthermore, the rotational inertia is not required when the excavator is in a stationary state because the rotational inertia is parameter related to swing dynamics. Therefore, the region between 5.5 s to 9.3 s in the working scenario (Fig. 6) was extracted and used in the simulation (total time was approximately 3.8 s). The swing acceleration changed dramatically after 7.5 s. Arbitrary noise with Gaussian distribution was applied to the velocity profile for reasonable performance evaluation, and the swing acceleration was estimated using the LKF. The following figures show the estimation results during rotation.

Case - 1) Without any material

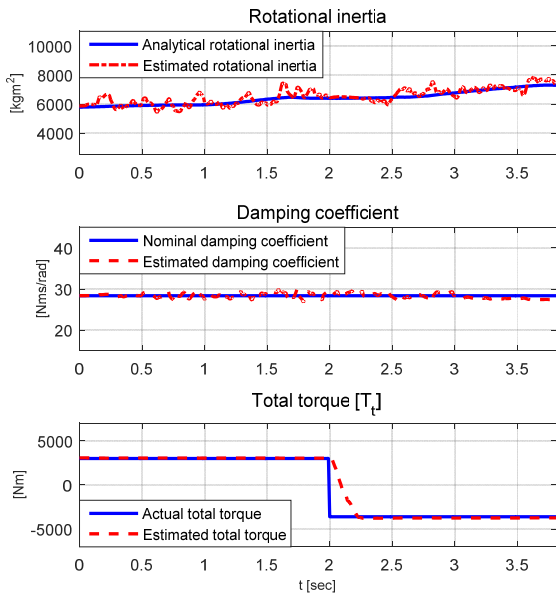


Fig. 11 Estimation results when no material was considered

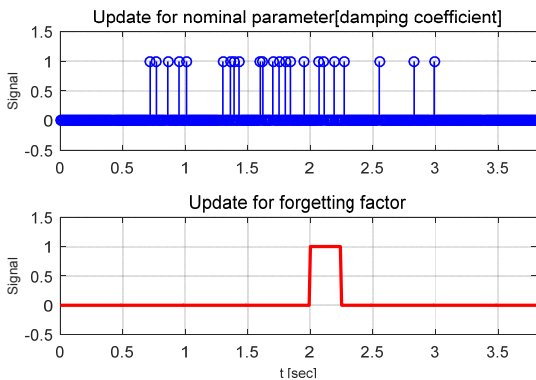


Fig. 12 Update of the damping coefficient and forgetting factor when no material was considered

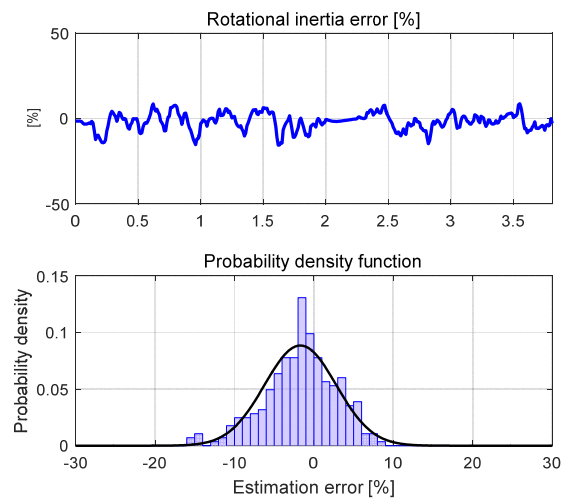


Fig. 13 Estimation error (rotational inertia) distribution when no material was considered

Case - 2) With material (sand, 288 kg)

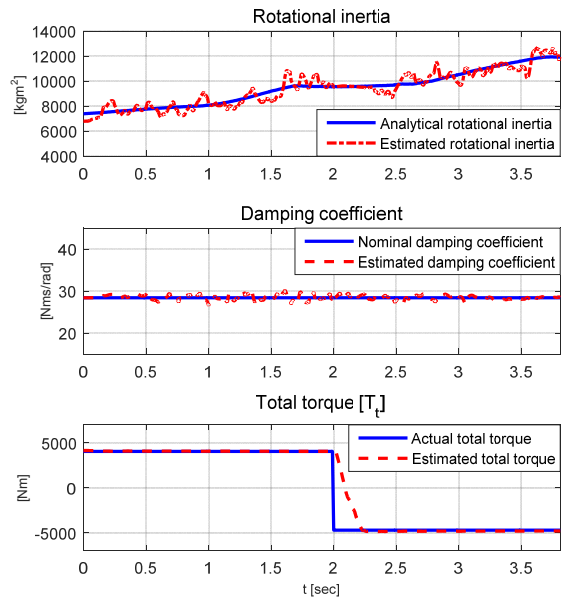


Fig. 14 Estimation results considering a material (sand, 288 kg)

