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A Multiple-Case Study of Preservice Secondary Mathematics Teachers' Teaching Demonstrations with Geometer's Sketchpad

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This is a multiple-case study of how preservice secondary mathematics teachers teach a particular mathematics using a technological tool. In a performance interview, the preservice teachers demonstrated how they would teach a specific mathematical topic using Geometer's Sketchpad. The results of this study showed that the preservice teachers designed diverse types of lesson plans and implemented different pedagogical and technological techniques in their teaching demonstrations. The findings suggest that preservice teachers' pedagogical content knowledge is an important factor in the integration of technology into their mathematics teaching. Thus, mathematics teacher educators should help preservice teachers gain a robust pedagogical content knowledge in order to effectively teach mathematics with technological tools.

Key Words: Preservice secondary mathematics teachers, Geometry, Teaching Demonstration, Dynamic geometry environment

Ⅰ. Introduction

Both at home and in school, students and teachers can have various experiences with technologies, and thus using technology to learn and teach mathematics is no longer a new experience for them. The National Council of Teachers of Mathematics' (NCTM, 2014) Principles to Actions: Ensuring Mathematics Success for All includes the guiding principles regarding the use of technology in mathematics teaching. Here, the NCTM (2014) states that the use of tools and technology can facilitate students' exploration and making sense of mathematical ideas, mathematical reasoning, and the communication of mathematical thinking. Diverse tools and technologies are emphasized as indispensable

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features of the classroom for the meaningful learning of mathematics. In addition, there are numerous technological resources for learning and teaching mathematics, such as computer software programs and the Internet or tablet applications. Thus, it is important for mathematics teachers to use these diverse resources appropriately to effectively teach mathematics.

In that sense, it is important for teacher education programs to cultivate teachers who effectively integrate technology into their teaching. Many teacher education programs in the US offer technology courses for preservice teachers (Kleiner, Thomas, & Lewis, 2007). In the literature, however, there is a lack of research on preservice teachers' technology use in their mathematics teaching, even though the mathematics education field emphasizes integrating technology into teaching. In this study, I examined how preservice secondary mathematics teachers teach a specific mathematical concept using technology, and focused on describing in detail how they implement it in their lessons.

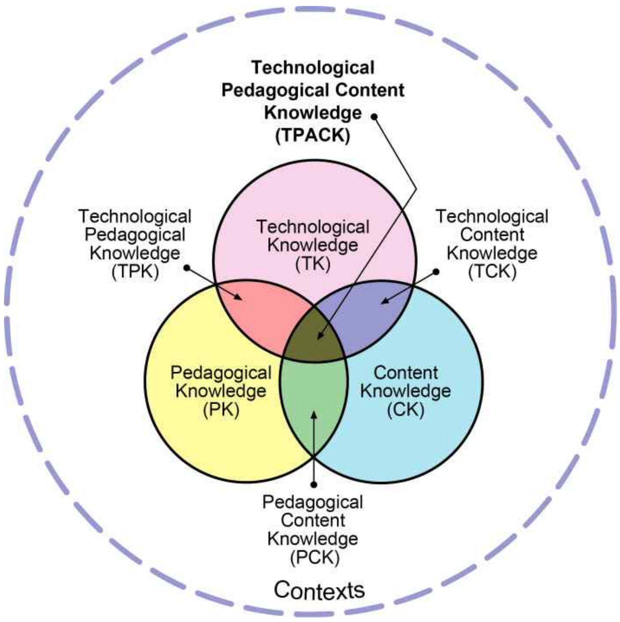
Ⅱ. Literature Review

1. Good Mathematics Teaching with Technology

What is good mathematics teaching with technology? How should teachers use technology to effectively teach mathematics? According to Koehler and Mishra (2005), good teaching with technology is "not simply adding technology to the existing teaching and content domain. Rather, the introduction of technology causes the representation of new concepts and requires developing a sensitivity to the dynamic, transactional relationship between all three components" (p. 134); technology, pedagogy, and content. Koehler and Mishra emphasized the role of teachers in teaching with technology. They argued that in order to effectively teach subject matter using technology, teachers should understand the representations of concepts using technologies, have pedagogical techniques that use technologies in constructive approaches to teach subject matter, know about what makes concepts difficult or easy to learn and how technology can help remedy some of the problems students may encounter, and have knowledge of how technologies can be used to build on existing or prior knowledge to develop new knowledge or strengthen old ones (Koehler & Mishra, 2009, p. 66). In terms of mathematics teachers, Niess (2006) stated that because teachers think about particular mathematical concepts or ideas, teachers need to simultaneously contemplate "how they might teach the important ideas embodied in the mathematical concepts in such a way that the technology places the concept in a form understandable by their students" (p. 196). Therefore, mathematics teaching with technology requires teachers to have a multifaceted understanding and knowledge about technology, pedagogy, and content, that is, Technological, Pedagogical, and Content Knowledge (TPACK).

2. Technological, Pedagogical, and Content Knowledge (TPACK)

What To effectively integrate technology into teaching, preservice and current teachers should have interwoven knowledge of technology, pedagogy, and content. Mishra and Koehler (2006) defined the construct TPACK to identify and describe the knowledge teachers should have to teach effectively while incorporating technology. Adding the 'technology' component to Shulman's (1986) Pedagogical Content Knowledge (PCK), Mishra and Koehler's framework has three main components: Content, Pedagogical, and Technological Knowledge (CK, PK, and TK) and the intersections between and among them, defined as Pedagogical Content Knowledge (PCK), Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), and Technological Pedagogical Content Knowledge (TPACK) (see Figure Ⅱ - 1).



[Figure Ⅱ-1] The TPACK framework and its knowledge components
(Source: http://tpack.org).

3. Status of Preservice Teachers' Teaching with Technology

With increased interest in the integration of technology into mathematics teaching, many researchers have investigated mathematics teachers' use of technology (e.g., Dwyer, Ringstaff, & Sandholtz, 1991; Ertmer, Ottenbreit-Leftwich, Sadik, Sendurur, & Sendurur, 2012; Kim, Kim, Lee, Spector, & DeMeester, 2013). Little research, however, has investigated preservice teachers' teaching with technology, such as their student teaching or the beginning of their teaching practice.

