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Original Article

Added Mass Estimation of Square Sections Coupled with a Liquid Using Finite Element Method

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ABSTRACT

Natural frequencies of immersed square sections decrease due to a contribution of added mass to the movement of square sections. In this study, natural frequencies of square sections are obtained as a function of gap size between the square section and a rigid square wall using the finite element method. Additionally, they are used to extract the added mass effect on translational and rotation motions. Published information and studies on the translational and torsional vibration of square beams are also examined for practical use. D coupling of a square section is also investigated for multiple square sections. The suggested added mass estimation can be applicable to the spent fuel storage design of a pressurized light water modulated nuclear power plant.

1. Introduction

It is generally known that the natural frequency of a structure in contact with, or immersed in, a liquid decreases significantly compared with that in a vacuum or in the air. This problem is referred to as a fluid–structure interaction problem. Many investigators have obtained some approximate solutions to this problem, which have been used to predict the changes in the natural frequencies of a structure in a liquid [1]. The dynamic interaction between an elastic structure and a liquid has been the subject of intensive investigations in recent years. However, analytical solution procedures are available only for simple problems. Therefore, numerical approaches that can be formulated in the time or frequency domain have to be employed. Since variational principles are employed to derive numerical solutions, many researchers have attempted to derive variational principles for different classes of fluid–structure interaction problems, which are stimulated using new technical applications and by the availability of powerful numerical tools based on the finite element and boundary element methods.

Typical examples of fluid–structure interaction problems are fuel assemblies in a nuclear reactor. When fuel assemblies are removed from the core, they must be transported to pools filled with water because they are still highly radioactive and continue to generate heat for a long time. They are stored on

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racks in the pool and submerged in the circulating water for cooling and protection against irradiation. As the amount of spent fuel assemblies increases, the spent fuel assemblies can be placed in metal containers the walls of which contain neutron-absorbing materials, in order to increase the storage density. High-density spent fuel storage racks in the pool should be designed to withstand dynamic motion without any loss of structural integrity during earthquakes. At the same time, they should provide continuous cooling by natural convection, and spent fuel assemblies should be protected against criticality. Prior to the seismic analysis of the fuel assemblies submerged in the pool, dynamic characteristics of the fuel assemblies must be taken into account [2]. Therefore, it is very important to identify the hydrodynamic coupling between the fuel assemblies and rack structures. In particular, the liquid gap between the spent fuel assemblies, or that between the fuel assemblies and the rack significantly affects the dynamic characteristics of the liquid-coupled system.

Similarly, fuel assemblies in the reactor core are also submerged in the circulating primary coolant during normal plant operation. The square sectional fuel assemblies are coupled with the coolant. The arrangement of the fuel assemblies in the core of the commercial power plant is basically composed of square sections with liquid gaps. This produces an increase in the added mass and diversity in coupled mode shapes.

This study will provide a hydroelastic vibration analysis of single or multiple square beams contained in a rigid square container with respect to translational or torsional motions, to identify the hydrodynamic effect. Natural frequencies of the square sections are obtained as a function of the gap size between the square section and the rigid square wall, using a commercial finite element analysis code. Simultaneously, the added masses on translational and torsional motions are extracted. The published information and studies on the translational and torsional vibration of the square beams are also re-examined for practical use. Dynamic coupling of the square sections owing to the presence of the liquid is also investigated for multiple square sections.

2. Analytical solution for added mass coefficient

Assuming that an infinitely long rigid structure is supported by an elastic spring, it can be regarded as a square section in a two-dimensional (2D) domain. The assumption excludes the axial movement of the liquid, and only the lateral motion is taken into account. When the length of the square sections is considered as a 3D problem, an equal length of the liquid must also be considered to obtain the added mass of the liquid. The added mass of the liquid may be overestimated when the square sections are short or when the axial mode number is larger.

The lateral motion w of a square section will be written as follows:

$$m\frac{d^2w}{dt^2} + kw = 0 \tag{1}$$

where m is the mass per unit length of the structure and k is the spring constant. The natural frequency in a vacuum, ω_a, is simply given as follows:

$$\omega_a = 2\pi f_a = \sqrt{\frac{k}{m}} \tag{2}$$

A liquid in contact with the structure cannot move according to an arbitrarily assigned law of liquid velocity. For the motion to be possible, it is evidently necessary that the continuity equation be satisfied. In particular, possible irrotational motions of a fluid are subject to a condition in which the velocity potential Φ shall satisfy the Laplace equation:

$$\nabla^2\Phi = 0 \tag{3}$$

When the structure is submerged in an infinite liquid, the corresponding equation of motion will become as follows, owing to the added mass of the liquid:

$$(m + m_f)\frac{d^2w}{dt^2} + kw = 0, \tag{4}$$

where m_f is an added mass of the liquid [3–6]. In addition, the natural frequency of the structure oscillating in the liquid is expressed as follows:

$$\omega = 2\pi f = \sqrt{\frac{k}{m + m_f}} \tag{5}$$

Comparing Eq. (2) with Eq. (5), we can obtain the normalized natural frequency as follows:

$$\frac{\omega}{\omega_a} = \frac{f}{f_a} = \sqrt{\frac{m}{m + m_f}} = \sqrt{\frac{1}{1 + (m_f/m)}}. \tag{6}$$

As the term m_f/m in Eq. (6) is greater than zero, the normalized nondimensional frequency, ω/ω_a, is always less than unity. That is, the liquid tends to reduce the natural frequency. Based on this simple example, it is obvious that the dynamic response characteristics of the system depend on the added mass coefficient for a single rigid structure in an ideal liquid. As long as the added mass coefficient is known, the dynamic response including the natural frequency in a liquid can easily be calculated.

Even though some papers on the added mass of circular sectional structures were published by the theoretical method, researches on the added mass of square sectional structures are rare. Generally, liquid corners in contact with the square sections provide mathematical singular points in the analytical formulation. Therefore, the numerical method based on the variational principle may be applicable to the problem. However, it would be a new work on this problem to provide some practical information that can be used in the structural design of nuclear engineering.