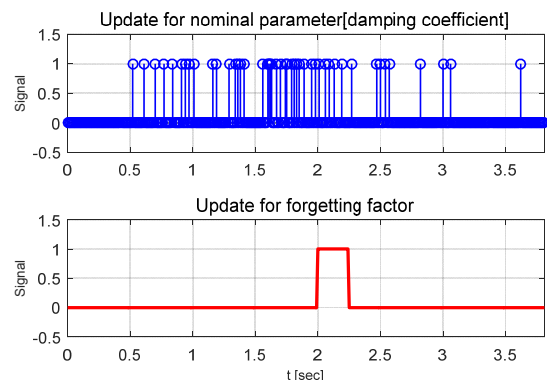


Fig. 15 Update of the damping coefficient and forgetting factor considering a material (sand, 288 kg)



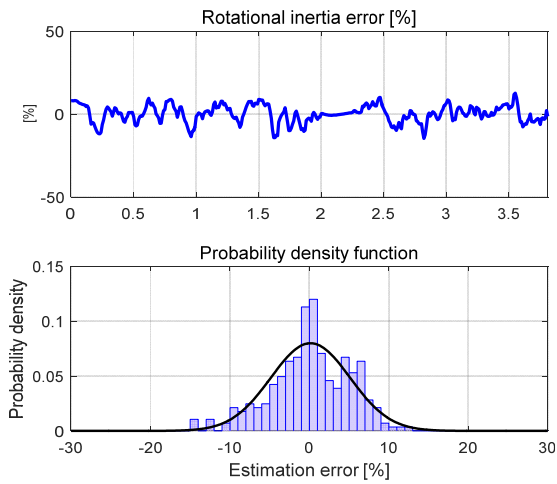


Fig. 16 Estimation error (rotational inertia) distribution considering a material (*sand, 288 kg*)

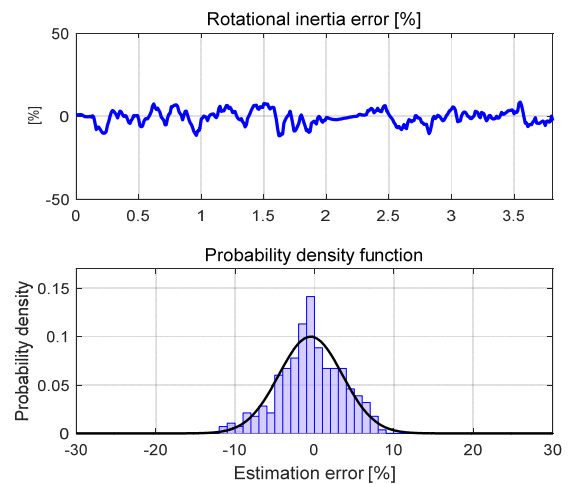


Fig. 19 Estimation error (rotational inertia) distribution considering a material (*sandstone, 423 kg*)

Case - 3) With material (*sandstone, 423 kg*)

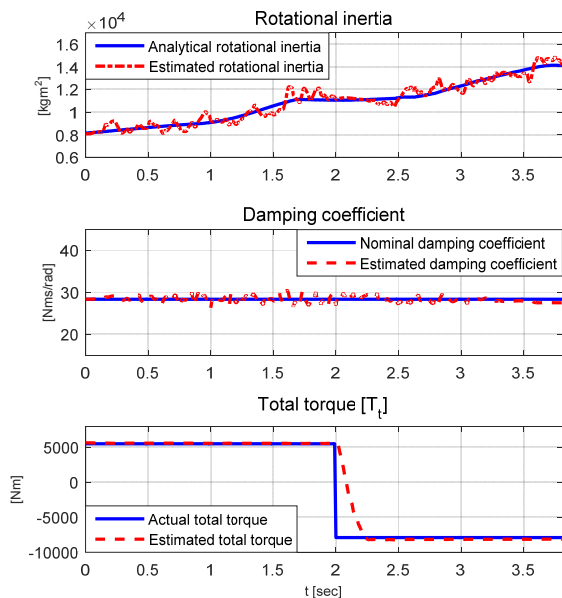


Fig. 17 Estimation results considering a material (*sandstone, 423 kg*)

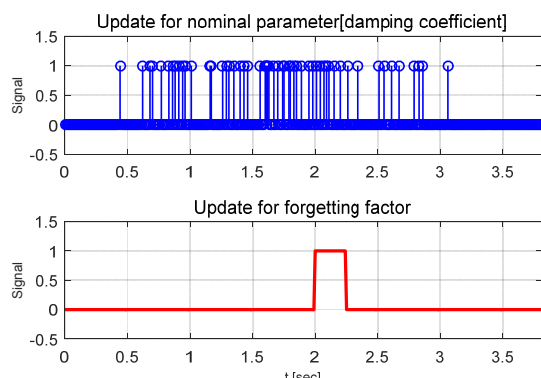


Fig. 18 Update of the damping coefficient and forgetting factor considering a material (*sandstone, 423 kg*)