Choy, Wong, and Gao (2009) examined preservice teachers' intentions and actions regarding technology integration in their student teaching. The researchers indicated that teachers had difficulties in integrating technology into their lessons even though they were willing to use it in their teaching, and the schools where they taught were well equipped with computers and Internet access. During their student teaching, the preservice teachers usually used technology to prepare handouts or record grades and attendance. Rather than to facilitate student-centered learning, most preservice teachers used technology as an instructional tool to convey information and gain students' attention, for instance by using PowerPoint or the Internet to display images or play videos.

Özgün-Koca, Meagher, and Edwards (2010) indicated that preservice secondary mathematics teachers' use of technology was superficial at the beginning of their research period. For instance, when one preservice teacher designed a task about the Pythagorean Theorem using a dynamic geometry environment (DGE), the task was no different from a traditional activity using pen and paper. The preservice teacher did not use the dynamic capabilities of DGE (e.g., the constructing or dragging features) to produce more exploratory-based tasks. Thus, the studies to date demonstrate that preservice teachers tend to use technology in traditional or teacher-centered approaches.

Ⅲ. Methodology

In order to thoroughly investigate and describe preservice secondary mathematics teachers' teaching demonstration with technology, I selected a qualitative research methodology, a multiple-case study.

1. Participants and Context

The participants were four undergraduate preservice secondary mathematics teachers who volunteered for this study. They were enrolled in a secondary mathematics pedagogy course at a university in the southern United States. During the course, they received instruction in learning and teaching geometry, probability, and sequences and series using technology. The participants were familiar with geometric topics and technological tools such as Geometer's Sketchpad 5 (GSP) and graphing calculators. All of the participants' names (John, Mia, Emily, and Leah) are pseudonyms.

2. Data Collection

Given that I focused on preservice secondary mathematics teachers' teaching with technology, I designed the performance interview for the participants to demonstrate how they would teach a specific mathematical topic using GSP (see Figure III - 1). Through this interview, I examined how participants design their lessons, make decisions based on their content knowledge and pedagogical reasoning, and use their knowledge about technology to teach geometry using GSP (Harris, Grandgenett, & Hofer, 2010). During the interview, I observed how participants actually implement a lesson on a specific mathematical concept using GSP. As the topic for the performance interview, I chose the polygon exterior angle sum theorem (the exterior angle theorem: the sum of the measures of the exterior angles of a convex polygon is 360 degrees). Given that the pedagogy course did not deal with this theorem, and that the participants could find diverse possible strategies to teach it, the theorem was appropriate for evaluating the participants' teaching with GSP. In the performance interview, each participant could use the Internet, a laptop with GSP, blank paper, and pencils. The interview took approximately one hour and was audio- and video-recorded using a computer-screen-capture software program.

3. Data Analysis

To assess participants' teaching demonstrations, I focused on three important categories: content, pedagogy, and technology. In the content category, I examined whether the participants had appropriate mathematical knowledge related to the exterior angle theorem, a deductive or inductive proof, an algebraic or geometric proof, or connections between mathematical ideas or concepts. For example, I examined whether the participants knew about definitions of mathematical concepts and figures, the sum of interior angles of polygons, or parallel line postulates. In the pedagogy category, I evaluated the pedagogical strategies that the participants used to teach the theorem and whether they could anticipate the students' mathematical thinking, understanding, or potential difficulties. For instance, I focused on whether the participants considered how the students learn mathematics, such as student-centered approaches or different learning styles. Lastly, in the technology category, I assessed the participants' knowledge about how to use GSP to implement certain mathematical tasks and considered how they used GSP to support or reflect pedagogical strategies in their teaching. For example, I examined whether the participants used GSP to facilitate students' exploration by using the dragging feature of GSP or provided various representations of mathematical concepts by fully using the diverse features of GSP.

Performance Interview

In this Performance Interview, I would like to see how you would teach a geometry topic using Geometer's Sketchpad. For this interview, you will prepare a lesson to teach a theorem about the exterior angles of a polygon using GSP.

Think about how to teach the following theorem:

The sum of the measures of the exterior angles of a convex polygon is 360.

I will conduct an interview 1 or 2 weeks later (depending on your schedule). During the interview, I will ask you to describe how you would teach the polygon exterior angle sum theorem and ask some questions about your lesson.

In your teaching, you should use GSP:

- You may prepare teaching materials in advance.
- You may make teaching materials during the interview.
- You may just use GSP during the interview.

[Figure Ⅲ-1] Performance Interview Instructions

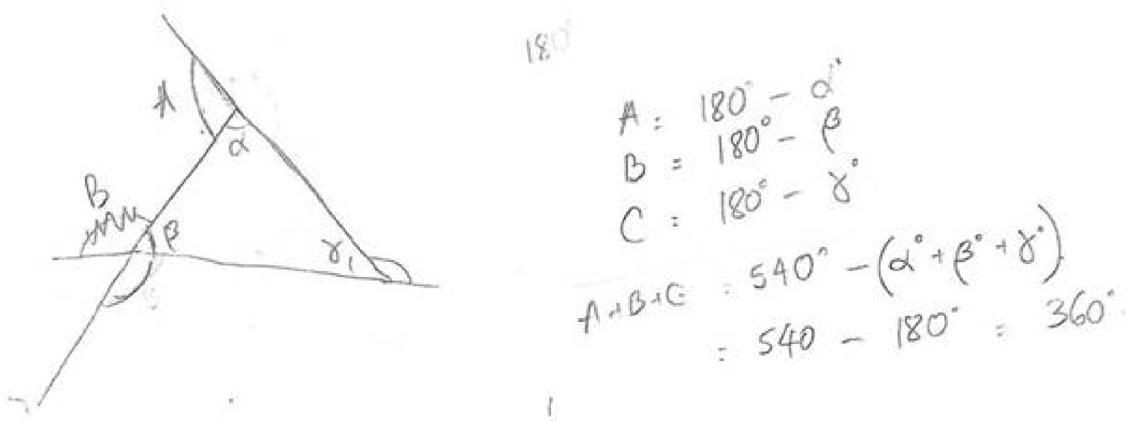
Ⅳ. Findings

Individual Case Studies

1. Mia

Mia was the only participant who did not prepare the lesson plan for the performance interview in advance. Mia, however, knew that she would teach a theorem about polygons. After reading thoroughly the performance interview guide sheet during the interview, she extemporarily described how she would teach the exterior angle theorem.