3. Added mass estimation

3.1. *Translational motion*

A square section surrounded by a liquid is supported using a spring, as shown in Fig. 1. The liquid is bounded by a larger rigid square container, which is concentrically arranged with

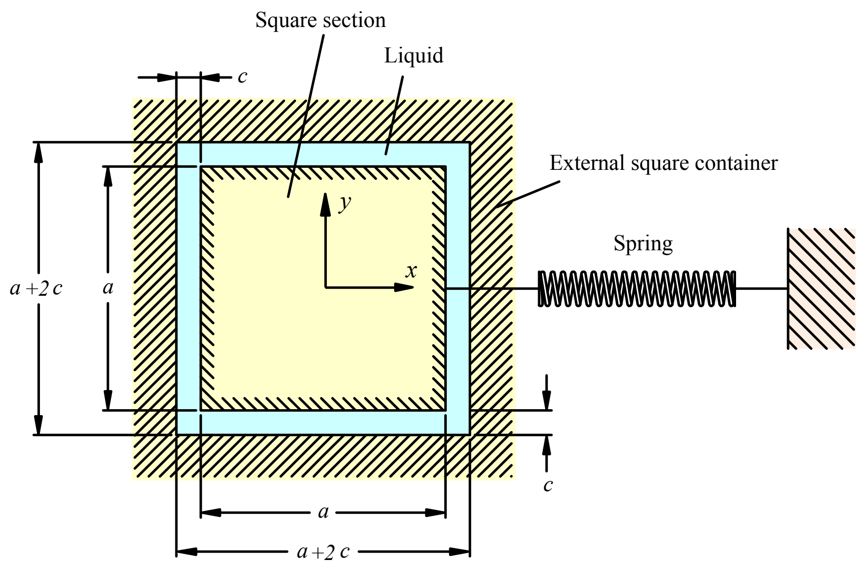

Fig. 1 — A square section model surrounded with a bounded liquid.

the square section. The side length of the square section is a, and the liquid gap between the inner section and the outer square container is c. The square section with a mass m is laterally supported by a spring with a spring constant k. The vibratory motion of the liquid-coupled system is significantly affected by the liquid gap. The liquid velocity potential of the square annular section should satisfy the Laplace equation with the corresponding liquid boundary conditions. However, it is very difficult to find an explicit solution that satisfies the Laplace equation and the liquid boundary conditions, owing to the presence of corners. Therefore, the finite element analyses, using the commercial computer code ANSYS (release 15; ANSYS, Inc., Canonsburg, PA, USA), were carried out for the square sections. To extract the added mass coefficient using ANSYS (ANSYS, Inc.), a 2D finite element model was constructed, as shown in Fig. 2.

A finite element model was constructed to extract natural frequencies of the liquid-coupled system (Table 1). The square section was meshed with an equal size of a 2D solid structure element (PLANE182) that can be used as either a plane element (plane stress, plane strain, or generalized plane strain) or an axisymmetric element. The element is defined by four nodes, with each node having two degrees of freedom: translations in the nodal x and y directions. Basically, axisymmetric elements can be applicable to cylindrical structures with an axis. A fixed circumferential mode number should be given in the axisymmetric finite element model for the finite element analysis. Therefore, modeling with axisymmetric elements

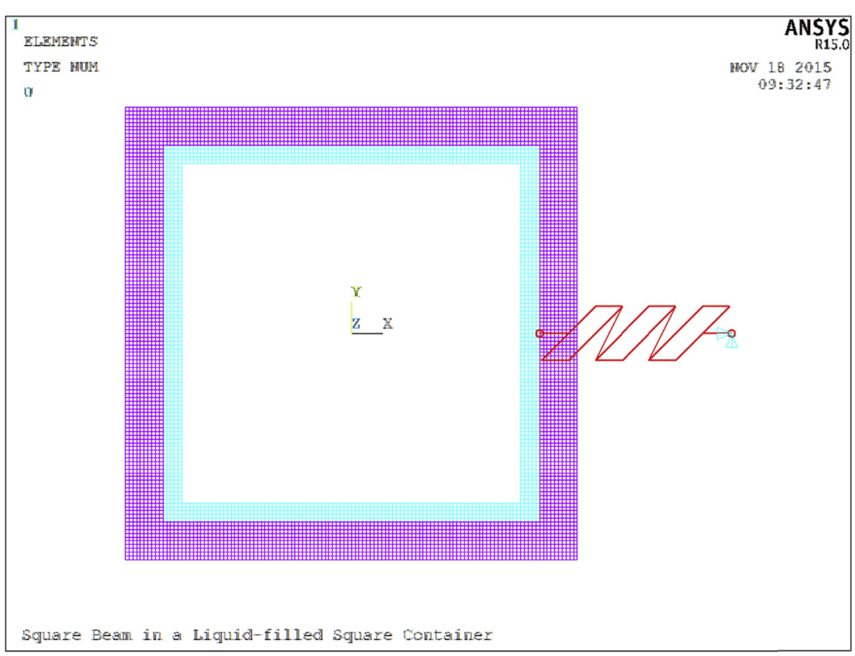

Fig. 2 — Two-dimensional finite element analysis model of a square section surrounded by a bounded liquid.

Table 1 – Physical and geometric input data for the two-dimensional finite element analysis model.

Material	Dimension or property	Value
Square section	Width of square section (mm)	100
	Material density (kg/m^2)	2,700
	Poisson's ratio	0.3
	Thickness (mm)	5.0
	Modulus of elasticity (GPa)	300.0
Liquid	Density (kg/m^2)	1,000
	Gap size (mm)	1–500
Spring	Spring constant (N/m)	5.3×10^5

may be inappropriate, and plane elements are used in this problem. The liquid region was modeled with an equal size of a 2D fluid element (FLUID29), which is used for modeling the fluid medium and the interface in fluid–structure interaction problems. Typical applications of the fluid element include sound wave propagation and submerged structure dynamics. The governing equation for acoustics, namely, the 2D wave equation, has been discretized, taking into account the coupling of the acoustic pressure and structural motion at the interface. The fluid element has four corner nodes with three degrees of freedom per node: translations in the nodal x and y directions, and pressure. The translations, however, are applicable only at nodes that are on the interface. The liquid region in the gap between the square section and the outer rigid container was divided into a number of identical liquid elements, and the square section was also meshed with a plane element, as shown in Fig. 2. A spring laterally attached to the square section was constructed using a spring–damper element (COMBIN14), which has a longitudinal or torsional capability in 1D, 2D, or 3D applications. The longitudinal spring–damper option is a uniaxial tension–compression element with up to three degrees of freedom at each node: translations in the nodal x, y, and z directions. The spring–damper element has no mass. The nodes of the liquid elements at $x = \pm(a/2 + c)$ and $y = \pm(a/2 + c)$ were constrained in the normal direction. By contrast, liquid movement along the container walls is not restricted to the tangential direction. Additionally, nodes of the liquid elements at $x = \pm a/2$ and $y = \pm a/2$ were constrained in the normal direction. The liquid element nodes adjacent to the outer surface of the wetted square section coincided with those of the inner square section. The element size of the model was 1.0 mm. A bulk modulus of 2.2 GPa was used to take into account the compressibility of water, which is equivalent to the speed of sound in water (1,483 m/s). The liquid gap in the model ranged from 1 mm to 500 mm. The natural frequencies obtained in the finite element analysis are listed in Table 2 as a function of the gap ratio $\eta = c/a$. Shell modes of the square section were not considered.