The simulation results obtained without considering any material (rotational inertia did not change significantly) show that the rotational inertia of the excavator was well estimated during rotation. In addition, the rotational inertia was well estimated when the rotational inertia changed significantly because of the presence of materials (sand weighing 288 kg and sandstone weighing 423 kg) in cases 2 and 3. In the updated results (Fig. 12, Fig. 15, and Fig. 18), “1” indicates that the update was performed, and “0” indicates that the update was not performed. Based on the designed updating rules for the nominal parameter (damping coefficient) and forgetting factors, estimation performance was secured despite the estimation error due to the convergence delay of the LKF. However, online convergence delay estimation is necessary to apply the proposed algorithm to a variety of situations and systems. Analysis of the estimation error showed that the error has Gaussian distributions and that an average of the estimation error for the rotational inertia is almost zero. The standard deviation is less than 5%. Table 1 lists the average and standard deviation of the estimation error for the rotational inertia.

Table 1 Estimation error (rotational inertia) analysis: Average and standard deviation

Division	Descriptive statistics for estimation error	
	Average [%]	Standard deviation [%]
Case 1	-1.7038	4.5112
Case 2	0.1314	4.9887
Case 3	-0.4894	3.9960

## 5. Conclusion

The aim of this study was to estimate the rotational inertia of an excavator by using RLS with updating rules and multiple forgetting and to evaluate the performance of the estimation algorithm. For this purpose, a recursive least square algorithm was developed, and appropriate multiple forgetting factors were defined to estimate the time-varying parameters, such as rotational inertia, which varies with the working posture. The updating rule for the forgetting factor considering the convergence delay of the LKF was applied for improving the estimation performance. The proposed estimation algorithm only requires information on swing velocity and damping coefficient for practical implementation to an actual system. The reference rotational inertia in a typical working scenario was derived analytically for verifying the performance of the developed estimation algorithms. Estimation performance was evaluated through simulations in MATLAB/Simulink environment with various material conditions (without any material, with sand (288 kg), and with sandstone (423 kg)). The simulation results show that the proposed estimation algorithm had high estimation performance with the designed updating rules. Based on the analysis of the estimation error with Gaussian distribution, the average of the estimation error was almost zero, and the standard deviation was less than 5%. However, the convergence delay used in the updating rule for the forgetting factor should be derived by online analysis to secure robust estimation performance for practical applications. In addition, the designed estimation algorithm cannot estimate the rotational inertia when the excavator is in a stationary state because the required information such as the swing velocity is zero. Therefore, application of online estimation of the convergence delay for swing acceleration and development of an estimation algorithm when the excavator is in a stationary state are being considered for future work. The proposed RLS-based algorithm for rotational inertia estimation can be applied to various mechanical systems, including excavators, for safety and automation. It is also expected that the proposed estimation algorithm and evaluation technique can be employed in the design stage.

## Acknowledgement

This work was supported by the KIMM Research Program (NK198C), the Brain Korea 21 Plus Project in 2016, the Institute of Advanced Machines and Design (IAMD), Seoul National University, and the National Research Foundation of Korea (NRF) grant (No. 2009-0083495) funded by the Ministry of Science, ICT and Future Planning (MSIP).

## References

- 1) S. Tafazoli, P. Lawrence and S. Salcudean, "Identification of Inertial and Friction Parameters for Excavator Arms", *IEEE Trans. on Robotics and Automation*, Vol. 15, pp. 863-885, 1999.
- 2) C. Tan, H. Zweiri, K. Althoefer and L. Seneviratne, "Online Soil Parameter Estimation Scheme Based on Newton-Raphson Method for Autonomous Excavation", *IEEE/ASME Trans. on Mechatronics*, Vol. 10, pp. 221-229, 2005.
- 3) S. Lee and P. Chang, "Modeling of a hydraulic excavator based on bond graph method and its parameter estimation", *Journal of Mechanical Science and Technology*, Vol. 26, pp. 195-204, 2012.
- 4) A. Vahidi, A. Stefanopoulou and H. Peng, "Recursive least squares with forgetting for online estimation of vehicle mass and road grade: theory and experiments", *Vehicle System Dynamics*, Vol. 43, pp. 31-55, 2005.
- 5) S. Dasgupta and Y. Huang, "Asymptotically convergent modified recursive least-squares with data-dependent updating and forgetting factor for systems with bounded noise", *IEEE Trans. on Information Theory*, Vol. 33, pp. 383-392, 1987.
- 6) X. Xu, H. He and D. Hu, "Efficient reinforcement learning using recursive least-squares methods", *Journal of Artificial Intelligence Research*, Vol. 16, pp. 259-292, 2002.
- 7) B. Pence, H. Fathy and J. Stein, "Recursive estimation for reduced-order state-space models using polynomial chaos theory applied to vehicle mass estimation", *IEEE Trans. on Control System Technology*, Vol. 22, pp. 224-229, 2014.
- 8) B. Yao, J., Zhang, D. Koehler and J. Litherland, "High performance swing velocity tracking control of hydraulic excavator", *Proceedings of American Control Conference*, Vol. 2, pp. 818-822, 1998.