Mia began the interview by drawing a triangle on paper and tried to figure out why the theorem is true. Mia knew the extended line of sides of polygon was needed to make the exterior angle, but she was not able to state the exact definition of exterior angle. With algebraic equations, Mia explained why the sum of exterior angles of triangle is 360 degrees using her knowledge that the sum of interior angles of triangle is 180 degrees and the sum of exterior and interior angle at the same vertex is 180 degrees. She found the sum of exterior angles of triangle is 180 × (The number of vertices of triangle) - (The sum of interior angles of triangle). Actually, this process was for her, not for teaching. She just checked and proved whether the theorem is really true for a triangle before teaching (see Figure IV - 1).



[Figure Ⅳ−1] Mia's proof of the exterior angle theorem for a triangle

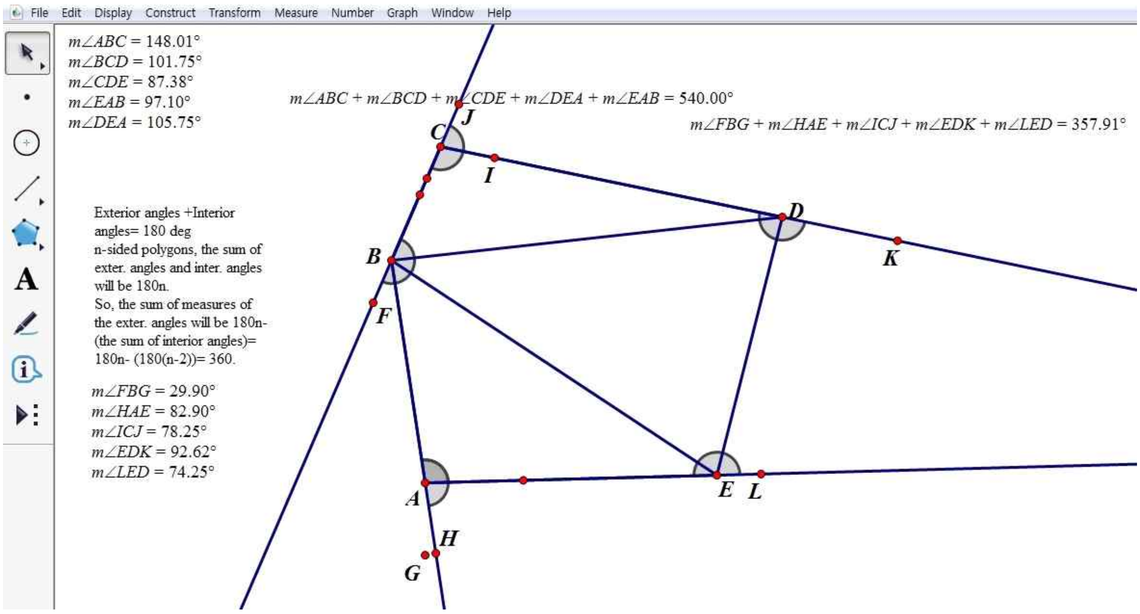
To teach the exterior angle theorem, Mia wanted to start out with a triangle. First, she would draw a triangle on the board and show and explain why the theorem is true for a triangle much like what she did on the paper in the beginning of the interview. Then, Mia said that she would have students construct any 4-sided polygon using GSP. Next, the students would find, measure, and add up the interior and exterior angles of the polygon, respectively. After checking that the theorem works for the 4-sided polygon using GSP, Mia would ask the students if it works for a triangle and quadrilateral, is it true that the sum of the measurements of the exterior angles is 360 degrees for a 5-sided polygon? Mia would let the students create, measure, and add up the interior angles of a 5-sided polygon by hand or using GSP. Mia would directly have the students find the sum of interior angles first rather than let the students explore the exterior angles or give them the opportunity to find the proof of the theorem themselves because she wanted the students to follow the process she did for a triangle case to prove why the exterior angle theorem is true for a 5-sided polygon. Mia had a hard time, however, finding the sum of interior angles for any polygon. She tried to find out a pattern between the sum of interior angles of an n-sided polygon and n, but she was not able to find it. After I gave her a hint by asking how many triangles are in the quadrilaterals, Mia was able to derive the pattern for the sum of interior angles of an n

-sided polygon as 180 × (n - 2). Finally, she could prove the theorem for any convex polygon. Return to her teaching demonstration, Mia would have the students come up with a formula about how to calculate the sum of interior angles of n-sided polygon. If the students do not know what to do, she would have them consider how many triangles are in the n-sided polygon and write down the process of finding the formula for the sum of interior angles of n-sided polygon she went through. After finding the formula, 180 × (n - 2), Mia would ask what the sum of the exterior and interior angle is and how to calculate the sum of the exterior angles. She hoped the students could answer her questions. As advanced tasks or questions, Mia said that she would ask the students what they learned from the activity and explore for a formula when you have a concave polygon.

When I asked about the students' thinking or misconceptions they may have, Mia stated that the students may not know what the exterior angle or the convex polygon is or how to construct the exterior angle using GSP. The students might think that the exterior and interior angle are the same, so they might say that the sum of exterior angle has to be 180 degrees because the sum of interior angles of triangle is 180 degrees. If the students do not know mathematical conceptions or have misconceptions, Mia said that she would explain what it is and correct their misconceptions. The definitions she provided during the interview, however, were not the precise definitions.

During the performance interview, Mia claimed she let the students find out whether the theorem is true for a quadrilateral and pentagon using GSP, but she provided directions step by step. She asked students to create polygons, measure interior and exterior angles, and calculate the sum of interior and exterior angles, respectively. Rather than having the students explore many cases to figure out whether the theorem is true or not using GSP, she was focusing on showing that the theorem is true. In addition, when Mia led the students to come up with the proof of the theorem, she seemed to give them lots of hints or ask direct questions. For example, she already showed the students the proof for the triangle case at the beginning of her teaching and asked a leading question such as what the sum of the exterior and interior angles at the one vertex is. Mia had the students consider a few examples (triangle, quadrilateral, and pentagon) and provided only the algebraic proof. Mia seemed to value explanation or transmission of her knowledge rather than students' opportunity to explore and find out why the theorem is true for any convex polygon themselves.