The natural frequency of the square section submerged in water, f_w, can be obtained using the added mass term, as written in Eq. (7):

$$f_w = \frac{1}{2\pi}\sqrt{\frac{k}{\rho A_e + C_m \rho_o a^2}} \tag{7}$$

where A_e is the cross-sectional area of the effective square section, ρ is the square section density, k is the spring constant,

Table 2 – Natural frequencies of a square section for various gap ratios.

Width of square section, c (mm)	Gap ratio, η (= c/a)	Natural frequency (Hz), f_w
Dry condition	–	51.123
1.0	0.01	4.457
2.0	0.02	6.250
3.0	0.03	7.593
4.0	0.04	8.699
6.0	0.06	10.489
8.0	0.08	11.928
10.0	0.10	13.137
15.0	0.15	15.512
20.0	0.20	17.295
30.0	0.30	19.829
50.0	0.50	22.772
80.0	0.80	24.920
120.0	1.20	26.225
200.0	2.00	27.220
500.0	5.00	27.903

and C_m is the added mass coefficient. The dimensionless added mass coefficient is the added mass divided by the displaced liquid mass. It is necessary to introduce a hydrodynamic coefficient in the fluid–structure interaction, because the added mass depends on the liquid density and geometry of a liquid-coupled system. Therefore, the added mass coefficient, as a normalized index, indicates a relative hydrodynamic effect on the structure in contact with a liquid. The cross-sectional area of the effective square section will be given as follows:

$$A_e = A_o - (a - 2h)^2 = a^2 - (a - 2h)^2 = 4(a - h)h \tag{8}$$

where h is the thickness of the square section and $A_o = a^2$. The added mass coefficient will be expressed as Eq. (9), as a function of the frequency ratio:

$$C_m = \frac{4(a - h)h}{a^2}\left(\frac{\rho}{\rho_o}\right)\left[(f/f_w)^2 - 1\right] \tag{9}$$

The added mass coefficients for the square and circular sections with a liquid gap is estimated based on Table 2, as shown in Fig. 3, as a function of gap ratio (η), where the Fritz equation for the added mass coefficient is included for the circular section [7]. Fritz [7] proposed a theory on the free vibration of two cylinders coupled with a gap of an ideal liquid. He suggested that the added mass coefficient for a circular section with an annular gap liquid is given as $C_{mc} = (R_o^2 + R_i^2)/(R_o^2 - R_i^2)$, where the radii of the moving inner and stationary outer cylinders are denoted by R_i and R_o, respectively. This is extremely large for a narrow gap, regardless of the configuration of the section. As the liquid gap increases, the added mass coefficients are drastically deceased, as illustrated in Fig. 3. In a region with a gap ratio of less than approximately 2.0, the added mass coefficient of a circular section is greater than that of a square section. In the region with a gap ratio of approximately 2.0, however, the situation is the reverse. The added mass coefficient of the square section is curve fitted as an exponential function. The equation obtained by exponential curve fitting can effectively be used for an estimation of the added mass with respect to an arbitrary square section with a narrow liquid gap:

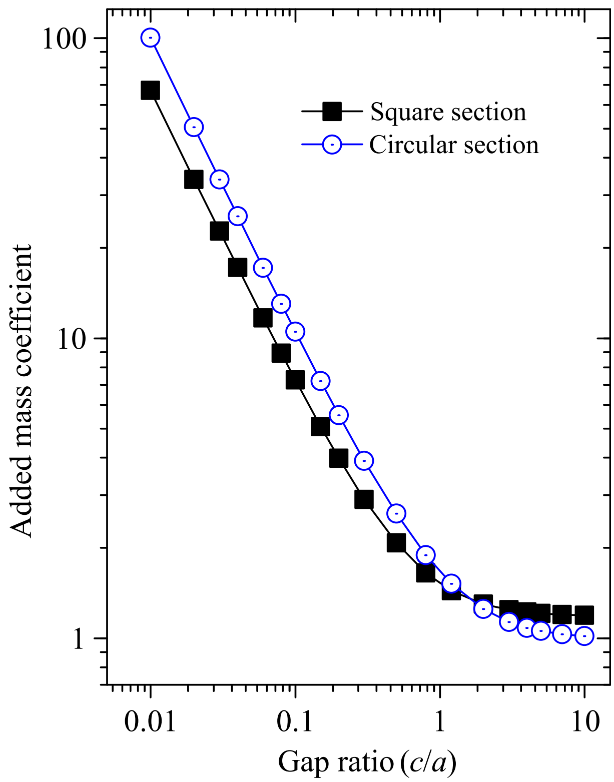

Fig. 3 – Translational added mass coefficients of a square and a circular section concentrically submerged in a square and a circular container.

$$C_m = 1.27483 + 50.45354 \exp\left(\frac{-0.01266 - \eta}{0.03266}\right) + 918.66757 \exp\left(\frac{-0.01266 - \eta}{0.00689}\right) + 7.00018 \exp\left(\frac{-0.01266 - \eta}{0.2255}\right) \quad (10)$$

The added mass of a square section in an infinite liquid, when the square section in the infinite liquid medium moves transversely, is listed in Table 14.1 of Blevins' paper [5] and Table 1 of Chung's paper [8].

$$M_{ma} = \frac{1.513\,\pi\,\rho_o}{4} a^2 = 1.188\,\rho_o\,a^2 \quad (11)$$

The added mass of a square section concentrically placed in a square liquid-filled rigid tube will converge to this value when the liquid gap increases infinitely. Fig. 3 shows that the added mass coefficient of a square section converges to an asymptotic value, 1.188 of Eq. (11).

3.2. *Torsional motion*

The torsional natural frequencies of a single circular cylinder submerged in a liquid are not affected by the presence of the liquid as long as it is inviscid. However, the presence of a liquid surrounding the square section affects the torsional motion of a single square section because the four corners of the square section produce an additional liquid flow during a motion. A schematic configuration of a square section concentrically submerged in a liquid-filled external square container is illustrated in Fig. 4. The section can rotate on an axis perpendicular to the section, and a torsional spring is attached to the pivot.

When the added mass inertia term owing to the ideal liquid is considered, the equation of motion for a solid square section submerged in an ideal liquid can be given as follows:

$$(J_c + J_f)\frac{d^2\theta}{dt^2} + k_s\theta = 0 \quad (12)$$

where J_c is the mass moment of inertia of the square section, J_f is the mass moment of inertia owing to the presence of the liquid, and k_s is the torsional spring constant. Therefore, the natural frequency of the square section for the torsional oscillation in a liquid is given as follows:

$$\omega = 2\,\pi\,f = \sqrt{\frac{k_c}{J_c + J_f}} \quad (13)$$

The torsional natural frequency of the square section under dry conditions is the following:

$$\omega_c = 2\,\pi\,f_c = \sqrt{k_c/J_c} \quad (14)$$

We can obtain the normalized natural frequency from Eqs. (13) and (14):

$$\frac{\omega}{\omega_c} = \frac{f}{f_c} = \sqrt{\frac{J_c}{J_c + J_f}} = \sqrt{\frac{1}{1 + (J_f/J_c)}} \quad (15)$$

As the term J_f/J_c in Eq. (15) is greater than zero, the normalized nondimensional frequency, $\omega/\omega_c = f/f_c$, is always less than unity. That is, the liquid also tends to reduce the torsional natural frequency. Based on this simple example, it is obvious that the dynamic response characteristics of the system depend on the added mass coefficient for a single rigid structure in an ideal liquid. As long as the added mass coefficient is known, the dynamic response including the torsional

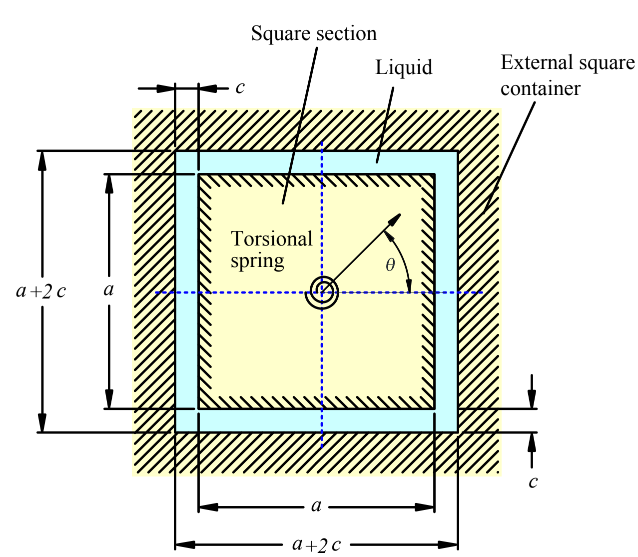

Fig. 4 – Torsional vibration model of a square section concentrically submerged in a square container.

natural frequency in a liquid can easily be calculated. The area moment of inertia of a hollow square section with thickness h is written as follows:

$$J_c = \frac{\rho}{6}\left[a^4 - (a-2h)^4\right] \quad (16)$$

The area moment of inertia displaced with a square section will be expressed as follows:

$$J_f = C_{ms}\,\rho_o\,a^4/6 \quad (17)$$

The torsional added inertia of moment coefficient for the square section can be extracted from the natural frequency ratio using Eqs. (15–17):

$$C_{ms} = \frac{\rho\left[a^4 - (a-2h)^4\right]}{\rho_o\,a^4}\left[\left(\frac{f_c}{f}\right)^2 - 1\right]. \quad (18)$$

From the calculation of torsional natural frequencies using the finite element analysis code ANSYS (ANSYS, Inc.), the rotational added mass coefficient can be obtained. The geometry and material properties are the same as those given in Table 1, except for the spring constant $k_c = 1{,}528.0\ \mathrm{kg\ m^2/s^2}$. Torsional natural frequencies and the added inertia of moment are estimated for various liquid gaps, and the added inertia of moment coefficient of the liquid for the torsional mode of the square section is plotted in Fig. 5 as a function of the liquid gap ratio. The torsional added inertia of moment coefficient also decreases with the liquid gap and finally converges at a specific value for an infinite liquid. The torsional added inertia of moment coefficient can be approximated using an exponential curve fitting.

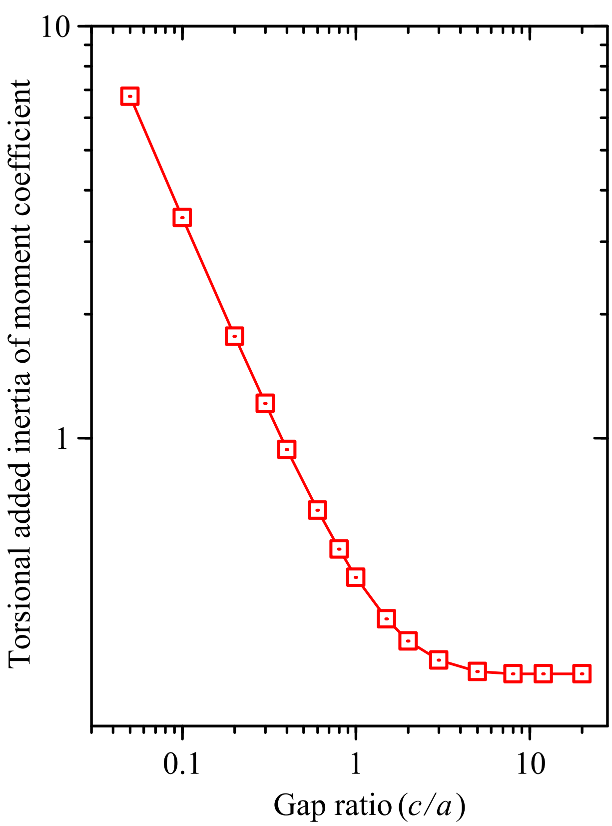

Fig. 5 – Torsional added inertia of moment coefficient of a square section with a liquid gap.

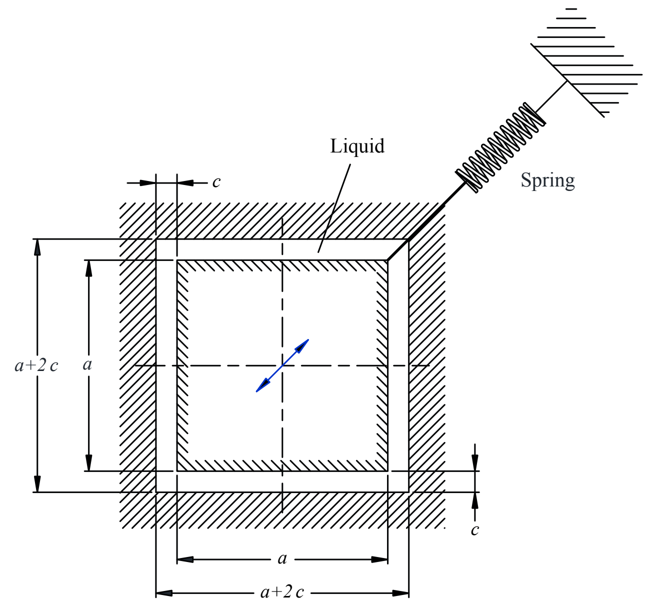

Fig. 6 – A square section coupled with a liquid gap for diagonal movement.