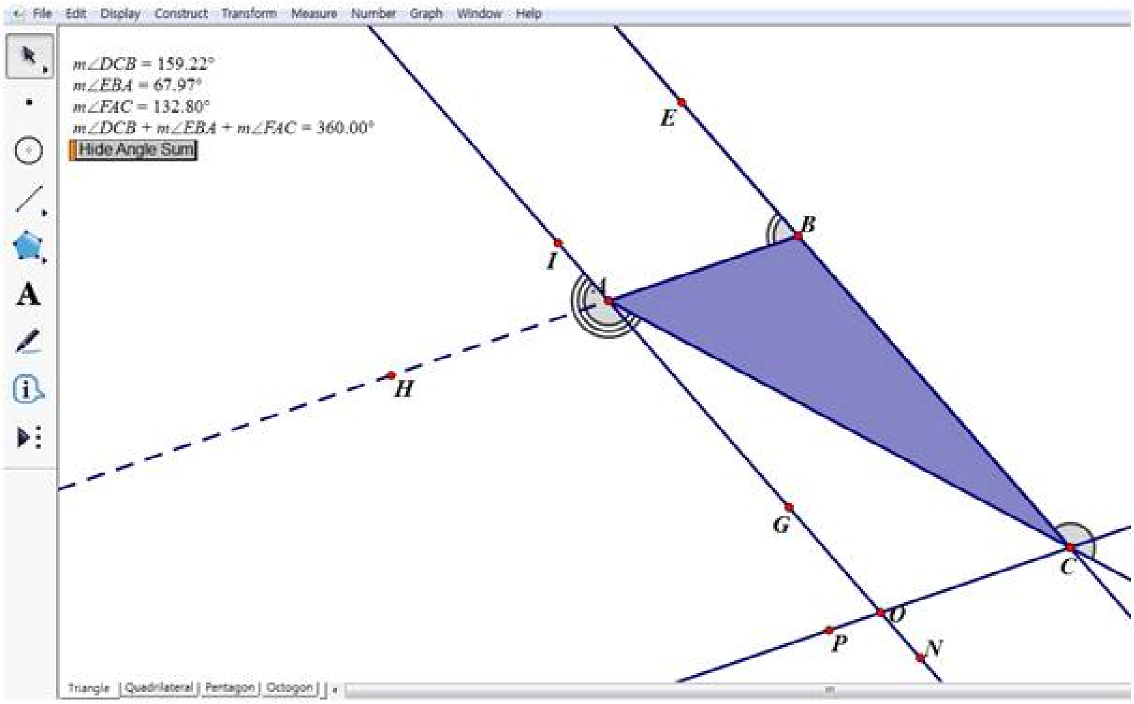
Using GSP, Mia was able to construct triangle, parallelogram, and pentagon, label vertices, mark and measure angles, calculate the sum of angles, and type letters. When Mia was constructing the extended sides of parallelogram, she used parallel lines to the sides. She was unable to correctly construct the extended line of sides of pentagon to find the exterior angles, however, because she actually did not know how to construct the extended line of the sides using GSP. Mia did not think the use of GSP makes it easier to deal with a large number sided polygon rather than pen and paper. She said that using GSP is appropriate for a small number sided polygon, but it is difficult for a large number sided polygon (a hundred sided or n-sided polygon) because she struggled to construct the extended line of sides of pentagon and her calculation of the sum of exterior angles was not correct (see Figure Ⅳ - 2). She was concerned that it would be a time consuming work if the students do not know how to create a large number sided polygons using GSP. Although the use of GSP helps students see that the sum of exterior angles always stays 360 degrees even if the polygon is changing, she said, she would not use GSP to teach this theorem for a large number sided polygons. Overall, Mia did not fully use the dragging feature of GSP to explore many examples or to examine her or students' conjectures until the interviewer prompted her dragging. Mia used GSP for convenience to construct polygons and measure angles, not for students' better understanding or pedagogical reason. Throughout the interview, Mia seemed to be more familiar with pen and paper work rather than the use of GSP. She believed that dealing with the proof or explaining processes of proof by hand is a familiar and easier way to teach the theorem than constructing conjectures and exploring many polygons using GSP.



[Figure Ⅳ-2] Mia's pentagon with her calculation of the sum of exterior angles

2. John

Unlike Mia, John prepared the pre-constructed GSP file consisting of several pages for his teaching. John described that he would give students the pre-constructed GSP file at the beginning of teaching so that the students could work themselves using GSP (see Figure Ⅳ - 3). On the first page of the GSP file, there were a triangle, exterior angles, measurements of the exterior angles, and a button “Show Angle Sum” for showing the sum of the exterior angles. First, John would make the students click and drag any point and see what is happening. By asking questions about the size of the exterior angles, John tried to have the students consider and find the pattern among the angles. After exploring, without hints or leading questions about the sum of exterior angles, John would let the students click the angle sum button and would ask whether the sum of exterior angles of this triangle was 360 degrees even though he had not addressed the theorem previously. Then, he would ask whether this was true for all triangles and would let the students make their own triangle by dragging vertices of the triangle. After checking it, John would move on to a quadrilateral and would let the students investigate the pattern of the exterior angles of quadrilateral using dragging feature. John anticipated the students would say that the quadrilateral’s exterior angles were smaller than a triangle’s because the quadrilateral had four sides.



[Figure Ⅳ-3] John’s pre-constructed GSP files

John said that if the students said the sum of exterior angles of quadrilateral is not 360 degrees when they explored the quadrilateral, then he would say, “Yes, because the [exterior] angles are smaller” (Interview 3). After that, John would let the students click the angle sum button to check whether it was true. John was unclear about what it means to be smaller. Were all exterior angles of quadrilateral smaller than all the angles of a triangle or some of them? If John and his students explored only regular polygons, they would easily notice that all exterior angles of quadrilateral smaller than triangles'. That would not always happen when exploring irregular polygons. Since John already knew that the sum of exterior angles of polygons is 360 degrees and 360 degrees should be separated into parts depending on the number of the sides of the polygon, he might have that sense. After doing the same activity with a pentagon, John would lead an investigation about a concave polygon because a concave polygon could be constructed by dragging. John would show that the sum of exterior angles of concave polygon is not 360 degrees and would let the students investigate a definition of convex. He did not state the definitions of convex and concave polygons, however, during the interview. Then, John thought the students could see that the theorem only works for convex polygons, not concave. Lastly, John wanted the students to construct their own convex polygon (a large number sided polygon) on the last blank page of GSP and explore the theorem using dragging, measuring, and calculating tools. Using GSP John constructed polygons using rays instead of constructing the extended lines of the sides of the polygon separately. He had the misconception that the rays should be in one direction. John said that he would address this early to let the students recognize it when they constructed polygons using GSP. John did not, however, state the meaning of the rays and the exact definition of exterior angle. He hoped the students would see that the theorem was true for any convex polygon through his activity. However, John did not prepare the proof part of the theorem. He only focused on exploration of many polygons without the formal proof. When I asked him what if students ask how we can know that the theorem is true for every convex polygon, John tried to prove the theorem. Although John had a clue that the sum of exterior and interior angles at each vertex is 180 degrees, he struggled to develop the proof using the clue. Finally, John proved why the theorem is true for a triangle using parallel line postulates, but he needed some help from the interviewer. He thought if it worked for a triangle, it would work for all polygons because every polygon is based on a triangle. John was unable, however, to connect what he already knew to prove the theorem for all convex polygons. When I asked about a further investigation or homework, John wanted the students to think about how you can prove the theorem and why the sum of exterior angles of concave polygon is greater than 360 degrees.

As students' misconceptions, John pointed out that the students may not know what convex and concave mean, what a polygon is, what an exterior angle is, or the need of a certain direction of rays. If most of the class did not know mathematical conceptions or had the same problem, he would stop the class to address or discuss it.

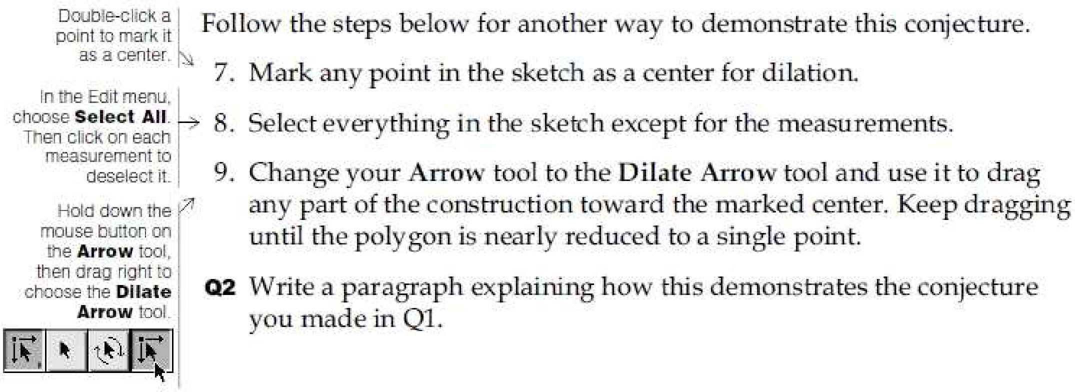
John was able to construct triangles, quadrilaterals, pentagons, and a 9-sided polygon using the rays, label vertices, mark and measure angles, calculate the sum of angles using the GSP. He also knew how to add a page on the GSP file and how to make a button that shows or hides a figure, measurement, or caption. In addition, rather than give the answers, John would try to ask many questions and give the students many opportunities to explore the theorem themselves using GSP even though he would have them directly consider the sum of exterior angles at the beginning of teaching. John would let the students use diverse tools of GSP such as constructing, dragging, measuring, and calculating tools. He believed that the use of GSP would add more fun because the students could work with their own figure and actually see that the theorem works for any convex polygon. John stated that he prefers to use GSP to teach the exterior angle theorem. He said:

> That [GSP] helps a lot. Because hand and paper you can just mess up, and I've done proofs where I've drawn figures and it's like ... you know, that residue ... that black/grey residue that it leaves on your paper. I guess it just makes it neater. And then you can check [a conjecture] yourself because if you point and drag it ... and 'Well, is it true? It should be' ... I just think it looks nicer and cleaner. (Interview 3)

And, he thought that the students could learn how to use GSP themselves through his activity.

3. Emily

For the performance interview, Emily brought a worksheet she found on the Internet to use for her lesson. The worksheet was about activities with GSP to discover the sum of the measures of the exterior angles in convex polygons. Unlike the worksheet that dealt with a pentagon, she wanted to start with a simple shape such as a triangle or square and then push the students to work with a hexagon or octagon. Thus, Emily started with a triangle in the beginning of her teaching and constructed it using rays on GSP. She explained the reason she would use rays was that it was easier to visualize exterior angles of the triangle. Rather than give the exterior angle theorem to the students at the beginning of lesson, Emily said that she would have the students measure the length of every side and angle of a polygon and would ask what the measures of the exterior angles of the different shapes are. Emily wanted the students to find the pattern of exterior angles by exploring various measurements of the polygon, but she did not ask appropriate questions to lead the students to consider the sum of the exterior angles to find the exterior angle theorem. She would let the students construct diverse polygons, measure the exterior angles of the polygons, calculate the sum of exterior angles, drag the vertices of the polygons, and observe the sum of exterior angle measures. The students would see that the sum of exterior angles of the polygons was still adding up to 360 degrees and could make their conclusion about the exterior angle measures. Emily did not state the exact definition of the exterior angle during the interview, however, and struggled to find correct exterior angles when she constructed and explored a quadrilateral as one of the examples. Emily correctly constructed the exterior angles when she tried again, but she still did not seem to know the exact definition of the exterior angle and the meaning of rays. She just said, "You make [the exterior angle] like where these [the rays] intersect ... That's your angle" (Interview 3). The last activity on the worksheet was about another way to visually demonstrate the theorem using the dilate arrow tool of GSP (see Figure IV – 4). The dilate arrow tool could make the size of the polygon shrink without changing the shape of the polygon and the size of the exterior angles. If you kept dragging until the polygon was nearly reduced to a single point, you could see that only the marked exterior angles remain and the gathered exterior angles would be a circle at the end. Emily tried to follow the steps in the worksheet, but she could not implement them correctly. Because Emily only focused whether the circle appeared, she did not care whether she correctly went through the processes to demonstrate the theorem (see Figure IV – 5). Emily kept trying to use the last way to demonstrate the exterior angle theorem for a triangle, quadrilateral, and seven-sided polygon, but she never succeeded. It may be due to her misunderstanding of the dilate arrow tool's role or careless reading. After doing the last activity of the worksheet, Emily said that we could do that with only GSP.


Double-click a point to mark it as a center.

In the Edit menu, choose **Select All**. Then click on each measurement to deselect it.

Hold down the mouse button on the **Arrow** tool, then drag right to choose the **Dilate Arrow** tool.

Follow the steps below for another way to demonstrate this conjecture.

7. Mark any point in the sketch as a center for dilation.
8. Select everything in the sketch except for the measurements.
9. Change your **Arrow** tool to the **Dilate Arrow** tool and use it to drag any part of the construction toward the marked center. Keep dragging until the polygon is nearly reduced to a single point.