$$C_{ms} = 0.27049 + 3.39038\,\mathrm{Exp}\left(\frac{-\eta}{0.1456}\right) + 14.65087\,\mathrm{Exp}\left(\frac{-\eta}{0.03387}\right) + 0.78929\,\mathrm{Exp}\left(\frac{-\eta}{0.70707}\right) \quad (19)$$

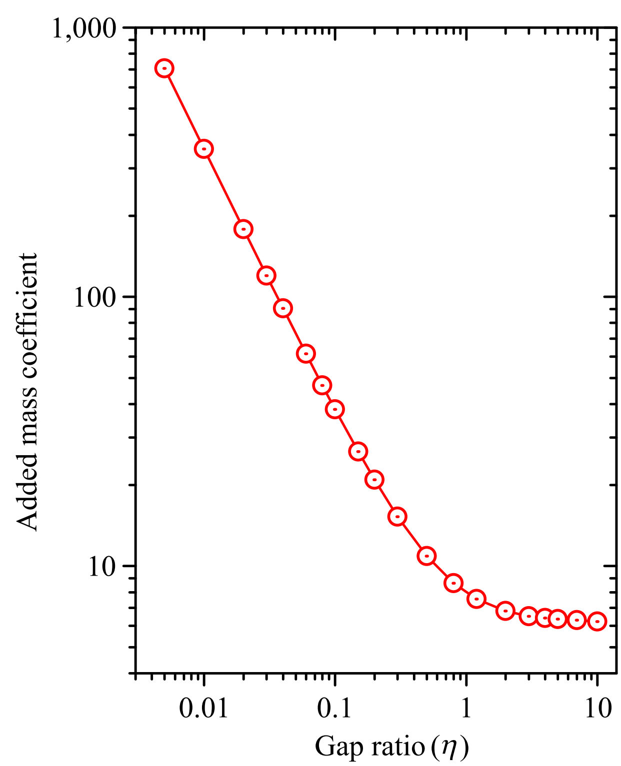

Fig. 7 – Diagonal added mass coefficient of a square section with a liquid gap.

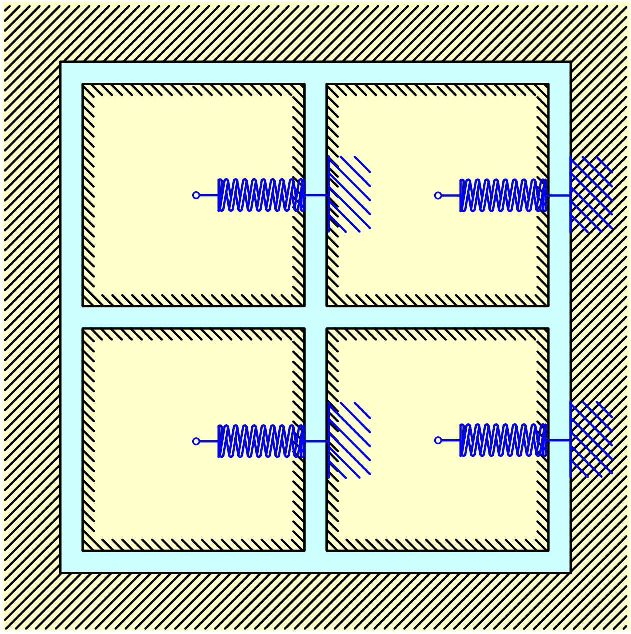

Fig. 8 – Four-square-section model coupled with a liquid gap.

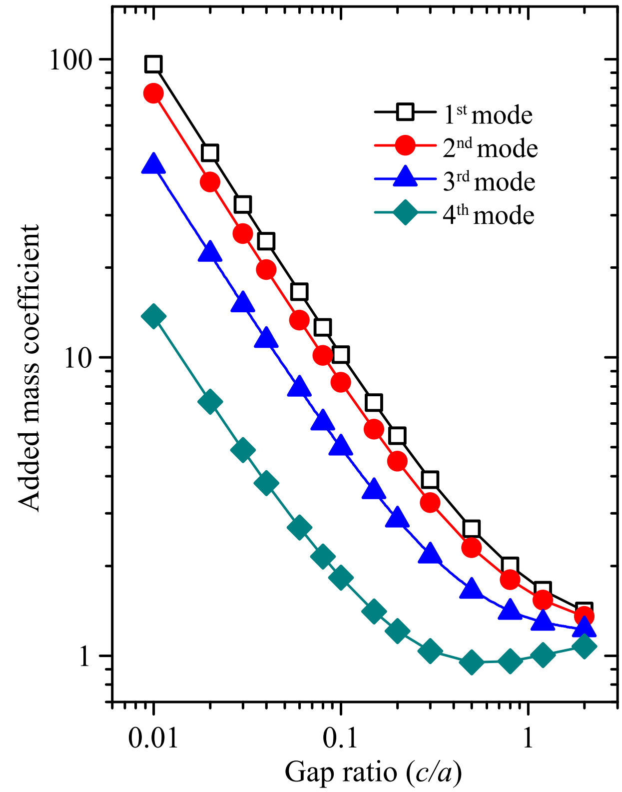

Fig. 10 – Added mass coefficients of four square sections submerged in a square container.

The torsional added inertia of moment coefficient per unit length of a square cross section in an infinite liquid, as the square section in the infinite liquid medium rotates along its center, is listed in Table 1 of Blevins' paper [5]:

$$C_{mr} = 0.2757. \quad (20)$$

The torsional added inertia of moment of a square section concentrically placed in a square liquid-filled rigid tube will be converged to this value when the liquid gap increases infinitely. Fig. 5 shows that the torsional added inertia of moment coefficient of the square section converges to an asymptotic value, 0.2757 of Eq. (20). The torsional added inertia of moment of a square section concentrically placed in a square liquid-filled rigid tube will infinitely be an asymptotic value according to an increase of the liquid gap.