Q2 Write a paragraph explaining how this demonstrates the conjecture you made in Q1.

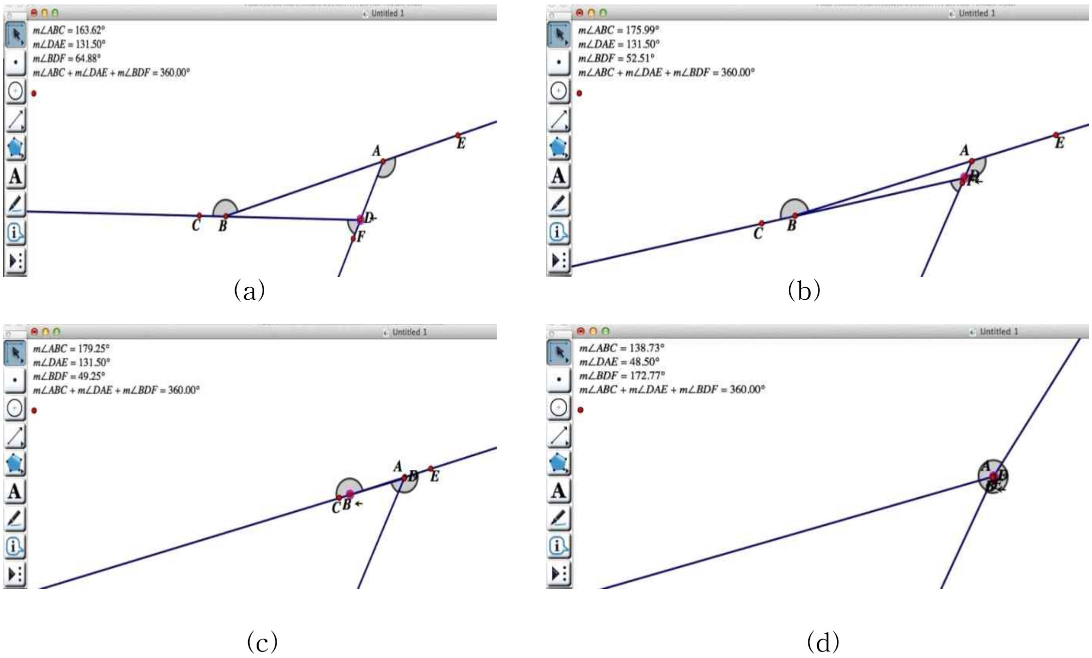
[Figure Ⅳ–4] The last activity on the worksheet that Emily brought

Like John, Emily did not prepare the proof part of the theorem and only focused on the exploration of many polygons without formal proof. When I asked how to prove the theorem, she said that she needs to split 360 degrees into n parts for an n-sided polygon. Actually, Emily needed to prove that the sum of exterior angles of any convex polygon is 360 degrees, but she already used that fact to prove it. And, Emily thought that she could prove the theorem for regular polygons using mathematical induction. She explained:

> If you have the first case and then you prove the one after and so then if the first case works and then the one after works. So we know our triangle and we know the square works because the square is the next step up. So, then from there you'll know the square works and then the pentagon has got to

work ... by induction. I mean, I didn't even prove like all the way mathematically, but you know what I am saying like it will be induction. (Interview 3)

After discussing the proof of the theorem with the interviewer, Emily said the homework she would assign would be “Prove why a regular polygon has exterior angles adding up to 360?” Then, the next day in class, she would work with the students on the proof for irregular polygons.



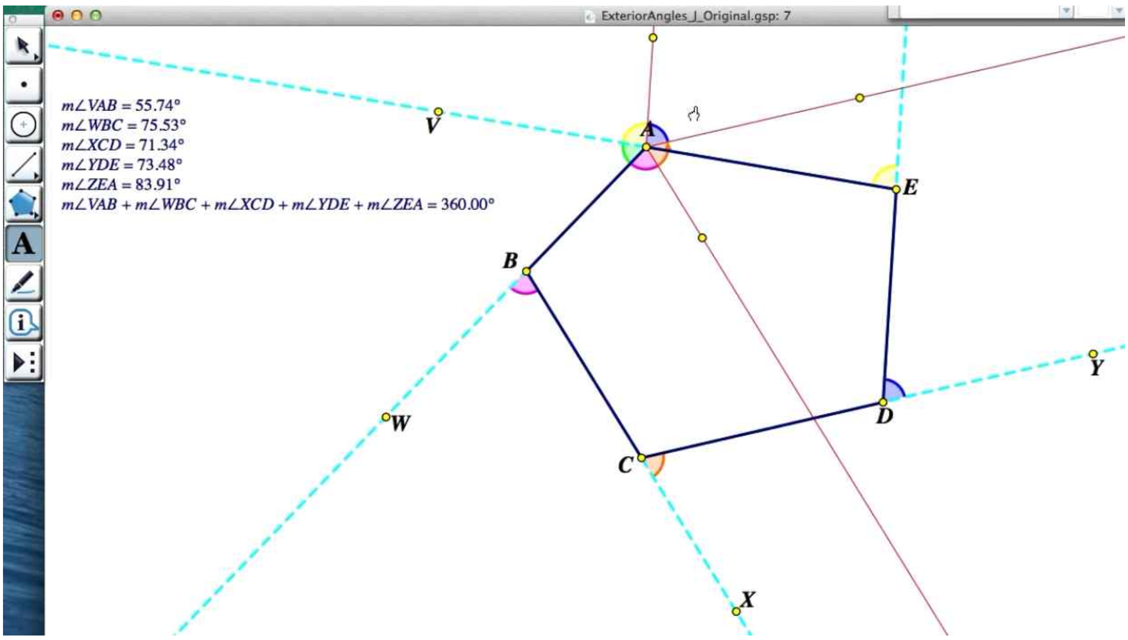
[Figure Ⅳ-5] Emily's performance on the last activity on the worksheet

When I asked what if the students have the misconception that a polygon with a greater number of sides has a larger sum of exterior angles, Emily said that she would let the students know that each exterior angle measure is going to be smaller if the number of sides is going to be bigger. Like John, Emily already knew that the sum of exterior angles is fixed at 360 degrees and did not seem to consider irregular polygons. Emily also thought that the students could make the same mistakes as she did in constructing exterior angles. If the students were confused or made mistakes when they made exterior angles, Emily said she would point out how to make the exterior angles asking, “How are you making your angles? What makes an angle?” (Interview 3)

Overall, Emily knew how to construct an equilateral triangle and quadrilateral using rays, mark and measure the exterior angles, and calculate the sum of the angles. She had the students examine whether the theorem is true for diverse polygons and tried to develop the worksheet by modifying it. Although she did not ask meaningful questions to help the students consider the sum of exterior angles, she tried not to directly give them the theorem or the answers to the questions. Emily said she would let the students explore many polygons using dragging, measuring, and calculating tools of GSP. She believed that it is valuable to drag a point and see that the sum of the exterior angles of the polygon is not changing even though the side lengths and angles of the polygon are changing so that the students can find a pattern and make their conjectures. The students could better understand what they were learning because they could see the exterior angles are gathered and make a circle. However, Emily was unable to develop the deductive proof and the visual demonstration of the theorem.

4. Leah

Like John, Leah brought a pre-constructed GSP file with several pages she made to the interview. First, Leah explained that she would give students the pre-constructed GSP file so that they could work individually. On the first page of the GSP file were questions about the sum of the exterior angles of a rectangle. To address the theorem, Leah said that she would ask, "What is the sum of exterior angles of the rectangle ABCD?" She wanted to start out with a rectangle because the students could see that the exterior angles are obviously 90 degrees and could be easily added together to arrive at the sum of 360 degrees. Then, Leah would ask the students to create the claim that this is true for any rectangle. She would let the students move around the rectangle and find that the exterior angles are always 90 degrees and the sum of them is always 360 degrees. Leah would not deal with a formal proof at this stage, but she would let the students discuss why that is true or not. She would have the students do the same activity with a triangle on the next page of the GSP file. The students would measure the exterior angles, calculate the sum, and drag vertices to explore diverse triangles to examine whether the claim they constructed is true or not. Leah expected that the students could come up with the proof for a triangle using that the interior angle and the exterior angle make a straight line and the sum of the interior angles of the triangle is 180 degrees and express this algebraically. Then, Leah would provide an octagon and let the students examine whether the claim is true for the octagon and then find a pattern. Because Leah wanted the students to explore and find the theorem, she said she would provide students with the claim: The sum of the measures of the exterior angles of any polygon is 360 degrees. Then, she would let the students construct a polygon in a blank space on the next page of GSP and test Leah's claim by dragging one vertex to create different shapes. Through this exploration of polygons, Leah expected the students could find out that the claim is not true for a concave polygon and could then revise the claim using the precise wording of the theorem: The sum of the measures of the exterior angles of any convex polygon is 360 degrees. Then, Leah would ask why this theorem is true. When dealing with a proof of the theorem, Leah said she would use an irregular pentagon. She expected that the students could use the same reasoning as they used in the triangle case. Leah would have the students discover the proof themselves first and then would discuss it as a class. Leah was the only participant who correctly demonstrated two different proofs of the exterior angle theorem (e.g., algebraic and geometric proofs). At first, Leah proved the theorem for a pentagon using algebraic equations, and she was also able to develop the proof for an n-sided polygon. Leah knew that the interior and exterior angles add up to 180 degrees and how to calculate the sum of the interior angles of a polygon. Using her knowledge, Leah was able to show that $180 \times n - \{(180 \times (n - 2)\} = 360$, where n = the number of the sides of a polygon. In addition, Leah said she would discuss another way to prove the theorem for the pentagon geometrically that the students could potentially come up with as a whole class. Leah provided an explanation of the second way using parallel line postulates to prove the theorem for the pentagon. Constructing parallel lines to each side going through one of vertices of the pentagon, she found and marked the same angles as each exterior angle of the pentagon at the vertex. Then, the same angles as each exterior angle around the vertex made a circle, and the sum of them was 360 degrees (see Figure Ⅳ - 6). She explained how the students could make a circle consisting of the exterior angles of the pentagon. Although this was not the proof for all polygon, the students could actually see that the sum of exterior angles of the pentagon is 360 degrees and better understand the theorem through this method.



[Figure Ⅳ-6] Leah's proof of the exterior angle theorem using parallel line postulates

After completing her teaching demonstration, I asked her, "What if the students cannot find out the proof?" Leah said that for the algebraic proof, she would go back to the triangle case to recall the reasoning they had used in the triangle case because they would be more familiar with a triangle and know the sum of the interior angles of the triangle. She would have the students focus on one of vertices to see that the interior angle and exterior angle make a straight line. Then, she would ask the students to use the same reasoning for other polygons. For the geometric proof, she would ask the students to recreate the angles at one vertex. If the students still did not understand, she would give a hint such as using parallel lines. Leah thought that if the students had that starting point, this would help them to do the proof. For advanced tasks or homework, Leah would ask about an example where the theorem is not true. She thought that the students could come up with a concave polygon. And, Leah would ask them to provide both algebraic and geometric proofs as they had done in the class for the different polygons.

Leah anticipated the students might have some misconceptions. One of them is where the exterior angle is. Leah said that it would be good to have the students find where the exterior angles are first to avoid their misunderstanding of it. She did not, however, state the exact definition of the exterior angle. The other misconception the students might have would be that the sum of exterior angles is going to be different depending on how many sides the polygon has.

Leah was able to use diverse GSP tools to construct triangles, rectangles, pentagons, and octagons using rays, label vertices, mark and measure angles, calculate the sum of angles, and type text. She also knew how to add a page to the GSP file and how to create parallel lines to the sides of the polygon. In addition, Leah wanted the students to explore diverse polygons themselves using diverse features of GSP. She thought that GSP is a nice tool to investigate the claim the students made because they can construct their own polygons, move them around, and see that the sum of exterior angles of the polygon is always going to be 360 degrees. When dealing with the octagon, Leah was concerned that the octagon could be made concave by dragging. Although the large number sided polygon may create issues for students, Leah said that she would use GSP to explore it because "It would be cool for them [the students] to see even this weird looking polygon still works for that one" (Interview 3). Leah stated that GSP is beneficial to dealing with, especially, the second proof because the use of GSP makes it more clear to see what is going on than drawing on the board by hand. She believed that GSP can help the students think of the idea or convince them that the sum of exterior angles is actually 360 degrees making a circle. Moreover, Leah was the only participant who considered aesthetic features of GSP. She used different colors for the students to see easily where the exterior angles or their corresponding angles are and why the theorem works (see Figure IV - 6). Leah also said that GSP lets the students accurately and quickly construct figures, manipulate colors, measure angles, and calculate the sum of angles. Therefore, Leah thought that GSP is an appropriate tool to learn the exterior angle theorem.