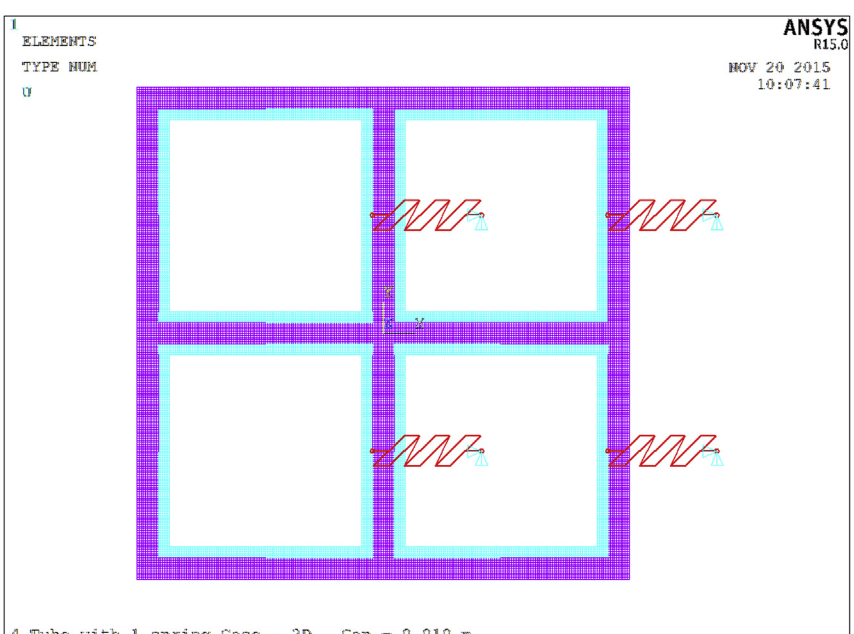

Fig. 9 – Finite element analysis model of four square sections coupled with a liquid gap.

3.3. Diagonal motion

Although the added mass of a circular section does not depend on the moving direction, the added mass of a square section depends on the oscillating direction. A schematic analysis model of a square section in a liquid with a diagonal movement is demonstrated in Fig. 6. The same process as described in Section 3.1 was carried out to estimate the added mass coefficient of a square section in a liquid with respect to the diagonal movement. Natural frequencies and added mass coefficients are estimated for various liquid gaps, and the added mass coefficient of the liquid for the diagonal movement of the square section is plotted in Fig. 7 as a function of the liquid gap ratio. The diagonal added mass coefficient also decreases with the liquid gap and finally converges at a specific value for an infinite liquid. The added mass coefficient can be approximated with an exponential curve fitting:

$$C_{md} = 7.08273 + 741.4387\ \mathrm{Exp}\left(\frac{0.01483-\eta}{0.01818}\right) + 87671.07157\ \mathrm{Exp}\left(\frac{0.01483-\eta}{0.00366}\right) + 70.86796\ \mathrm{Exp}\left(\frac{0.01483-\eta}{0.13542}\right) \quad (21)$$

4. Applications and discussion

4.1. Four square sections

A liquid-coupled system of four square sections in a rigid container is illustrated in Fig. 8. The square sections are identical in dimensions and material properties, and each section is connected with a separated identical lateral spring (spring constant k). The liquid gap between the sections and rigid walls has the same dimensions (c). The 2D square sections connected with springs can simulate the bending motion of square cylinders such as fuel assemblies. To investigate the coupling effect of the four square sections, finite element analyses were carried out for several liquid gaps using ANSYS (ANSYS, Inc.). Fig. 9 shows a 2D finite element model meshed with an equal sized 2D solid structure element (PLANE182) and 2D fluid element (FLUID29). The material properties of the system are identical to the case of a single square section.

Several modes were observed owing to the coupling effect of the liquid. Natural frequencies and mode shapes were extracted, and added mass coefficients were estimated using Eq. (9). They are also plotted as a function of the gap ratio, as illustrated in Fig. 10. Each added mass coefficient in a narrow liquid gap shows a large difference, whereas it converges

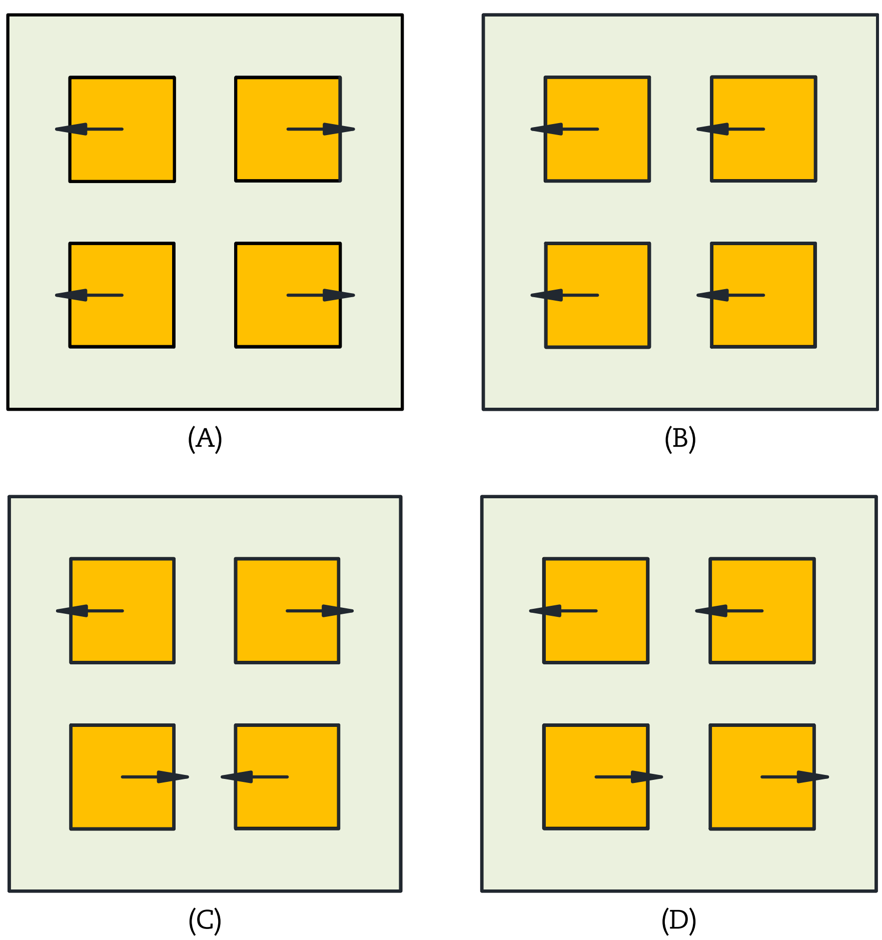

Fig. 11 – Coupled mode shapes of four square sections submerged in a liquid-filled square container. (A) First mode. (B) Second mode. (C) Third mode. (D) Fourth mode.

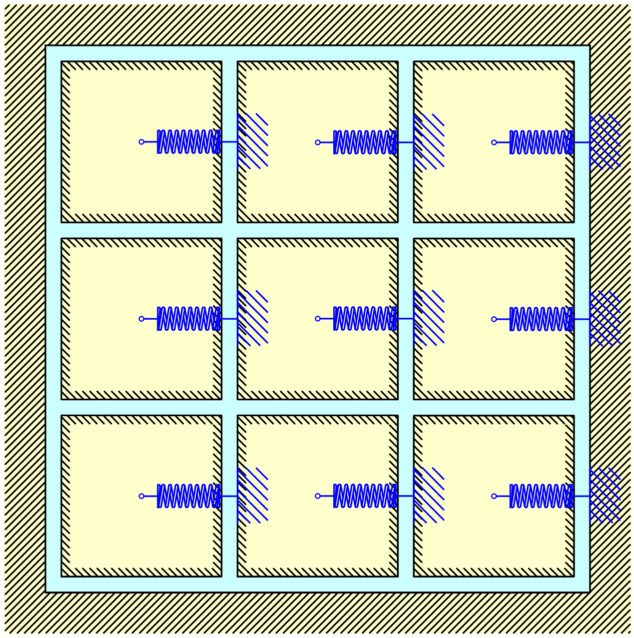

Fig. 12 – Nine-square-section model coupled with a liquid gap.

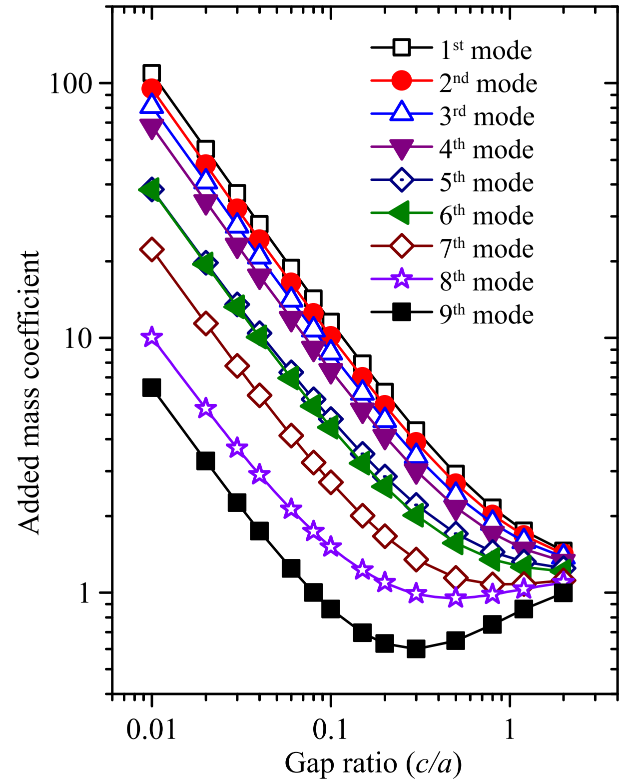

Fig. 14 – Added mass coefficients of nine square sections submerged in a liquid-filled square container.

to the added mass coefficient value of a single square section submerged in an infinite liquid as the liquid gap increases. It is generally well known that the added mass coefficient decreases for the liquid gap owing to a squeeze effect mitigation. It is remarkable that the added mass coefficient of the fourth mode decreases as the gap ratio increases, until a certain gap ratio, but gradually increases to the added mass coefficient value of a single square section submerged in an infinite liquid. Fig. 11 shows the coupled mode shapes of four square sections. Several combinations of the in-phase and out-of-phase modes are presented. The first one is an out-of-phase mode, in which both the upper and the lower square section oscillate such that they are moving in opposite directions. The second one is an in-phase mode, in which both the upper and the lower square section oscillate in the same direction. The

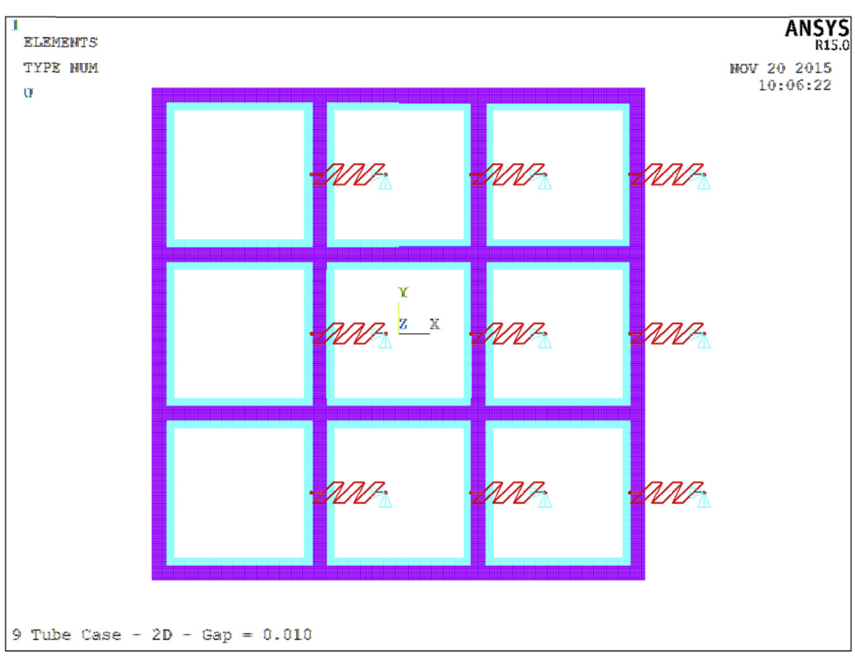

Fig. 13 – Finite element analysis model of nine square sections coupled with a liquid gap.

third and fourth ones show, respectively, the out-of-phase and in-phase modes laterally with the vertical out-of-phase oscillation. It is clear that the liquid-coupled system is prone to be excited with the in-phase mode (2nd mode) with respect to a lateral excitation, such as an earthquake. By contrast, the liquid-coupled system can be excited with any modes shown in Fig. 11, with respect to the propagated excitation such as a pump pulsation.

4.2. *Nine square sections*

A liquid-coupled system of nine square sections in a rigid container is illustrated in Fig. 12. The square sections are identical in dimensions and material properties, and each section is connected with a separated identical lateral spring (spring constant *k*). The liquid gap between the sections and rigid walls has the same dimensions (*c*). The 2D square sections connected with springs can simulate the bending motion of a bundle of square beams. To investigate the coupling effect of the nine square sections, finite element analyses were carried out for several liquid gaps using ANSYS (ANSYS, Inc.). Fig. 13 shows a 2D finite element model meshed with an equal size of a 2D solid structure element (PLANE182) and 2D fluid element (FLUID29). Material properties of the system are also identical to the case of a single square section.