Ⅴ. Discussion and Implication

After analyzing all participants' cases of teaching demonstrations using GSP, I found there were different types of teaching styles and materials. In this section, I classify participants' teaching demonstrations and describe the similarities and differences between them.

First, I classified participants' teaching demonstrations with GSP into two different types based on how they prepared their teaching materials. Most of the participants – Mia, John, and Leah – designed their own lesson using GSP by themselves. John and Leah prepared pre-constructed GSP files for their teaching, planned how to teach the exterior angle theorem and constructed their teaching materials. Although Mia did not prepare pre-constructed GSP files, she constructed her own teaching materials during the interview rather than searching for materials on the Internet. Only Emily used materials she gathered from the Internet: she searched for teaching materials and found a worksheet with GSP activities about the exterior angle theorem. Emily followed most of the steps on the worksheet, but also modified the worksheet based on her students' ability levels.

Second, there were two different types of lesson introduction. In order to teach the exterior angle theorem, Mia and Leah directly addressed the exterior angle theorem for a triangle or rectangle. Although they did not provide a completed theorem, they started with the core idea of the exterior angle theorem: the sum of the exterior angles of a specific polygon is 360 degrees. On the other hand, John and Emily did not provide any information about the theorem at first; rather, they would have their students explore and measure the size of the angles and find the pattern among them.

Third, based on whether to consider proof of the exterior angle theorem, I divided participants' teaching demonstrations into two types. During the teaching, Mia and Leah tried to have students come up with the proof of the exterior angle theorem, and could also provide proofs with or without the interviewer's prompt. However, John and Emily did not consider or prepare the proof during their teaching. In addition, because of their lack of mathematical knowledge, they could not provide correct proofs even though the interviewer tried to help them come up with the proofs.

Lastly, there was a difference in how they used GSP in their teaching. John, Emily, and Leah were aware of the advantages of using GSP in mathematics teaching and were willing to use it to teach the exterior theorem. They utilized the diverse features of GSP and fully used the dragging feature to aid students' exploration and discovery. On the other hand, Mia preferred to use pen and paper rather than GSP in her teaching, and had relatively insufficient knowledge of GSP. Because Mia valued that technology enables students to work quickly, she usually used GSP to measure angles and calculate the sum of them and rarely used the dragging feature.

John and Emily demonstrated very similar teaching styles. In their teaching demonstrations, both emphasized students' exploration and discovery of patterns and the theorem rather than focusing on the proof of the theorem. Both, however, lacked pedagogical content knowledge, meaning they could not lead the students to the core ideas and could not provide appropriate questions in order to have the students find the theorem themselves. Eventually, they directly asked the students to measure the sum of the exterior angles. John and Emily also lacked knowledge about how to prove the theorem. Mia and Leah, however, demonstrated quite different teaching styles, despite having similar introductions in their teaching and considering the proof of the theorem as the important part of the lesson. Mia also lacked pedagogical content knowledge and had a traditional view of teaching and learning. Thus, she focused more on conveying the proof of the exterior theorem rather than providing the students with opportunities to explore many examples. In addition, Mia said she would provide her method about how to find the proof and ask direct questions or explain procedures when the students encountered any difficulty. Leah, like Mia, said she would give the students concrete directions regarding what to do next. However, the difference between Leah and Mia is that Leah would provide the students with opportunities to explore many different polygons themselves, and encourage the students to create and examine their own claims. Meanwhile, Leah would try to facilitate the students' understanding of the theorem and its proof through organized task questions rather than by simply giving the students the answer or showing the processes of proofs.

In this study, I examined and analyzed four preservice secondary mathematics teachers' teaching demonstrations. The findings suggest the aspects that mathematics teacher educators (MTEs) should consider in aiming to improve preservice mathematics teachers' effective mathematics teaching with technology. According to the results of this study, preservice secondary mathematics teachers' pedagogical content knowledge is important to implement effective mathematics teaching with technology. Although preservice teachers had knowledge of how to use technology, they could not provide appropriate tasks or questions to facilitate students' mathematical learning if they lacked pedagogical content knowledge. For example, John and Emily's teaching demonstrations were based on students' exploration, but they could not connect that exploration with their conceptual understanding of the mathematical concept. In addition, Mia tended to teach using the traditional or teacher-centered approaches that focus on the delivery of mathematical knowledge. Mia did not consider students' exploration and their own mathematical learning, meaning her lesson was traditional and little different from the teaching without GSP. Therefore, in order to improve preservice teachers' mathematics teaching with technology, MTEs should focus on developing preservice teachers' pedagogical content knowledge, even though it does not include technology-related knowledge. According to 이광호 (2007), professional development for teaching with technology should be designed in the manner that teachers are challenged in their pedagogical content knowledge. In addition, the approach to mathematics teaching with technology should be developed based on solid mathematical learning theories (김동중, 2010).

As I only collected data from one teaching demonstration by preservice teachers, the results of this research are limited in fully examining and understanding preservice teachers' teaching. Thus, future studies in diverse mathematical areas and with a greater number of teaching demonstrations are needed.

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예비 고등 수학교사들의 Geometer’s Sketchpad 를 이용한 수업 시연에 관한 다중 사례 연구

김소민[2)]

초록

이 연구는 예비 고등 수학 교사들이 수학 개념을 테크놀로지를 이용하여 어떻게 가르치는지에 관한 다중 사례 연구이다. 예비 교사들은 퍼포먼스 인터뷰에서 Geometer's Sketchpad (GSP) 를 사용해 특정 수학 개념을 어떻게 가르칠 것 인지를 실제 수업처럼 시연하였다. 이 연구의 결과에 따르면, 예비 교사들은 다양한 종류의 학습지도안을 디자인 했으며, 그들의 수업 시연에서 각기 다른 교수학적 그리고 공학적 기술을 보여주었다. 이 연구의 결과는 예비 교사들의 교수학적 내용 지식이 테크놀로지 통합 수학 수업의 중요한 요인임을 제안한다. 그러므로 수학 교사 교육자들은 예비 교사들이 테크놀로지를 이용해 효과적으로 수학을 가르칠 수 있도록 하기 위해, 그들이 강력한 교수학적 내용 지식을 얻을 수 있도록 도와주어야 한다.

주요 용어: 예비 고등 수학 교사, 기하학, 수업 실연, 동적 기하학 환경

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