Various coupled vibration modes were observed owing to the dynamic coupling effect of the liquid. The added mass coefficients of the coupled modes are also plotted as a function of the gap ratio, as illustrated in Fig. 14. Each added mass coefficient in a narrow liquid gap shows a large divergence, whereas it converges to the added mass coefficient value of a single square section submerged in an infinite liquid as the

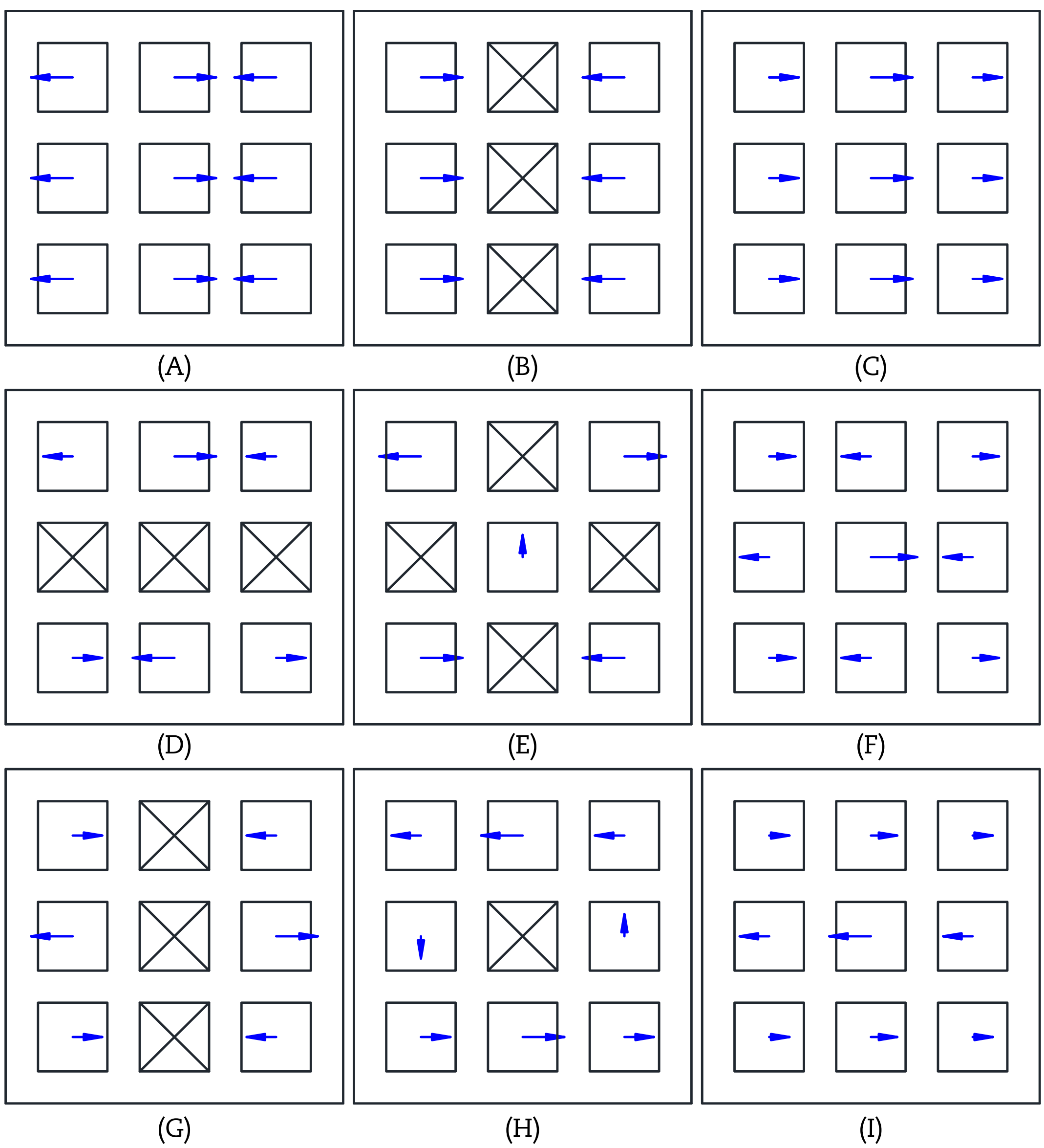

Fig. 15 – Coupled mode shapes of nine square sections submerged in a liquid-filled square container. (A) First mode. (B) Second mode. (C) Third mode. (D) Fourth mode. (E) Fifth mode. (F) Sixth mode. (G) Seventh mode. (H) Eighth mode. (I) Ninth mode.

liquid gap gets large enough. It is generally well known that the added mass coefficient decreases for a liquid gap owing to a squeeze effect. It is remarkable that the added mass coefficients of the seventh, eighth, and ninth modes decrease according to the increase in the gap ratio, until a certain gap ratio, but gradually increase up to the added mass coefficient value of a single square section submerged in an infinite liquid. Fig. 15 shows the coupled mode shapes of nine square sections. Several combinations of in-phase, out-of-phase, and rotational modes are presented. The mode shapes of the nine square sections are more complicated than those of four square sections. The first mode oscillates with all out-of-phase movement in the x direction, but it maintains all in-phase movement in the y direction. The second one shows an out-of-phase mode, in which the six peripheral sections oscillate with all out-of-phase movement in the x direction, but maintains all in-phase movement in the y direction, whereas the central three sections are all stationary. The third one is an in-phase mode, in which all square sections oscillate in the same direction. The other ones show a combination of the out-of-phase and in-phase modes. It is remarkable that the fifth and eighth modes show y-directional movement of specific sections in spite of the x-directional vibration system. Generally, the specific x-directional movement of adjacent sections can induce the x-directional movement of other sections. Exceptionally, the eighth one shows a rotational mode with respect to the center of the system in spite of a laterally connected spring–mass system. This is caused by the coupling effect phenomenon of the liquid. It is clear that the liquid-coupled system is prone to excitation with the in-phase mode (3^{rd} mode) with respect to a lateral excitation such as an earthquake. Conversely, the liquid-coupled system can be excited with any modes of Fig. 15 with respect to the propagated excitation such as a pump pulsation.

5. Conclusions

To estimate the added mass of a square beam, the free vibration of single or multiple square sections contained in a liquid-filled rigid square container, with respect to translational, diagonal, and rotation motions, was investigated. In this study, natural frequencies of square sections were obtained as a function of gap size between the square section and the rigid square wall using the commercial finite element analysis code. Natural frequencies of a single square section decrease owing to the hydrodynamic mass contribution to the movement of a square section regardless of the moving direction. It is evident that the contribution of the added mass to the section is the largest for a diagonal movement, whereas the added mass effect on the section is smallest for a rotational movement. Dynamic coupling of square sections was also investigated for multiple square sections. Multiple added mass coefficients were extracted based on the liquid coupling effect. The added mass coefficient for the in-phase mode might be useful in applying the seismic response among the various coefficients. The suggested added mass estimation can be applicable to the spent fuel storage of a pressurized light water modulated nuclear power plant.

The correlation between in-phase and out-of-phase modes and the coupling effect of 3D multiple structures coupled with a liquid will be studied in further research to show more realistic engineering applications.

Conflicts of interest

All authors declare no conflicts of interest.

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